Series of Exercises 01

Exercise 1

Show that the following numerical series are convergent and calculate their sums:

❶ $\sum_{n=2}^{+\infty} \frac{1}{n(n-1)}$.

❷ $\sum_{n=0}^{+\infty} \frac{n^2}{n!}$.

❸ $\sum_{n=0}^{+\infty} (-1)^{n+1} \frac{\cos(nx)}{2^n}$, $x \in \mathbb{R}$.

Exercise 2

Study the nature of the following series:

1) $\sum_{n=1}^{+\infty} n \sin\left(\frac{1}{n}\right)$ 2) $\sum_{n=1}^{+\infty} \arctan\left(\frac{1}{n^2}\right)$ 3) $\sum_{n=1}^{+\infty} \frac{\alpha^n}{\alpha^{2n} + \alpha^n + 1}$ $(\alpha \geq 0)$

4) $\sum_{n=1}^{+\infty} \left(\frac{n+a}{n+b}\right)^{n^2}$, a et $b \in \mathbb{R}$ 5) $\sum_{n=1}^{+\infty} \frac{\ln(n)}{n^2+2}$ 6) $\sum_{n=1}^{+\infty} \frac{1 \times 3 \times \ldots \times (2n-1)}{2 \times 4 \times \ldots \times (2n)}$

7) $\sum_{n \geq 1} \frac{2^n}{n^2} \sin^{2n}(\theta)$, $\theta \in \left[0, \frac{\pi}{2}\right]$ 8) $\sum_{n \geq 1} \frac{\exp(inx)}{n}$, $x \in \mathbb{R}$.

Exercise 3

Let θ be a real number. Study, according to the value of θ, the absolute convergence, the semi-convergence and the divergence of the following numerical series:

1) $\sum_{n=1}^{+\infty} \frac{(-1)^n}{n^\theta}$, 2) $\sum_{n=1}^{+\infty} \left(\frac{2n-1}{n+1}\right)^{2n} \theta^n$, 3) $\sum_{n=1}^{+\infty} \frac{\cos(n)}{n^\theta + \cos(n)}$.