

Chapter 2: Reminders on the fundamental laws of electricity

2. Sinusoidal regime

Today, all power networks operate with alternating currents and voltages of sinusoidal form. Sinusoidal quantities are special types of periodic quantities whose study is essential in electronics and electrical engineering [3].

2.1. Representation of Sinusoidal Quantities

The real representation of sinusoidal quantities (current and voltage) is given by:

$$u(t) = U_M \cdot \sin(\omega t + \varphi_u)$$

$$i(t) = I_M \cdot \sin(\omega t + \varphi_i)$$

U_m and I_m are the maximum (peak) values of $u(t)$ and $i(t)$ respectively.

$\omega t + \varphi$: Instantaneous phase.

ω : Angular frequency in radians per second (rad/s), with $\omega = 2\pi f = \frac{2\pi}{T}$

f : Frequency in hertz (Hz), and T : Period in seconds (s).

φ_u, φ_i : Initial phases of $u(t)$ and $i(t)$, respectively.

$\varphi = \varphi_u - \varphi_i$: Phase difference between $u(t)$ and $i(t)$,

2.2. Average and RMS Values

Average Value:

A periodic function $u(t)$ with a period T has an average value U_{avg} given by:

$$U_{avg} = \frac{1}{T} \int_0^T u(t) dt$$

The average value of a sinusoidal signal is zero.

RMS Value (effective value):

In general, for a periodic function $u(t)$ with period T , the root mean square (RMS) value is given by:

$$U_{rms} = \sqrt{\frac{1}{T} \int_0^T [u(t)]^2 dt}$$

2.3.Vector Representation of a Sinusoidal Quantity

A sinusoidal quantity (current or voltage) can also be represented by a **rotating vector** in the Oxy plane, rotating counterclockwise with an angular velocity ω . This rotating vector is called the **Fresnel vector** associated with the sinusoidal quantity.

To simplify the representation of Fresnel vectors, they are usually drawn at $t = 0$, which does not affect the final result.

The magnitude of the Fresnel vector U_{rms} of the quantity $u(t)$ is equal to its **RMS value**.

$$u(t) = \sqrt{2} U_{rms} \sin(\omega t + \varphi)$$

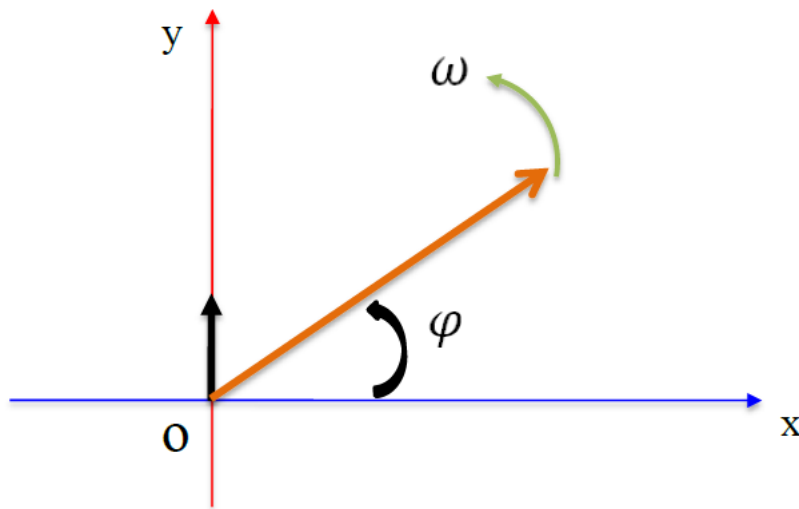


Figure 2.1: Fresnel vector.

Example

Let the two sinusoidal quantities be:

$$u(t) = 5\sqrt{2} \sin(\omega t)$$

$$i(t) = 2\sqrt{2} \sin\left(\omega t - \frac{\pi}{6}\right)$$

Give the vector representation of the two quantities $u(t)$ and $i(t)$?

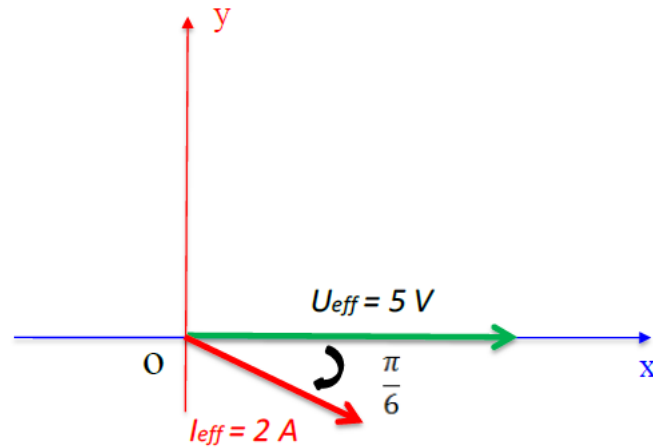


Figure 2.2: Fresnel representation.

2.4. Complex Impedance

Electrical impedance measures the opposition that an electrical circuit (or a component) presents to the flow of an alternating current.

Let a circuit be traversed by a sinusoidal alternating current of the form:

$$i(t) = I_{eff} \sqrt{2} \sin(\omega t + \varphi)$$

$$i(t) = I_{eff} \sqrt{2} \cdot e^{j\varphi}$$

By applying Euler's formula, it can also be written in complex form as:

$$i(t) = I_M \cdot \sin(\omega t) + j I_M \cdot \cos(\omega t)$$

where:

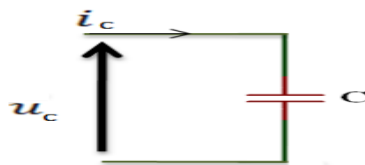
$$I_{eff} \sqrt{2} = I_M$$

2.5. Complex Impedance of Dipoles R, L, and C

2.5.1. Capacitor

Let a voltage be applied to the capacitor is $U_c = E e^{j\omega t}$, and we assume that the phase $\varphi = 0$.

