COURSE

3 Sampling and Estimation

Below are detailed solutions (step-by-step, with full algebraic and numerical calculations) for the exercise series provided in the file "Série$_{de}$$_T$$D2$". $All answers are given in English and presented using the same LaTeX template style you specif$

1 Exercise 1 — Elementary Set and robability Operations

Statement

An experiment yields three events A, B and C. We are given:

$$P(A) = 0.15, \quad P(B) = 0.3, \quad P(C) = 0.4, \quad P(A \cup B) = 0.42, \quad P(A \cap C) = 0.05,$$

and it is stated that B and C are incompatible (mutually exclusive), i.e. $P(B \cap C) = 0$. Compute:

1. $P(\overline{A})$
2. $P(B \cup C)$
3. $P(A \cap B)$
4. $P(A \cap \overline{C})$
5. $P(\overline{A} \cap \overline{B})$

Solution

We use standard set and probability identities.

1. Complement of A:

$$P(\overline{A}) = 1 - P(A) = 1 - 0.15 = 0.85.$$

2. Union $B \cup C$: since B and C are incompatible, $P(B \cap C) = 0$. Thus

$$P(B \cup C) = P(B) + P(C) - P(B \cap C) = 0.3 + 0.4 - 0 = 0.7.$$

3. Intersection $A \cap B$: use

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Rearrange to get

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.15 + 0.3 - 0.42 = 0.03.$$

4. $A \cap \overline{C}$: note that A decomposes as $A = (A \cap C) \cup (A \cap \overline{C})$ and these two are disjoint, hence

$$P(A) = P(A \cap C) + P(A \cap \overline{C}).$$

Therefore
$$P(A \cap \overline{C}) = P(A) - P(A \cap C) = 0.15 - 0.05 = 0.10.$$

5. $\overline{A} \cap \overline{B}$: by De Morgan,
$$\overline{A} \cap \overline{B} = \overline{A \cup B},$$
so
$$P(\overline{A} \cap \overline{B}) = 1 - P(A \cup B) = 1 - 0.42 = 0.58.$$

Summary of results:

$$P(\overline{A}) = 0.85, \quad P(B \cup C) = 0.70, \quad P(A \cap B) = 0.03,$$
$$P(A \cap \overline{C}) = 0.10, \quad P(\overline{A} \cap \overline{B}) = 0.58.$$

2 Exercise 2 — Law of Total Probability and Bayes' Rule

Statement

A batch of test tubes is supplied by three companies A, B and C with proportions
$$P(A) = 0.50, \quad P(B) = 0.30, \quad P(C) = 0.20.$$

Defective rates are:
$$P(D \mid A) = 0.02, \quad P(D \mid B) = 0.03, \quad P(D \mid C) = 0.04.$$

A tube is chosen at random from the batch.

1. What is the probability that the chosen tube is defective, $P(D)$?
2. Given that the chosen tube is defective, what is the probability it came from company A, i.e. $P(A \mid D)$?

Solution

1. Using the law of total probability:
$$P(D) = P(A)P(D \mid A) + P(B)P(D \mid B) + P(C)P(D \mid C).$$

Compute each term precisely:
$$P(A)P(D \mid A) = 0.50 \times 0.02 = 0.010,$$
$$P(B)P(D \mid B) = 0.30 \times 0.03 = 0.009,$$
$$P(C)P(D \mid C) = 0.20 \times 0.04 = 0.008.$$

Sum:
$$P(D) = 0.010 + 0.009 + 0.008 = 0.027.$$

Hence $P(D) = 0.027$ (which is 2.7%).

2. Posterior probability $P(A \mid D)$ via Bayes' theorem:

$$P(A \mid D) = \frac{P(A)P(D \mid A)}{P(D)} = \frac{0.50 \times 0.02}{0.027} = \frac{0.010}{0.027}.$$

Compute the quotient exactly (to reasonable precision):

$$\frac{0.010}{0.027} \approx 0.37037037037\ldots$$

So

$$P(A \mid D) \approx 0.37037 \quad \text{(about 37.037\%)}.$$

3 Exercise 3 — Binomial Probability (Germination)

Statement

A seed has probability $p = 0.8$ to germinate when planted. If the researcher plants $n = 10$ seeds, compute the probability that at least 8 of them germinate:

$$P(\text{at least } 8) = P(X \geq 8) \quad \text{where } X \sim \text{Bin}(n = 10, p = 0.8).$$

Solution

We compute

$$P(X \geq 8) = P(X = 8) + P(X = 9) + P(X = 10).$$

The binomial probability mass function is

$$P(X = k) = \binom{10}{k} p^k (1-p)^{10-k}.$$

Compute term by term with exact arithmetic and then numeric evaluation.

