

Chapitre 01

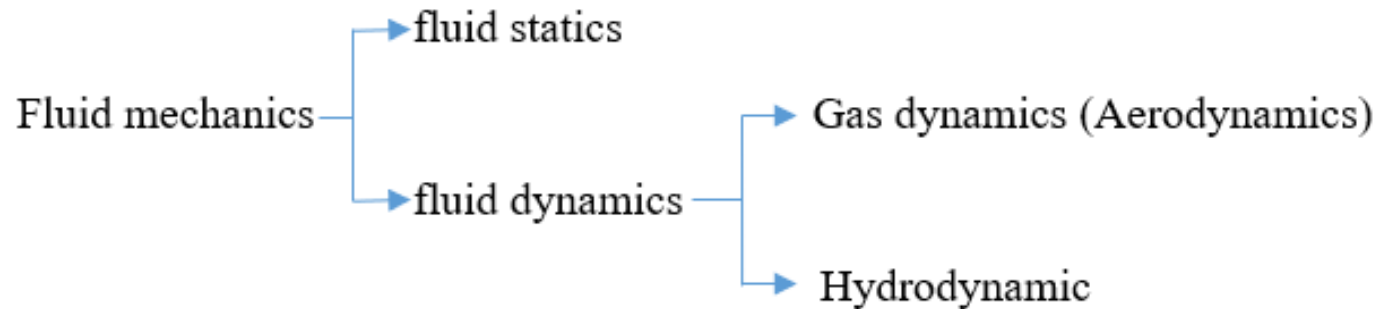
Reminders:

- *Propriétés des fluides.*
- *Statiques des fluides.*
- *Dynamiques des fluides parfaits.*

Chapitre 01

1- Introduction

Fluid mechanics is a branch of engineering which is concerned with the study of fluids at rest (i.e. fluid statics) as well as the study of fluids in motion or the study of fluid flow (i.e. kinematics and fluid dynamics).



Fluid mechanics has a wide range of applications, including urban hydraulics (e.g., water distribution networks), air conditioning & heating, hydroelectric power, urban aerodynamics, vibration analysis, aeronautics, the chemical and food industries, and meteorology.

Applications of Fluid Mechanics



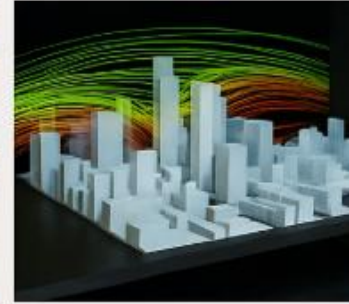
Urban Hydraulics



Air Conditioning
& Heating



Hydroelectric Power



Urban Aerodynamics



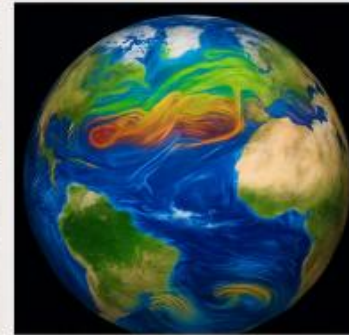
Vibration Analysis



Aeronautics



Chemical and
Food Industries



Meteorology

In this chapter, we highlight the definition of a fluid, both ideal and real, as well as the distinction between compressible and incompressible fluids. In this way, we summarize their various properties.

States of matter

Matter can exist in three different states:

✚ **Solids:** low temperature material

- ✓ Ordered molecules, very close together, linked.
- ✓ Invariable form
- ✓ Constant volume

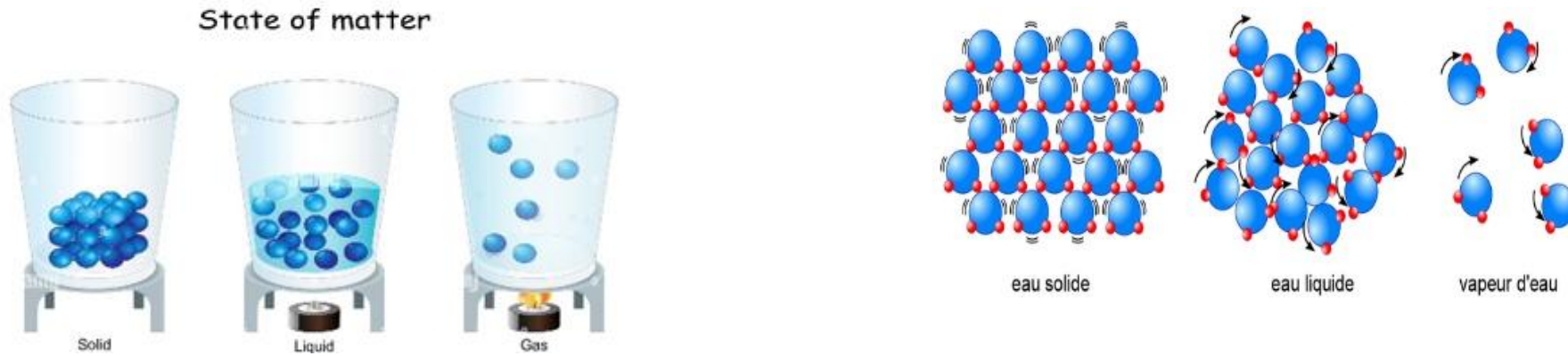
✚ **Liquid:** materials at medium temperature and sufficiently high pressure.