The impedance of the capacitor is given by: $Z_c = \frac{U_c}{i_c}$



$$u_c = \frac{1}{C} \int i_c(t) \cdot dt \longrightarrow \frac{du_c}{dt} = \frac{1}{C} i_c(t)$$

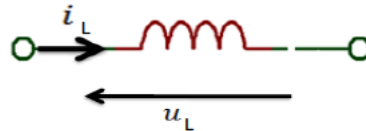
$$i_c(t) = C \cdot \frac{du_c}{dt} = C \frac{d(Ee^{j\omega t})}{dt} = C \cdot E \cdot j \cdot \omega \cdot e^{j\omega t}$$

$$\begin{cases} u_c = E e^{j\omega t} \\ i_c(t) = C \cdot E \cdot j \cdot \omega \cdot e^{j\omega t} \end{cases}$$

$$Z_c = \frac{u_c}{i_c} = \frac{E e^{j\omega t}}{C \cdot E \cdot j \cdot \omega \cdot e^{j\omega t}} \rightarrow Z_c = \frac{1}{j\omega C}$$

2.5.2. Inductor (coil)

Let an inductor be traversed by a current: $i_L(t) = I e^{j\omega t}$



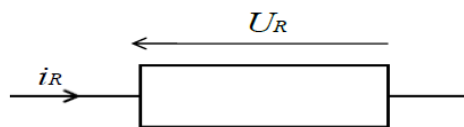
The elementary law gives:

$$u_L = L \frac{di}{dt} = L \frac{d(I e^{j\omega t})}{dt} = L \cdot I \cdot j \omega \cdot e^{j\omega t}$$

$$Z_L = \frac{u_L}{i_L} = \frac{L \cdot I \cdot j \omega \cdot e^{j\omega t}}{I e^{j\omega t}} = j\omega L \rightarrow Z_L = j\omega L$$

2.5.3. Resistance

By using Ohm's law:



$$Z_R = \frac{u_R}{i_R} = R \rightarrow Z_R = R$$

2.6. Powers in the Sinusoidal Regime (Instantaneous, Active, Reactive, Apparent)

Power in the alternating current (AC) regime is the power in an electrical circuit operating under a sinusoidal alternating regime.

In this section, we address the concepts of active power, apparent power, and reactive power.

2.6.1. Instantaneous power

In sinusoidal steady-state conditions, we have:

$$\begin{cases} u(t) = U_M \cdot \sin(\omega t + \varphi) = U_{eff} \sqrt{2} \cdot \sin(\omega t + \varphi) \\ i(t) = I_M \cdot \sin(\omega t) = I_{eff} \sqrt{2} \cdot \sin(\omega t) \end{cases}$$

The instantaneous power is:

$$P = u(t) * i(t) = U\sqrt{2} \cdot \sin(\omega t + \varphi) * I\sqrt{2} \cdot \sin(\omega t)$$

$$P = 2UI \sin(\omega t + \varphi) * \sin(\omega t)$$

Using trigonometric identities, the power is then equal to:

$$\sin a \cdot \sin b = \frac{1}{2} [\cos(a - b) - \cos(a + b)]$$

$$P = UI \cos \varphi - UI \cdot \cos(2\omega t + \varphi)$$

It is observed that the instantaneous power is the sum of a constant term $UI \cos \varphi$ and a periodically varying term $UI \cos(2\omega t + \varphi)$.

2.6.2. Active power

Represents the average power consumed by the dipole. It is expressed in watts (W).

$$P = UI \cdot \cos(\varphi), [Watts]$$

2.6.3. Reactive power

Is represented by the product $UI \sin \varphi$ and is denoted by the symbol Q .

$$Q = UI \cdot \sin(\varphi), [VAR]$$

Q is expressed in Volt-Amperes Reactive [V.A.R].

2.6.4. Apparent Power

The apparent power is the power supplied by the source. Mathematically, the product UI is expressed in Volt-Amperes (V.A). It is denoted by the symbol S .

$$S = UI, [VA]$$

2.7. Boucherot's Theorem

Boucherot's theorem states the conservation of active and reactive powers.
 In any electrical installation (comprising several receivers of different types), we have:

$$P_T = P_1 + P_2 + P_3 + \dots + P_n = \sum_{i=1}^n P_i$$

$$Q_T = Q_1 + Q_2 + Q_3 + \dots + Q_n = \sum_{i=1}^n Q_i$$

However, apparent powers are not conserved ($S \neq S_1 + S_2 + \dots + S_n$)

To apply Boucherot's method to a circuit or an installation, it is necessary to establish the balance of active and reactive powers. This balance can be presented in the form of a table.

DIPOLES	ACTIVE POWER (W)	REACTIVE POWER (VAR)
Receiver 1	P_1	$Q_1 = P_1 \tan \varphi_1$
Receiver 2	P_2	$Q_2 = P_2 \tan \varphi_2$
Receiver 3	P_3	$Q_3 = P_3 \tan \varphi_3$
INSTALLATION	$P = P_1 + P_2 + P_3$	$Q = Q_1 + Q_2 + Q_3$