University Center of MILA 2025/2026
Faculty of sciences and technology Course:Maths3
Second year (GC, Hy, Ing GC)

Series of exercises N^o01
Simple and multiple integrals

Exercise1 Evaluate the following integrals
$I_1 = \int \frac{x}{(1+4x^2)^3} dx, \quad I_2 = \int \frac{1}{x^2-5x+6} dx, \quad I_3 = \int \frac{dx}{(x-1)^2(x+3)}$
$I_4 = \int \frac{1}{4x^2-4x+5} dx, \quad I_5 = \int \frac{dx}{(x^2+5)(x+3)}, \quad I_6 = \int \frac{x^3-4}{x^2-x-2} dx.$

Exercise2 Evaluate the following integrals
$J_1 = \int x^2 sinx dx, \quad J_2 = \int_2^4 x sin\left(x^2\right) dx, \quad J_3 = \int cosx e^x dx$
$J_4 = \int \sqrt{x^2+6x-5} dx, \quad J_5 = \int \frac{sinx}{\sqrt[3]{cos^4 x}} dx, \quad J_6 = \int \frac{coshx-1}{coshx+1} e^x dx.$
Exercise3 Let D be a domain

$$D = \left\{(x,y) \in \mathbb{R}^2; x \geq 0; y \geq 0, x+y \leq 1\right\}$$

Calculate $\int\int_D f(x,y)\, dxdy$ in each case

1. $f(x,y) = x^2 + y^2$
2. $f(x,y) = xy(x+y)$

Exercise4 Let D be a domain

$$D = \left\{(x,y) \in \mathbb{R}^2; -1 \leq x \leq 1 \text{ and } x^2 \leq y \leq 4 - x^3\right\}$$

Find the area of D.
Exercise5 Calculate double integrals $\int\int_D f(x,y)\, dxdy$ over regions bounded by given curves.

1. $f(x,y) = x$ $D = \left\{(x,y) \in \mathbb{R}^2; y \geq 0; x-y+1 \geq 0; x+2y-4 \leq 0\right\}$
2. $f(x,y) = cos(xy)$ $D = \left\{(x,y) \in \mathbb{R}^2; 1 \leq x \leq 2; 0 \leq xy \leq \frac{\pi}{2}\right\}$
3. $f(x,y) = \frac{-1}{(x+y)^3}$ $D = \left\{(x,y) \in \mathbb{R}^2; 1 < x < 3; y > 2; x+y < 5\right\}$
4. $f(x,y) = xy\sqrt{x^2+2y^2}$ $D = \left\{(x,y) \in \mathbb{R}^2; x > 0; x^2+y^2 < 1\right\}$.

Exercise6 Calculate $\int\int_D \frac{xdxdy}{1+x^2+y^2}$ such that

$$D = \left\{(x,y) \in \mathbb{R}^2; x^2+y^2 \leq 1 \text{ et } x \geq 0\right\}.$$

Exercise7 Let

$$D = \left\{(x,y) \in \mathbb{R}^2; \frac{1}{2} \leq x^2+y^2 \leq 3 \text{ et } x \geq 0\right\}$$

1. Sketch the region of integration D
2. Calculate the following integral

$$\int\int_D \left(x^2+y^2\right) dxdy.$$

Exercise8 Let

$$D = \left\{(x,y) \in \mathbb{R}^2; x^2+y^2-2x \leq 0\right\}$$

1. Prove that D is a disc.
2. Calculate $\int\int_D \sqrt{x^2+y^2} dxdy$.

Exercise9 Calculate $\int\int\int_\Delta f(x,y,z)\, dxdydz$ such that
$f(x,y,z) = 1-2yz$ and $\Delta = \left\{(x,y,z) \in \mathbb{R}^3; x^2+y^2 \leq 1; 0 \leq z \leq 3\right\}$.
Exercise10 Calculate the volume of V

$$V = \left\{(x,y,z) \in \mathbb{R}^3; x = 4-y^2; x+z = 4; x = 0; z = 0\right\}.$$

Exercise11 By using the cylindrical coordonates, calculate the following triple integral

$$\int\int\int_D \left(x^2+y^2+z\right) dxdydz$$

such that Δ is the cylinder defined by

$$\Delta = \left\{(x,y,z) \in \mathbb{R}^3; x^2+y^2 \leq 9; -5 \leq z \leq 5\right\}.$$