- ✓ molecules disordered, close, together, linked.
- ✓ Variable form.
- ✓ Constant volume.

✚ **Gaz:** material at sufficiently high temperature at low pressure

- ✓ disordered, spaced out, very agitated molecules
- ✓ variable form
- ✓ variable volume

The term fluid includes gases and liquids, because the latter have similarities.



The main differences between liquids and gases are:

- ✓ Liquids are practically incompressible, whereas gases are compressible and must often be treated accordingly.
- ✓ Liquids occupy well-defined volumes and have free surfaces, whereas a given mass of gas expands to fill the entire volume of its container. Gases do not form free surfaces.

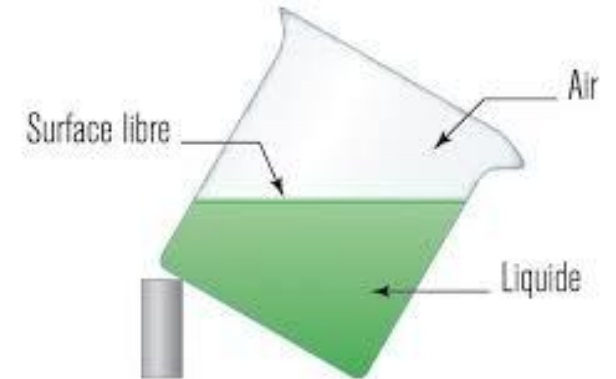


Figure I.1: the free surface of a liquid

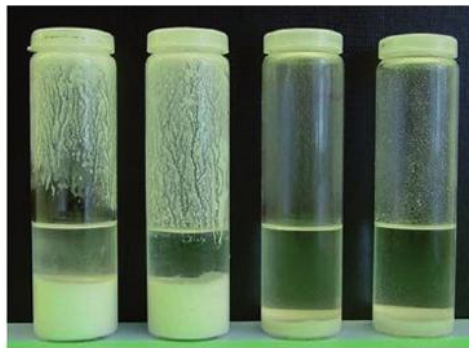
- ❑ Not all fluids are pure liquids or gases. We often encounter fluids in which two phases coexist in thermodynamic equilibrium. Compared to pure liquids, the presence of particles (such as gas bubbles, solid particles, or droplets) introduces a multitude of interfaces between the liquid (continuous phase) and the particles (dispersed phase), which can radically alter the nature of the mixture. The following types can be distinguished:

- ***The dispersions:***



Dispersions that do not sediment (colloids)

- **Suspensions:** coarse particles in a liquid (saturated soil, etc.)



Exemple de suspension



Exemple d'émulsion

- **Foams:** gas bubbles in a liquid (foam, etc.).



- **Emulsions:** these are mixtures of fine droplets of a liquid in another (vinaigrette, etc.).

In Fluid Mechanics (FM), the phenomena of liquid and gas flows are generally treated from a macroscopic point of view using Newton's laws of mechanics. In this context, the flow medium is considered as continuous.

Several fluid problems do not involve motion. They mainly concern the distribution of pressure in a fluid at rest and its effect on the solid walls of floating or submerged objects. When the fluid velocity is zero, known as the *hydrostatic condition*, the pressure variation is solely due to the weight of the fluid. If the fluid is known in a given gravitational field, the pressure can be easily calculated by integration.

Physical definition of a fluid

✚ A fluid can be constituted as being made up of a large number of material particles, very small and free to move relative to each other. A fluid is therefore a continuous material medium, deformable, without rigidity and which can flow. The fluid is an isotropic material.

- **Continuous:** its properties vary continuously.
- **Deformable:** it has no proper shape; the molecules can easily slide over each other, this mobility causes the fluid to take the shape of the container that contains it.
- **Capable of flowing:** A fluid can flow more or less easily from one container to another or through a pipe. However, **frictional forces** (internal resistance) may arise due to the fluid's **viscosity**, which opposes the relative motion of its particles.
- **Isotropic:** At any given point within the fluid, its physical properties are identical in all directions.

- **The fluid particle:** it is a portion of fluid to which correspond, at an instant, a pressure, a temperature, a density.... etc.

Fluids can be classified as follows:

+ Perfect fluid:

Is a fluid in which internal friction (tangential shear stresses) is zero, meaning there is no viscosity ($\mu = 0$). In such a fluid, the contact forces are always normal (perpendicular) to the surface elements on which they act.

