Chapter 1: Generalities on Vibrations

1. Definition of a Periodic Motion

A motion is said to be **periodic** if it repeats itself identically at equal time intervals.

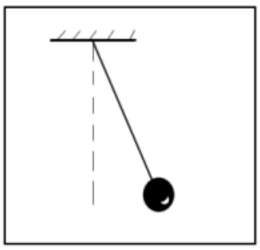
Exemples :

- **Heartbeat:** A heartbeat is a sequence of contractions and relaxations of the heart muscles, which operate valves and cause the circulation of blood throughout the body.
- **Moon's revolution:** The Moon completes a full cycle of revolution around the Earth in approximately 29 days.

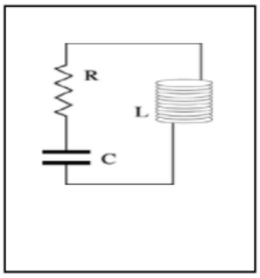
1. Definition of Vibrations

A motion that repeats over time is called a *vibratory motion* or an *oscillatory motion*.
Any mechanical system that includes a mass and a flexible element (such as a spring), or their equivalents in an electrical system (an inductor, a capacitor), can exhibit vibratory motion. Examples of such motion include the simple pendulum, the oscillating electrical circuit, and the mass-spring system.

- **Simple Pendulum:** Composed of a mass attached to a string, displaced from its equilibrium position and then released, it performs a back-and-forth motion that repeats over time.

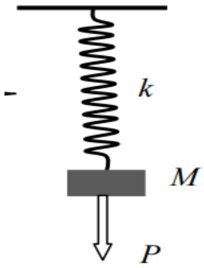


- **Oscillating Electrical Circuit:** A linear circuit containing an electrical resistance, a capacitor (capacitance), and a coil (inductance) that can generate electrical oscillations.



- **Mass-Spring System:**

  Composed of a mass attached to a spring, displaced from its equilibrium position and then released, it performs a motion that repeats over time. As soon as the body is moved away from the equilibrium position, a force appears to try to bring it back to equilibrium. This force is called the *restoring force*.



3. Definition of a Sinusoidal (Harmonic) Motion

A motion that repeats itself at equal time intervals is called a *periodic motion*.

The simplest periodic motion is *harmonic motion*, in which the amplitude remains constant (dissipative forces are negligible).

The motion of a point along a circular path with constant angular velocity, and the motion of a mass attached to a spring, are examples of harmonic motion.

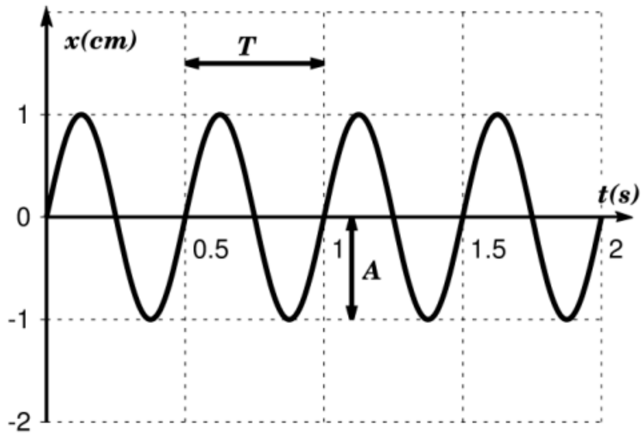
Harmonic motion can be mathematically represented using sine or cosine function

$$x(t)=A\sin(\omega t+\varphi) \text{ or } x(t)=A\cos(\omega t+\varphi).$$

A is called the amplitude: it is the maximum deviation of the vibrating system from its equilibrium position.

ω : (omega) is the angular frequency (or pulsation).

φ : (phi) is the initial phase (in radians).



Period (T) in seconds (s): It is the time interval of one complete cycle.

Frequency (f) in Hertz (Hz): The number of repetitions (cycles) per second.

$$f = \frac{1}{T}$$

where : T : is the period

Angular Frequency (ω) in radians per second (rad/s): $$\omega = \frac{2\pi}{T} = 2\pi f$$

4. Degrees of Freedom

The *degree of freedom (dof)* is defined by the following relation:

$$\mathbf{dof = N - R}$$

Note: The degree of freedom is the number of equations that need to be studied.

N : Number of independent or dependent generalized coordinates

R : Number of constraints (relations) between the coordinates

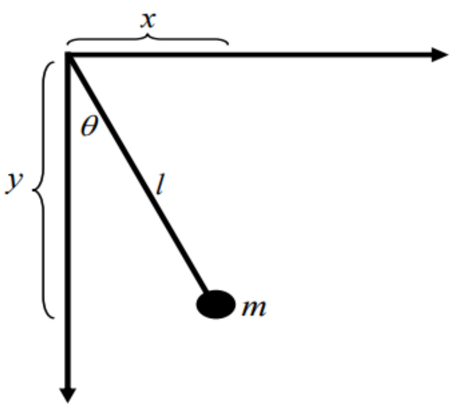
Exemples :

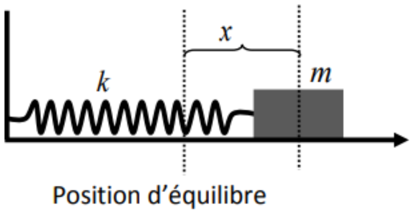
- **Simple Pendulum:**
  The system shown in the figure can be studied (i.e., the position of the mass *m* can be known at any moment) by knowing just one coordinate, *x* or *y*, because they are linked by the relation: $x^2 + y^2 = l^2$. Knowing one of the two coordinates means the other can be directly deduced from the above equation.