For $k = 8$:

$$\binom{10}{8} = 45, \quad p^8 = 0.8^8, \quad (1-p)^2 = 0.2^2 = 0.04.$$

Thus

$$P(X = 8) = 45 \times 0.8^8 \times 0.04.$$

Numerically,

$$0.8^2 = 0.64,\ 0.8^4 = 0.4096,\ 0.8^8 = (0.4096)^2 \approx 0.16777216.$$

So

$$P(X = 8) = 45 \times 0.16777216 \times 0.04 = 45 \times 0.0067108864 \approx 0.3019898880.$$

For $k = 9$:

$$\binom{10}{9} = 10, \quad p^9 = 0.8^9 = 0.8 \times 0.8^8 \approx 0.8 \times 0.16777216 = 0.134217728,$$

$$(1-p)^1 = 0.2.$$

Hence

$$P(X = 9) = 10 \times 0.134217728 \times 0.2 = 10 \times 0.0268435456 \approx 0.2684354560.$$

For $k = 10$:

$$\binom{10}{10} = 1, \quad p^{10} = 0.8^{10} = 0.8 \times 0.8^9 \approx 0.8 \times 0.134217728 = 0.1073741824,$$

$$(1-p)^0 = 1.$$

So

$$P(X = 10) \approx 0.1073741824.$$

Sum:

$$P(X \geq 8) \approx 0.3019898880 + 0.2684354560 + 0.1073741824 = 0.6777995264.$$

Therefore the probability that at least 8 seeds germinate is approximately

$$\boxed{0.6777995264 \ \approx \ 0.6778\ (67.78\%)}.$$

4 Exercise 4 — Normal Probability (Newborn Weights)

Statement

Newborn weights are approximately normally distributed with mean $\mu = 3.4$ kg and standard deviation $\sigma = 0.5$ kg. Compute the probability that a newborn weighs less than 3 kg:

$$P(X < 3) \quad \text{for } X \sim \mathcal{N}(3.4,\, 0.5^2).$$

Solution

Standardize using the standard normal variable Z:

$$Z = \frac{X - \mu}{\sigma}.$$

Compute the z-score:

$$z = \frac{3 - 3.4}{0.5} = \frac{-0.4}{0.5} = -0.8.$$

Thus

$$P(X < 3) = P\big(Z < -0.8\big) = \Phi(-0.8).$$

Using the standard normal CDF value:

$$\Phi(-0.8) \approx 0.2118553986.$$

Therefore

$$\boxed{P(X < 3) \approx 0.2118554\ (\approx 21.19\%)}.$$

(Equivalently, $P(X \geq 3) = 1 - 0.2118554 \approx 0.7881446$.)

5 Exercise 5 (Supplementary) — Normal ail Probability

Statement

The concentration of a chemical in a biological sample is normally distributed with mean 25 mg/L and standard deviation 5 mg/L. Compute the probability that a randomly chosen concentration exceeds 30 mg/L:

$$P(X > 30) \quad \text{for } X \sim \mathcal{N}(25, 5^2).$$

Solution

Standardize:

$$z = \frac{30 - 25}{5} = \frac{5}{5} = 1.$$

Thus

$$P(X > 30) = P(Z > 1) = 1 - \Phi(1).$$

Using the standard normal table (or numerical value),

$$\Phi(1) \approx 0.8413447461,$$

so

$$P(X > 30) \approx 1 - 0.8413447461 = 0.1586552539.$$

Hence

$$\boxed{P(X > 30) \approx 0.1586553 \ (\approx 15.87\%)}.$$

Notes:

- All normal distribution quantiles and probabilities above were computed to at least 7 decimal places and then rounded for presentation; you may increase precision using statistical software if needed.
- For Exercise 3 (binomial), computations were shown step-by-step (powers of 0.8 explicitly computed) to ensure exact numeric clarity.
- If you would like these solutions translated back into Arabic, inserted into your exact chapter LaTeX file, or exported as a compiled PDF, tell me which option you prefer and I will produce the file for download.