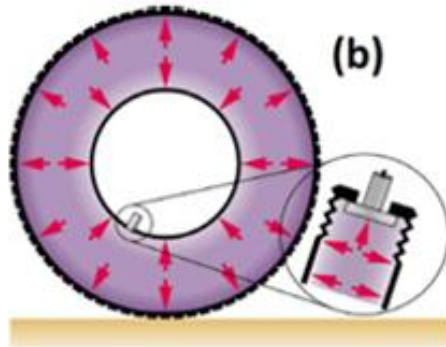
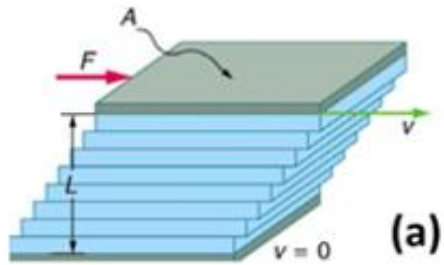
- *The term "perfect fluid" is generally used to describe this type of fluid (without viscosity, thermal conductivity, etc.).*

+ Real fluid:

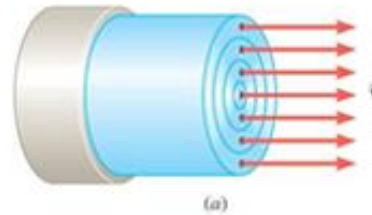
In real fluids, tangential internal friction forces — which oppose the relative motion between fluid layers — must be taken into account ($\mu \neq 0$). This viscous phenomenon becomes significant when the fluid is in motion. **In static conditions**, however, **real fluids behave like perfect fluids**, and their statics are identical.

In static conditions; a fluid — whether real or ideal — is at rest in a given reference frame. In this case, there is no relative motion between adjacent fluid layers, so the **shear (tangential) stresses** that characterize viscous effects in real fluids **vanish**.

Consequently, the internal forces in a fluid at rest are **purely normal stresses** (i.e., pressure forces), and the **mechanical behavior of a real fluid in static equilibrium becomes indistinguishable from that of a perfect (inviscid) fluid**. Therefore, in fluid statics, **the governing equations** — such as the hydrostatic pressure gradient $\nabla P = \rho \vec{g}$ apply equally to real and ideal fluids.

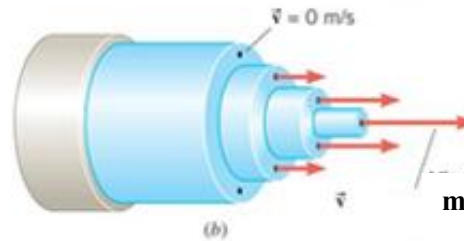


(a) Friction force (b) pressure force



Flow of a perfect fluid:

- Constant velocity
- Zero viscosity ($\mu = 0$)



Flow of a real fluid:

- variable velocity with radius
- no zero viscosity ($\mu \neq 0$)

(a) Perfect fluid (b) Real fluid

✚ Incompressible fluid ($\rho = cste$):

a fluid is said to be incompressible when the volume occupied by a given mass does not vary as a function of external pressure ($\rho = cste$). liquids can be considered as incompressible fluids (water, oil, etc.)

- *Gases, on the other hand, cannot be considered incompressible except under special conditions (low speeds, low pressure gradients).*

✚ Compressible fluid ($\rho \neq cste$):

A fluid is said to be compressible when the volume occupied by a given mass changes significantly with variations in external pressure — that is, its density ρ is not constant.

Gases are typical examples of compressible fluids, as their density can vary considerably with pressure and temperature changes.

Comparison table: Newtonian vs. non-Newtonian fluids

Characteristic	Newtonian Fluid	Non-Newtonian Fluid
Definition	Fluid in which shear stress is proportional to the velocity gradient.	Fluid in which shear stress is not proportional to the velocity gradient.
Constitutive law	$\tau = \mu \frac{du}{dy}$	$\tau \neq \mu \frac{du}{dy}$, depends on other factors (time, velocity, etc.)
Viscosity	Constant (independent of shear rate)	Variable (depends on shear rate, time, etc.)
Common examples	Water, air, light oil, alcohol	Blood, toothpaste, paint, ketchup, mud, polymers
Shear stress vs. shear rate	Straight line (linear relationship)	Nonlinear curve or threshold behavior
Mathematical simplicity	Easy modeling (classical Navier–Stokes equations)	More complex modeling (Bingham, Ostwald–de Waele models, etc.)

Physical properties of fluids

The characteristics of fluids allow us to describe their physical state, which we refer to as **fluid properties**.

Among the most important properties are:

- **Density (ρ)**
- **Specific weight (ϖ)**(weight per unit volume)
- **Relative density (ν) (or specific gravity)** and **viscosity**.