We say that. The two coordinates are not independent. The study of this system requires only one coordinate; therefore, the system has **one degree of freedom**.

- **Mass-Spring System:**

To know the position of the mass at any moment, it is sufficient to know the horizontal position x. Thus, the study of this system requires only one coordinate, so the system also has **one degree of freedom**.



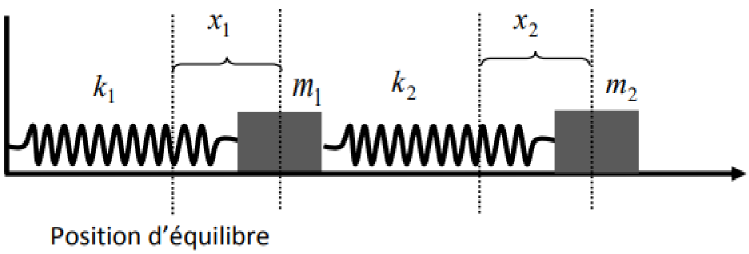


Pendule simple **Système masse-ressort**

- **Two Masses and Two Springs:**

To determine the positions of the masses m_1 and m_2 at any moment, it is necessary to know the coordinates x_1 and x_2. These two coordinates are independent — for example, x_1 can take values independently of x_2.

The study of this system therefore requires two coordinates, which means the system has **two degrees of freedom**.



Two Degrees of Freedom System – Two Masses and Two Springs

Exercise:

A vibratory motion is characterized by the following displacement:

$$x(t) = 4\cos(25t + \pi/2)$$

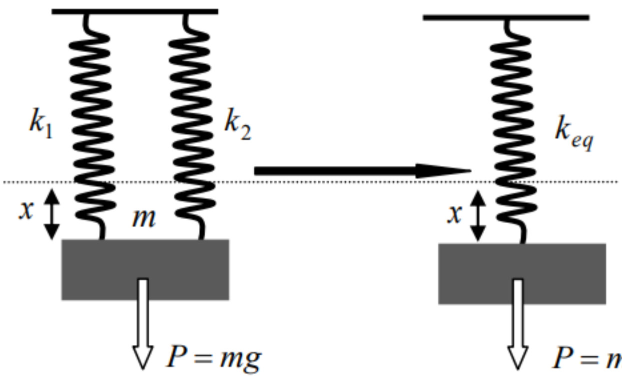
Where **x** is in centimeters, **t** in seconds, and the **phase** in radians.

1. Determine the maximum amplitude
2. Give the angular frequency (natural pulsation), the frequency, and the period of the motion
3. Express the initial phase (phase shift at origin)
4. Calculate the displacement, velocity, and acceleration at the moments t = 0 s and t = 0.5 s

5. Spring Connections

5.1. Springs in Parallel

Let **two springs**, k_1 and k_2, have the same natural (unstretched) length l_0 and undergo the same extension x. When a **mass m** is attached to the ends of both springs, the **equivalent spring** of stiffness k_{eq} has the same elongation. At equilibrium, we have:



$$mg = k1x + k2x$$

$$mg = keq\ x$$

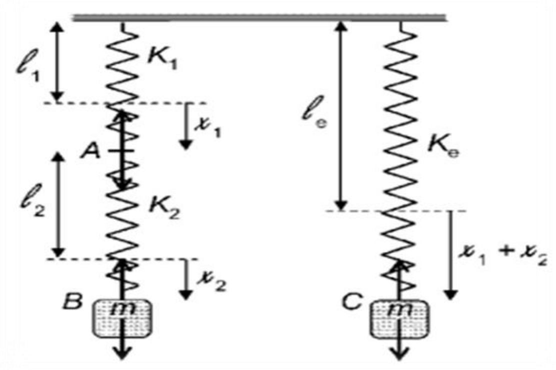
So $$keq = k1 + k2$$

5.2. Springs in Series

Consider two springs k_1 and k_2, with their respective elongations x_1 and x_2, The equivalent spring of stiffness keq has an elongation $x = x_1 + x_2$, such that $\boldsymbol{k_1x_1 = k_2x_2}$

$\boldsymbol{mg = k_2x_2}$

$\boldsymbol{mg = (x_1 + x_2)}$



$x_1 = k_2\, k_1\, x_2$

$k_2 x_2 = (x_1 + x_2) \Rightarrow$

$k_2 x_2 = keq(\frac{k1}{k2}\, x_2 + x_2) \Rightarrow$

$$keq = \frac{k1k2}{k1+k2}$$

Or $$\frac{1}{keq} = \frac{1}{k1} + \frac{1}{k2}$$

6.Equivalent Mass:

From the total kinetic energy of the mechanical system, we can find the equivalent mass and the equivalent moment of the system as follows:

$T\ (system)_{tot} = 1/2\ (equivalent\ mass)^{2}$

Exemple :

Given the following system, find the equivalent mass and the equivalent spring.