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Institute of Science and Technology
Department of Civil-Eng and Hydraulic
Rational Mechanics

2^{nd} Year

Series 01:

EXERCISE 1

Vectors $\vec{A}$ and $\vec{B}$ have scalar product -6 and their vector product has magnitude +9. **What is** the angle between these two vectors.

EXERCISE 2

We have the vectors $\mathbf{V}_1$ and $\mathbf{V}_2$ shown in the figure 1.
(*a*) **Determine** the magnitude S of their vector sum $\mathbf{S} = \mathbf{V}_1 + \mathbf{V}_2$
(*b*) **Determine** the angle α between $\mathbf{S}$ and the positive x-axis
(*c*) **Write S** as a vector in terms of the unit vectors **i** and **j** and then write a unit vector **n** along the vector sum **S**
(*d*) **Determine** the vector difference $\mathbf{D} = \mathbf{V}_1 - \mathbf{V}_2$

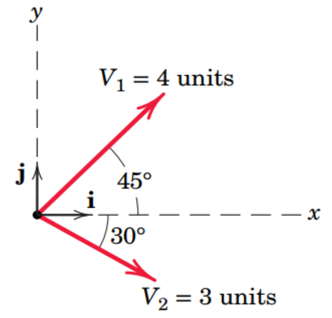


Figure.1

EXERCISE 3

A steel beam is lifted as shown in Fig.2. **Determine** the components of the tension T in the rope, parallel and perpendicular to the axis of the beam.

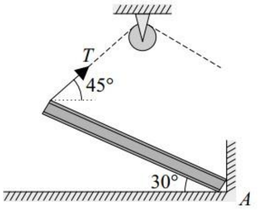


Figure.2

EXERCISE 4

Find the components of each force as shown in Fig.3.

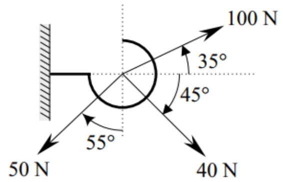


Figure.3

EXERCISE 5

Two smooth spheres having weight W and radius r rest in a thin-walled circular cylinder (weight Q, radius $R = 4r/3$) as shown in Fig. 4. **Find** the magnitude of Q required to prevent the cylinder from falling over.

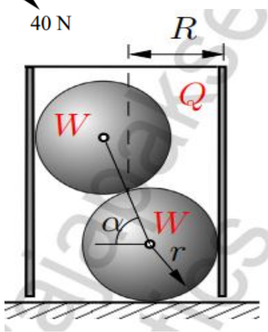


Figure.4

EXERCISE 6

The forces $\boldsymbol{F_1}$, $\boldsymbol{F_2}$, and $\boldsymbol{F_3}$, all of which act on point $\boldsymbol{A}$ of the bracket, are specifi ed in three different ways (Fig.5.).
Determine the $\boldsymbol{F_x}$ and $\boldsymbol{F_y}$ scalar components of each of the three forces.

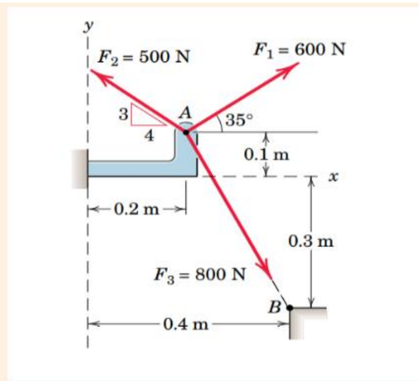


Figure.5

EXERCISE 7

Determine the resultant of the system of three concurrent forces pass through the origin and points (10, -5 ,6), (-5,5,7), (6, -4, -3) respectively. The respective magnitudes of the forces are 1500N, 2500N and 2000N.

EXERCISE 8

Two forces act on the hook shown in Fig.6. Specify the magnitude of and its coordinate direction angles of that the resultant force $\boldsymbol{F_R}$ acts along the positive y axis and has a magnitude of 800 N.