These physical quantities are fundamental for analyzing the behavior of fluids in both static and dynamic conditions.

1- Temperature

Relative temperature scale

Celsius scale ($^{\circ}\text{C}$): In the International System of Units (SI), the Celsius scale divides the temperature interval between the freezing and boiling points of water into 100 equal degrees.

Absolute temperature scale

The Kelvin is the official SI unit for thermodynamic temperature.

Absolute zero corresponds to **0 K = -273.15°C** , which is the minimum energy state of a thermodynamic system.

The relationship between Celsius and Kelvin is: **$T(K) = T(^{\circ}\text{C}) + 273.15$**

2- Density or masse density (ρ)

The **density** of a substance is defined as the ratio of its **mass (m)** to the **volume (V)** it occupies: $\rho = \frac{m}{V}$

- ρ is expressed in ($\text{kg} \cdot \text{m}^{-3}$)
- **m** is the mass in (**kg**)
- **V** is the volume in (**m³**).

For **gases**, density depends on **temperature and pressure**. For an **ideal (perfect) gas**, the density can be determined from the **ideal gas law**:

$$P V = m R T \Rightarrow P = \rho R t \Rightarrow \rho = \frac{P}{R T} \quad \text{Where :}$$

P : Absolute pressure in (Pa)

T : absolute temperature in (K)

R : specific gas constant in ($\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$), defined as: $R = \frac{r}{M_{gas}}$

r: universal gas constant ($r = 8.314 \text{ kJ} \cdot \text{kmol}^{-1} \cdot \text{K}^{-1}$)

M: molar mass of gas in ($\text{kg} \cdot \text{mol}^{-1}$)

- For air, the specific gas constant is: $R = 287 \text{ J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$

3- Specific volume (ν)

The **specific volume** of a substance, denoted by ν , is defined as the **inverse of the density**:

$$\nu = \frac{1}{\rho} = \frac{V}{m}$$

It represents the **volume occupied by a unit mass** of the substance.

Specific volume (ν) is expressed in $(m^3.kg^{-1})$

4- Specific weight or weight density (ϖ)

The **specific weight**, denoted by ϖ , is defined as the **weight per unit volume** of a substance. It is given by the formula:

$$\varpi = \rho g = \frac{m g}{V}$$

ϖ is expressed in $(N.m^{-3})$

m: mass in (kg)

g: acceleration of gravity in $(m.s^{-2})$

V: volume in (m^3) .

5- Specific gravity (δ)

The **specific gravity** (δ) of a fluid, denoted δ , is the **ratio of the density of the fluid (ρ)** to the **density of a reference fluid**, both measured under the **same temperature and pressure conditions**:

$$\delta = \frac{\text{density of fluid}}{\text{density of a standard fluid}} = \frac{\rho}{\rho_{\text{réf}}}$$

Specific gravity is **dimensionless** (no units), as it is a ratio of two densities.

Typically, the reference fluid is:

- ✓ Water for liquids $\left(\delta = \frac{\rho_{\text{liquid}}}{\rho_{\text{water}}} \right)$
- ✓ Air for gases $\left(\delta = \frac{\rho_{\text{gas}}}{\rho_{\text{air}}} \right)$.

6- Viscosity of a fluid

- Viscosity is a property of fluids that arises from **molecular cohesion** and **intermolecular interactions**, which **resist relative motion and deformation** between adjacent fluid layers. This property becomes significant **whenever the fluid is in motion**, particularly in **shear flow**, where internal friction develops due to velocity gradients within the fluid.
- Viscosity measures a fluid's internal resistance to flow.
- For Newtonian fluids, the shear stress is proportional to the velocity gradient $\tau = \mu \frac{du}{dy}$

✚ (According to Newton's law, if a fluid satisfies the relation $\tau = \mu \frac{du}{dy}$, the fluid is said to be Newtonian (examples: air, water).)

 Practical examples:

✓ Newtonian fluid: water

- ✓ If the stirring speed is increased, the stress increases linearly.
- ✓ The viscosity remains constant.

✓ Non-Newtonian fluid: ketchup

- ✓ When stirred slowly, it remains thick (high viscosity).
- ✓ If shaken vigorously, it becomes more fluid → pseudo-plastic behavior.

There are two types: ✓ Dynamic viscosity μ ($Pa.s$) ✓ Kinematic viscosity in ν (m^2/ρ) in (m^2/s)

6.1- Dynamic viscosity (μ)

To establish a relationship for viscosity, consider a volume of fluid confined between two **infinite, parallel, horizontal plates** separated by a distance h .

The **lower plate is fixed**, while the **upper plate is mobile** and moves at a **constant velocity U** .

To maintain this steady motion, a **constant tangential force F** must be applied to the upper plate.

