Series N 1

Exercice 1:

1. Determine a simple equivalent of the following sums of Riemann:

$$\sum_{k=1}^{n} \frac{1}{2n+k}, \qquad \sum_{k=1}^{n} \frac{1}{\sqrt{n^2+2kn}}, \qquad \sum_{k=1}^{n} \frac{k}{n^2+k^2}$$

2. Calculate the following limits:

$$\lim_{n\to+\infty} \sum_{k=1}^{n} \frac{n}{n^2+k^2}, \qquad \lim_{n\to+\infty} \sum_{k=1}^{n} \frac{1}{3n+k}$$

Exercise 2:

1. Calculate the following double integrals:

$$\iint_D x\,dxdy \qquad D = \{(x,y) \in \mathbb{R}^2, x \geq 0, \quad y \geq x, \quad x+y \leq 2\}$$

$$\iint_\Omega xy^2\,dxdy \qquad \Omega = \{(x,y) \in \mathbb{R}^2, 1 \geq \lfloor x \rfloor, \quad y \in [0,2]\}$$

$$\iint_D \frac{1}{\sqrt{x^2+y^2}}\,dxdy \qquad \phi = \{(x,y) \in \mathbb{R}^2, 1 \leq x^2+y^2 \leq 4\}$$

2. Let $E = \{(x,y) \in \mathbb{R}^2, x \geq 0,\ y \geq 0,\ x^2+y^2 \leq 2,\ y \leq x^2\}$.

Consider the function $f(x,y) = x \cdot y$ on the domain E. Compute the double integral in Cartesian coordinates.

$$\iint_E f(x,y)dxdy$$

Exercise 3:

1. Calculate the area of the domain A, delimited by the curves:

$$y = \frac{8}{x}, \qquad y = x-1 \; et \; x = 1$$

2. Calculate the area of the domain A, delimited by the curves:

$$y = x^2 \; et \; y = 1-x^2$$

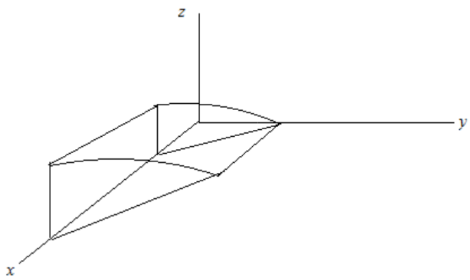
Exercice 4:

1. Calculate the following triple integral:

$$J_1 = \iiint_D (x+y+z)dxdydz \quad, \quad D = \{(x,y,z) \in \mathbb{R}^3 \mid 0 < x < 1, 0 < y < x, 0 < z < y\}$$

$$J_2 = \iiint_D \frac{1}{\sqrt{x^2+y^2+z^2}}dxdydz \quad, \quad D = \{(x,y,z) \in \mathbb{R}^3 \mid x^2+y^2+z^2 \leq 1\}$$

2. Compute the triple integral of $f(x,y,z) = z$ over the domain V located in the first octant, defined by $x \geq 0, y \geq 0, z \geq 0$ and bounded by the planes $y = 0, z = 0, x+y = 2, x+2y = 6$, and the cylinder $y^2+z^2 = 4$.



Exercise 5:

Let E be a solid homogeneous ellipsoid centered at the origin of a Cartesian coordinate system with semi-axes a,b,c along the coordinate axes. Its mass density is ρ=1.

Let (r, θ, φ) be the ellipsoidal coordinate system defined as:

$$x = a \cdot r \cdot \sin\theta \cdot \cos\varphi$$

$$y = b \cdot r \cdot \sin\theta \cdot \sin\varphi$$

$$z = c \cdot r \cdot \cos\theta$$

Using these ellipsoidal coordinates, the ellipsoid can be described as:

$$E: 0 \leq r \leq 1, 0 \leq \theta \leq \pi, 0 \leq \varphi \leq 2\pi$$

Using only this coordinate system:

a) Compute the determinant of the Jacobian of the given ellipsoidal transformation.

b) Compute the volume of the ellipsoid E.