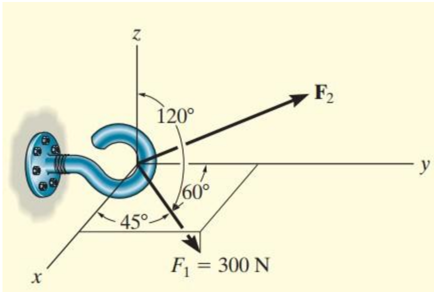
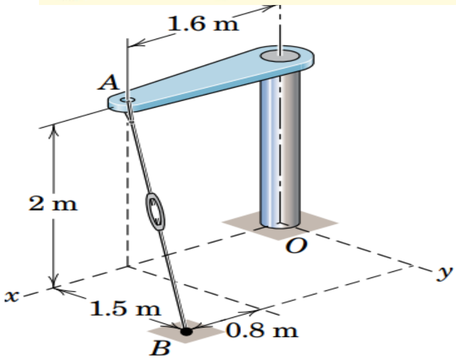


Figure.6

EXERCISE 9

The turnbuckle is tightened until the tension in cable ***AB*** is 2.4 kN Fig.8.
Determine the moment about point ***O*** of the cable force acting on point *A* and the magnitude of this moment.

Figure.7

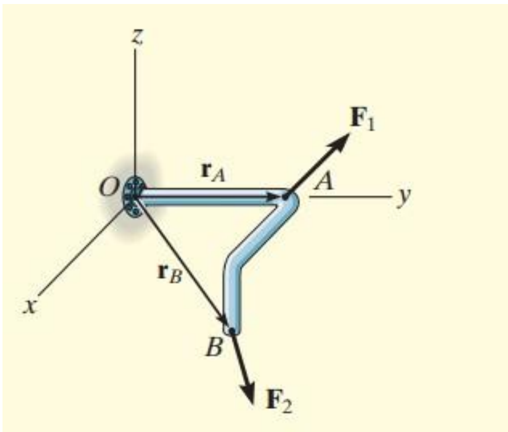


EXERCISE 10

Two forces act on the rod shown in Fig.8.
Determine the resultant moment they create about the flang at O. Express the result as a Cartesian vector.

$$\vec{F_1} = -6\vec{\imath} + 4\vec{\jmath} + 2\vec{k},$$
$$\vec{F_2} = 8\vec{\imath} + 4\vec{\jmath} - 3\vec{k}$$
$$\vec{r_A} = 4\vec{\jmath}$$
$$\vec{r_B} = 4\vec{\imath} + 5\vec{\jmath} - 2\vec{k}$$

Figure.8



EXERCISE 1

$$\vec{A}\cdot\vec{B} = |A||B|\cos\theta = -6$$
$$\vec{A}\times\vec{B} = |A||B|\sin\theta\,\hat{n} = 9\hat{n}$$
$$\tan\theta = \frac{9}{-6} = -\frac{3}{2} = -1.5$$
$$\theta = \tan^{-1}(-1.5) = -56.31^\circ$$

$$(i+j+k)\cdot(\hat{i}-\hat{j}-\hat{k}) = 1+0+0+0-1+0+0-1$$
$$= -1.$$
$$(\hat{i}+\hat{j}+\hat{k})\cdot(\hat{i}+\hat{j}+\hat{k}) = \hat{i}^2+\hat{j}^2+\hat{k}^2 = 3$$
$$|\hat{i}+\hat{j}+\hat{k}| = \sqrt{3}$$
$$(\hat{i}-\hat{j}-\hat{k})\cdot(i-j-k) = 3 \quad \therefore |i-j-k| = \sqrt{3}$$
$$\therefore (\hat{i}+\hat{j}+\hat{k})\cdot(\hat{i}-\hat{j}-\hat{k}) = |\sqrt{3}||\sqrt{3}|\cos\theta = -1$$
$$\theta = \cos^{-1}-\frac{1}{3} = 109.4^\circ$$

EXERCISE 2

Solution (*a*) We construct to scale the parallelogram shown in Fig. *a* for adding $\mathbf{V}_1$ and $\mathbf{V}_2$. Using the law of cosines, we have

$$S^2 = 3^2 + 4^2 - 2(3)(4)\cos 105^\circ$$
$$S = 5.59 \text{ units} \qquad \textit{Ans.}$$