This results in a **viscous interaction** between the fluid and the plates:

➤ The fluid exerts a **drag force** on the moving plate, And the plate applies a **shear force** on the fluid.

Due to the **no-slip condition**, the **fluid in contact with the upper plate** moves at velocity U , while the **fluid in contact with the lower plate** remains **stationary** (velocity = 0).

A **linear velocity gradient** is thus established between the two plates.

The **shear stress τ** is defined as the **tangential force per unit area** acting on the fluid: $\tau = \frac{F}{S}$ Where:

τ : The shear stress in ($N.m^{-2}$ or Pa)

S : area of the surface in contact with the fluid in (m^2).

F : tangential (shear) force applied to the surface, in (**N**)

When the fluid moves in **parallel layers**, the flow is said to be **laminar**. In this case, the **velocity of the fluid varies linearly** from 0 (at the fixed plate) to U (at the moving plate).

The **velocity profile** is given by: $u(y) = \frac{y}{h} U$

where:

- $u(y)$: fluid velocity at a distance y from the stationary plate.
- h : distance between the two plates,
- U : velocity of the upper moving plate.

While the **velocity gradient** is: $\frac{du}{dy} = \frac{U}{h}$

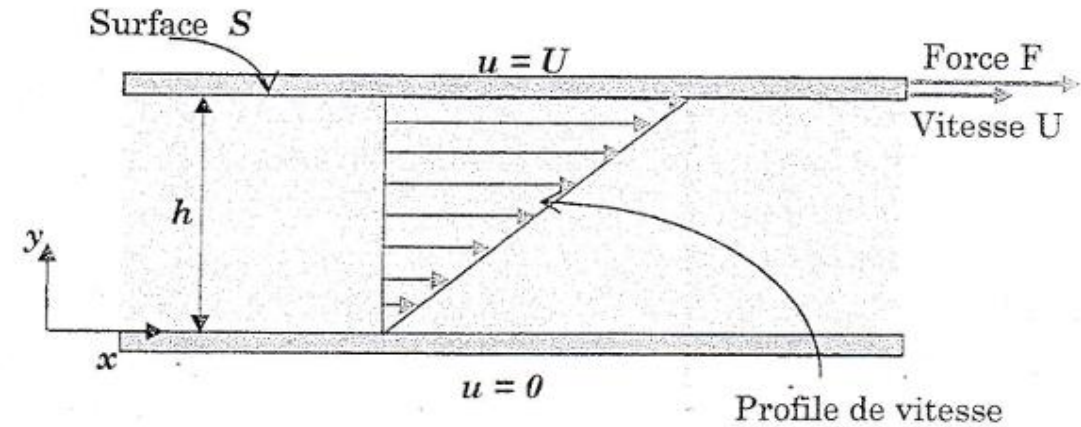


Figure I.3: Behaviour of a fluid in laminar flow between two parallel plates.

The **tangential force** F required to maintain the motion depends on the **area** S of the plate and the **velocity gradient**.

According to **Newton's law of viscosity**, the **shear stress** is proportional to the velocity gradient: $\tau = \mu \frac{du}{dy}$

and since $\tau = \frac{F}{S}$ we also have: $F = \tau S = \mu S \frac{du}{dy}$ The force F depends on the surface area s and the velocity gradient.

Where:

- μ : dynamic viscosity (in $\text{Pa}\cdot\text{s}$) or **poiseuille(pl)**,  In the international system, the unit of **dynamic viscosity** μ is the (**$\text{Pa}\cdot\text{s}$**) or **poiseuille(pl)**.
- S : surface area (in m^2)
- h : distance between the plates (in m)
- U : velocity of the moving plate (in m/s).

knowing that: $1\text{Pa}\cdot\text{s} = 1\text{N}\cdot\text{s}\cdot\text{m}^{-2} = 1\text{Pl} = \text{kg}\cdot\text{m}^{-1}\cdot\text{s}^{-1}$ other units:
 $1\text{Pl} = 10 \text{ poise}$.

6.2- Variation in viscosity (ν)

Kinematic viscosity is the ratio of the dynamic viscosity to the density ρ of a fluid.

$$\nu = \frac{\mu}{\rho}$$

In the international system, *kinematic viscosity* (ν) is expressed in ($\mathbf{m^2.s^{-1}}$). Other units: $1\mathbf{stockes} = 10^{-4} \mathbf{m^2.s^{-1}}$.

7- Compressibility

The compressibility of a fluid can be defined as its resistance to a change in volume for a constant mass. Liquids have very low compressibility. In contrast, the compressibility of gases is very high.

II. Basic law of hydrostatics

$$\text{We have : } \frac{\partial P}{\partial x} = 0 \ ; \ \frac{\partial P}{\partial y} = 0 \ ; \ \frac{\partial P}{\partial z} = -\rho g \quad (\text{Hydrostatic variations})$$

This shows us that the pressure does not depend on the **x** and **y** directions; on the other hand, it only depends on direction **z**.

