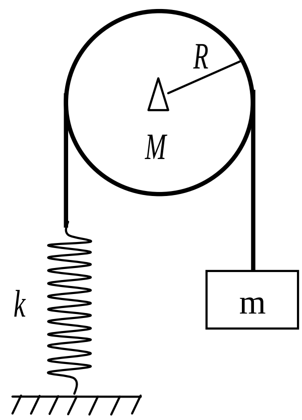
University Centre of Mila

Institute of Science and Technology **Departement of GM & EM**

Series N°2
Free Vibrations of Systems with One Degree of Freedom

Exercise 1:

An inextensible string carries a mass m=0.75 kg and it unwinds without slipping over a pulley of mass M, which can rotate around a fixed axis Δ. The other end of the pulley is connected to a spring of stiffness k $=10^4$ Nm^{-1}.



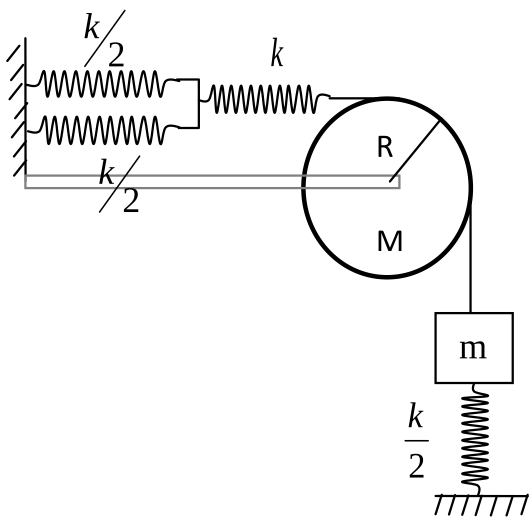
For the two cases:

(1) M=0 and **(2)** M=0.5 kg, find for this system:

1. The kinetic energy**,** potential energy, and the Lagrangian**.**
2. Deduce the differential equation of motion**.**
3. If at t=0 ($x = 5cm,\ \dot{x} = 1cms^{-1}$), find the natural (proper) frequency**,** the amplitude of the motion, the initial phase, and the total energy**.**

Exercise 2:

A mass m is connected on one side to a spring system via an inextensible string that passes over a disk of mass M and radius R, and on the other side to a spring that is placed on the ground. The disk is free to rotate with an angle θ around its fixed axis (see the adjacent diagram).



The mass m is displaced slightly from its equilibrium position by a small distance x, then released to oscillate.

Find for this system:

1. The **natural frequency** of oscillation.
2. Under the initial conditions

   (t=0 ; $x = 0cm,\ \dot{x} = 2cms^{-1}$)

– The **maximum angle** θmax made by the disk.

– The **total energy** of the system.

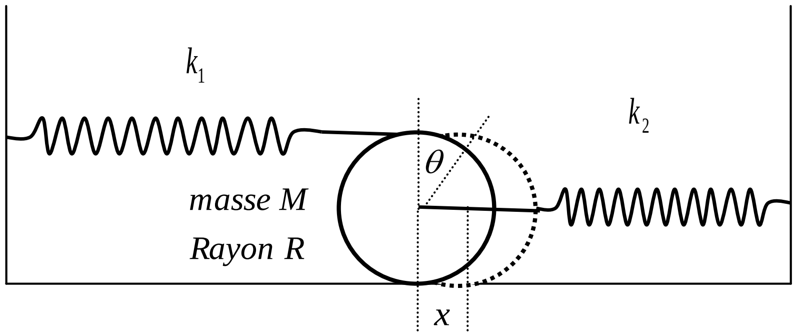
$(m = 0.5kg,\ \ M = 1kg,\ R = 2cm\ k = 10^5\, Nm^{-1})$

Exercise 3:

A pulley of mass M and radius R can both **rotate** and **slide** at the same time. It is connected to a frame by means of two springs (see the figure below).

The pulley is displaced from its equilibrium position by a small angle θ.

Find the natural frequency of oscillation **of the system**.



Exercise 4:

Consider two mechanical systems (Figures 1 and 2), each consisting of a **massless rod of length l**. The rod can undergo **vibrational rotational motion** around the **Z-axis**, and carries a **mass m** at its free end. The rod is connected to a fixed frame via a **spring of stiffness k**.

In both cases:

1. Air resistance on the mass is **negligible**.
2. $\vec{f} = -c\dot{\vec{x}}$

Find for both systems:

- The degree of freedom.
- The potential energy and kinetic energy.
- The Lagrangian.
- The natural frequency of oscillation.

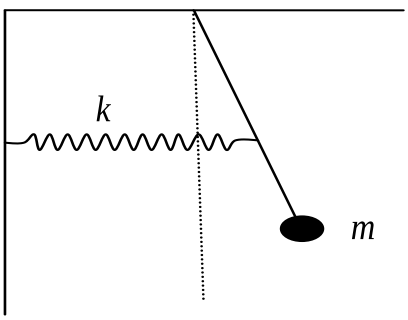


Figure 1

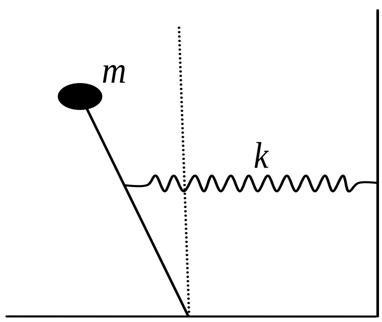
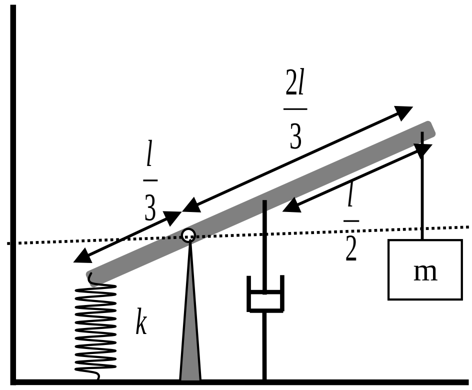


Figure 2

Exercise 5:

Determine the natural frequency of the system shown in the adjacent diagram (the bar has a mass M).



1. The axis of rotation is at the center of mass of the bar.
2. The axis of rotation is located at (see the diagram).
3. Find the natural frequency of the system represented by this diagram.