

University Abd Elhafid Boussouf -Mila
Faculty of Science and Technology
Second year (GC, Hy, Ing GC)
Mathematics 3

Series No. 2
Improper integrals

Exercise 1

Study the convergence of the following integrals:

$$I_1 = \int_1^{+\infty} \frac{t+1}{t(t^2+1)} dt, \quad I_3 = \int_1^{+\infty} \frac{\sin^2 t}{t^2} dt,$$
$$I_4 = \int_0^\pi \frac{\cos(1/t)}{t} dt, \quad I_5 = \int_0^{\frac{1}{2}} \frac{\ln(1-t^2)}{t^2} dt.$$

Exercise 2

Study the absolute convergence of the following integrals:

$$J_1 = \int_1^{+\infty} \frac{1 - \cos t}{t^2} dt, \quad J_2 = \int_0^{+\infty} \frac{t^2}{t^{\frac{17}{5}} + 1} dt.$$

Exercise 3

Let a and b be two real parameters. Discuss, according to their values, the convergence of

$$\int_2^{+\infty} \frac{dt}{t^a (\ln t)^b}.$$

You may:

1. When $a \neq 1$, use Riemann's convergence rules.
2. When $a = 1$, compute explicitly

$$\int_2^x \frac{dt}{t(\ln t)^b}$$

for a real x tending to $+\infty$.