Hence the differential equation to solve to know the pressure at any point of

$$\text{the fluid at rest: } \frac{dP}{dz} = -\rho g$$

The fundamental equation of statics can be established in a more general way, without involving a particular reference. $\rho \vec{g} - \overrightarrow{\text{grad}} P = 0$; $\{\vec{g} = -g\vec{k}\}$

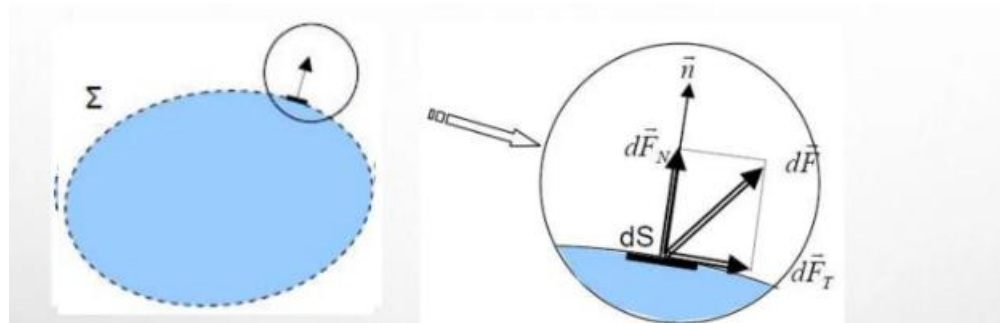
either: $\overrightarrow{\text{grad}} P = -\rho g\vec{k}$, fundamental vector relation of fluid statics.

II.1- Hydrostatic pressure in an incompressible fluid

Let $d\vec{F}$ be the elementary force exerted by the particles of fluid medium 2 on the particles of fluid medium 1 through a surface element dS

This force can be decomposed into two components:

- A tangential component $d\vec{F}_T$ (associated with shear stresses).
- A normal component $d\vec{F}_N$ (associated with normal stresses, i.e., pressure).



- ✚ In the case of a perfect (ideal) fluid at rest, the tangential component is zero because there is no shear stress (there is no relative motion between adjacent layers, hence no viscous effects). Only the normal component remains, and it is directed perpendicular to the surface, pointing toward the interior of the fluid.
- ✚ In a fluid at rest, the pressure is **isotropic** and produces only **normal forces**. Tangential (shear) forces occur only when viscosity is present or when there is relative motion between fluid layers. Pressure in a fluid is a **normal stress** that acts equally in all directions.

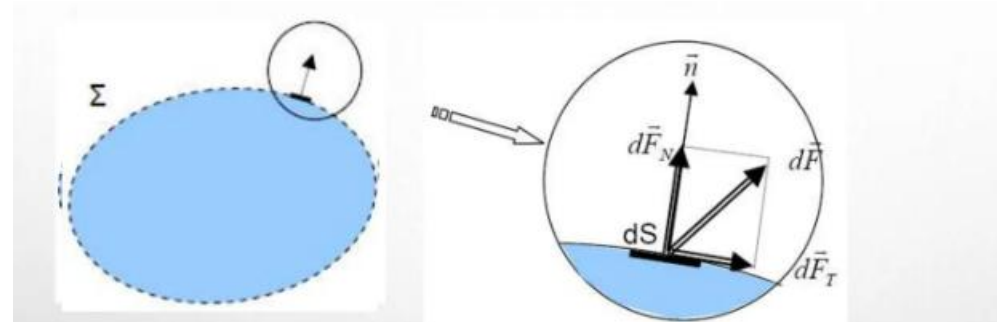


Figure II.1 the components $d\vec{F}_T$ and $d\vec{F}_N$ of the force $d\vec{F}$ on a fluid surface element.

The tangential component $d\vec{F}_T$ acting on the surface element dS is called **shear stress**, and the normal component $d\vec{F}_N$ acting on the same surface element is called **normal stress**.

In **fluid statics**, there are no friction forces in the fluid. The **tangential component** vanishes because it is associated with **viscosity**. Consequently, all the forces that develop on the surfaces are due to **pressure (a normal stress)**:

$$P = \frac{dF_N}{dS}$$

Knowing that the unit of pressure in the international system is the pascal (Pa):

$$1Pa = 1N.m^{-2} = 1Kg.m^{-1}s^{-2}$$

$$1bar = 10^5 Pa$$

$$1atm=760mmHg$$

$$1atm=1.013 \cdot 10^5 Pa$$

$$1 atm= 10 m de colonne d'eau (mce)$$

$$1Kgf.cm^{-2} = 98070Pa.$$

In a fluid at rest, **pressure** is defined as the normal force per unit area exerted on a surface element. It is a **scalar quantity** and, according to Pascal's principle, does not depend on the orientation of the surface element.