(*b*) Using the law of sines for the lower triangle, we have ①

$$\frac{\sin 105^\circ}{5.59} = \frac{\sin(\alpha + 30^\circ)}{4}$$
$$\sin(\alpha + 30^\circ) = 0.692$$
$$(\alpha + 30^\circ) = 43.8^\circ \qquad \alpha = 13.76^\circ \qquad \textit{Ans.}$$

(*c*) With knowledge of both S and α, we can write the vector $\mathbf{S}$ as

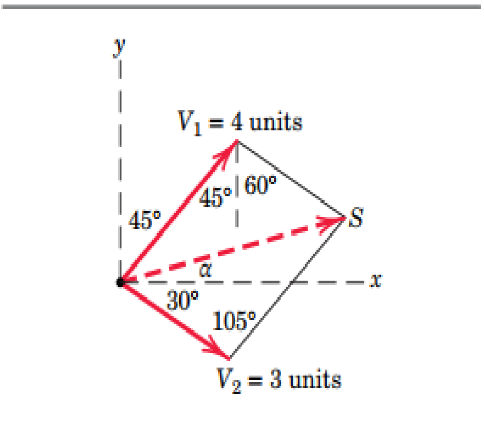
$$\mathbf{S} = S[\mathbf{i}\cos\alpha + \mathbf{j}\sin\alpha]$$
$$= 5.59[\mathbf{i}\cos 13.76^\circ + \mathbf{j}\sin 13.76^\circ] = 5.43\mathbf{i} + 1.328\mathbf{j} \text{ units} \qquad \textit{Ans.}$$

Then $$\mathbf{n} = \frac{\mathbf{S}}{S} = \frac{5.43\mathbf{i} + 1.328\mathbf{j}}{5.59} = 0.971\mathbf{i} + 0.238\mathbf{j} \quad ② \qquad \textit{Ans.}$$

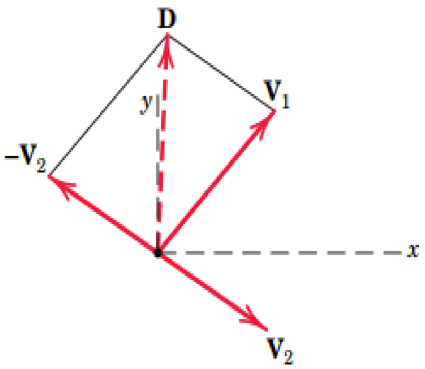
(*d*) The vector difference $\mathbf{D}$ is

$$\mathbf{D} = \mathbf{V}_1 - \mathbf{V}_2 = 4(\mathbf{i}\cos 45^\circ + \mathbf{j}\sin 45^\circ) - 3(\mathbf{i}\cos 30^\circ - \mathbf{j}\sin 30^\circ)$$
$$= 0.230\mathbf{i} + 4.33\mathbf{j} \text{ units} \qquad \textit{Ans.}$$

The vector $\mathbf{D}$ is shown in Fig. *b* as $\mathbf{D} = \mathbf{V}_1 + (-\mathbf{V}_2)$.



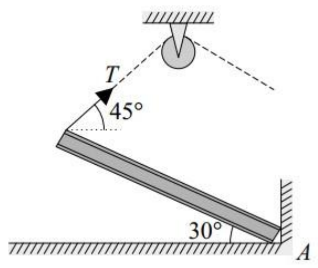
(*a*)



(*b*)

EXERCISE 3

We see that the inclination of the rope with respect to the beam is 45°+30°=75°. Hence, the component of the tension parallel and perpendicular to the axis of the beam.



$T_{\text{par}} = T \cos 75°$

$T_{\text{per}} = T \sin 75°$

EXERCISE 4

The calculations are summarized in the table below:

Force	*Magnitude*	*Inclination with respect to X-axis*	*X-component (N)*	*Y-component (N)*
$\vec{F}_1$	100 N	35°	100 cos 35° = 81.92	100 sin 35° = 57.36
$\vec{F}_2$	40 N	45°	40 cos 45° = 28.28	–40 sin 45° = –28.28
$\vec{F}_3$	50 N	35°	–50 cos 35° = –40.96	–50 sin 35° = –28.68