You correctly consider that the only forces acting on the fluid element are:

1. The pressure force exerted by the neighboring surfaces.
2. The weight of the fluid.

❖ **Suggestion:** Specify that the studied element is **an infinitesimal tetrahedron** (or a small inclined volume) in order to naturally introduce the three pressures P_1, P_2, P_3

To do this, let us take any portion of a fluid at rest, as shown by **figure**.

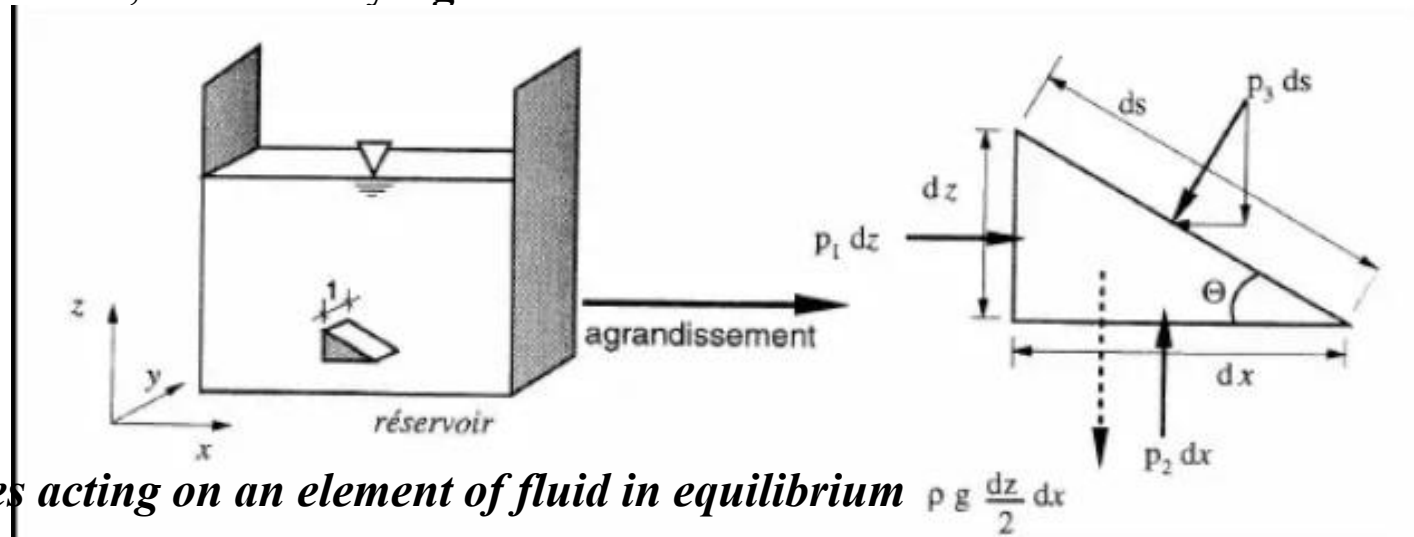


Figure II.2 Forces acting on an element of fluid in equilibrium

We call P_1, P_2, P_3 the pressures in the 3 surfaces in the plane $(\mathbf{ox}, \mathbf{oz})$. The only forces which act on the volume are those of **pressure** and the **weight of the fluid**.

$$P_1 = P_2 = P_3 = P$$

This proves that **at a point in a fluid at rest, the pressure is the same in all directions** — a direct consequence of Pascal's principle

II-2 Application of the fundamental hydrostatic equation to the incompressible fluid

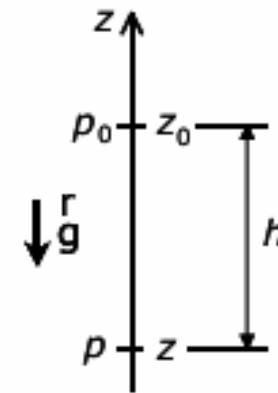
For an **incompressible fluid**, the density ρ is constant throughout the fluid ($\rho = \text{const}$).

We also assume that the gravitational acceleration $\mathbf{g}$ is constant (valid when altitude variations are small).

From the hydrostatic equilibrium condition in a fluid at rest: $\frac{dP}{dz} = -\rho g = \text{cste}$

And by integral : $P(z) = \int dp(z) = -\rho g z + \text{cste}$

So : $P(z) + \rho g z = \text{cste}$ *Fundamental equation of hydrostatics*



Either : $P(z) + \rho g z = P_0 + \rho g z_0$ Where P_0 is the pressure at altitude z_0

$$P(z) = P_0 + \rho g(z_0 - z)$$

If $h = z_0 - z$ is the depth measured from the reference point z_0 , then: $P(z) = P_0 + \rho g h$

Such a pressure field is called a **hydrostatic pressure field**.

Where **h** is the height of the fluid below the reference level.

Most of the time, we take $z_0 = 0$, with the reference level corresponding to the free surface of the fluid, where $P_0 = P_{atm}$.

For numerical applications, the standard atmospheric pressure is: $P_{atm} = 1.013 \cdot 10^5 Pa$.

Level Surfaces

Definition

A *level surface* is defined as the locus of points in the fluid subjected to the same pressure.

We have: $\frac{P}{\rho g} + z = \text{Cst}$

If **P= Cste**, then the equation of the level surface is **z= Cste**.

Therefore, level surfaces are *horizontal planes*.

Free Surface of a Liquid

At the separation surface between the liquid and the surrounding air, the pressure is **P_a** . This surface is therefore horizontal. It is called the *free surface* of the liquid.

the free surface of a liquid in equilibrium **is flat and horizontal (an isobaric surface)**, the pressure at any point on this surface is equal to the atmospheric pressure, which is constant: **$P = P_{atm} = 1.013 \cdot 10^5 Pa$** .

We apply the **fundamental equation of hydrostatics** to calculate the pressure at a point **M** located below the free surface. Knowing that the pressure at the free surface is equal to **P_{atm}** .

We obtain: $P_M + \rho g Z_M = P_{atm} + \rho g Z_0$

$\Rightarrow P_M = P_{atm} + \rho g (Z_0 - Z_M)$ hence: $h = (Z_0 - Z_M)$

Z_M : altitude of point M.

Z_0 : altitude of the free surface.

Therefore: $P_M = P_{atm} + \rho g h$

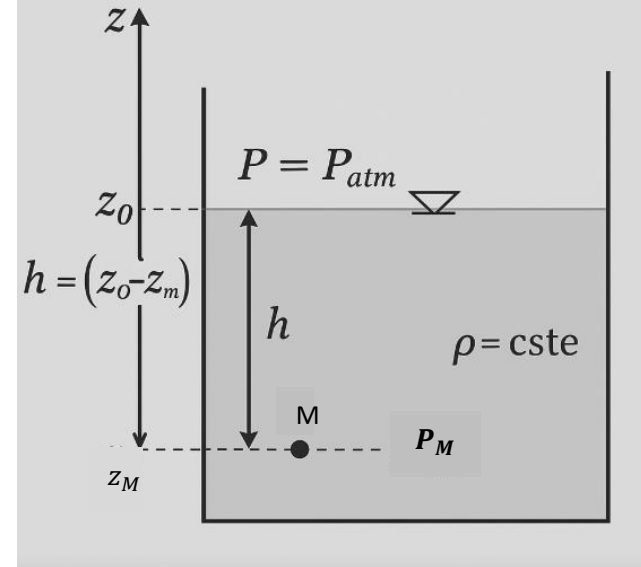


Figure II.3 One-point pressure

III. Bernoulli's Theorem and Its Applications

The **Bernoulli equation** is a fundamental relation in fluid mechanics. It expresses the principle of **conservation of mechanical energy** for a fluid that is incompressible, inviscid (frictionless), and flowing steadily along a streamline.

III.1 Establishment of the Bernoulli Equation

In a **steady flow** of a **perfect and incompressible fluid** under the influence of gravity, we consider an **infinitesimally narrow flow tube**, so that pressure, velocity, density, and elevation can be regarded as **uniform within the same cross section**. Furthermore, we assume that the flow is **one-dimensional**, i.e., directed along a single streamline.

Hypotheses :

- The fluid is perfect (i.e., non-viscous) and incompressible.
- The flow is steady (stationary in time).

The volume force density is derived from a potential field: $\vec{F}_V = -\nabla U$

General form of Bernoulli's equation:

$$\frac{P}{\rho} + \frac{1}{2}v^2 + gz = cste (*)$$
 All terms are expressed in (J/Kg) or (m²/s²)

“Bernoulli's equation therefore reflects the conservation of energy along a streamline”.

“Bernoulli's theorem shows that the **total energy remains constant along a streamline** (no energy loss)”

$\frac{P}{\rho}$: Pressure potential energy per unit mass.

$\frac{1}{2}v^2$: Kinetic energy per unit mass.

gz : Potential energy per unit mass

III.2 Bernoulli's Equation (with work exchange)

We consider a fluid flowing through a hydraulic machine (pump, fan, turbine) when it passes (during its movement) from **(A)** to **(B)**.. The machine exchanges a power $P_{\text{machine,fluid}}$ with the fluid, with efficiency η .

In head form **(en forme de charge)** :

$$\frac{P_A}{\rho g} + \frac{1}{2g} v_A^2 + z_A + \frac{P_{\text{machin, fluid}}}{\rho g Q_v} = \frac{P_B}{\rho g} + \frac{1}{2g} v_B^2 + z_B$$

With :

Q_v : Volume flow in m³/s

$P_{\text{machin, fluid}}$: Power exchanged with the fluid in Watt.