

Chapter 2: Reminders on the fundamental laws of electricity

1. Steady-State Operation

In direct current (DC) steady-state operation, the current and voltage quantities remain constant over time.

1.1. Electric Dipole

An electric dipole is a single component or a set of components connected to two (02) terminals (see Figure 1).

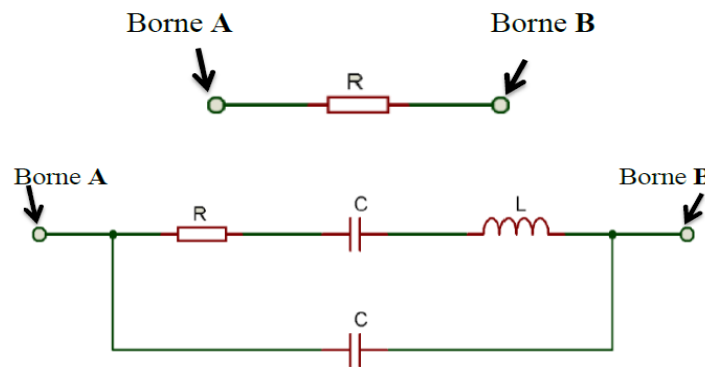


Figure 1: Electric Dipole.

We assign a direction for the current.

Receiver convention: the current i and the voltage u are oriented in opposite directions.

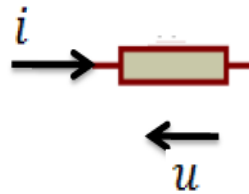


Figure 2: Receiver convention.

Generator convention: the current i and the voltage u are oriented in the same direction.

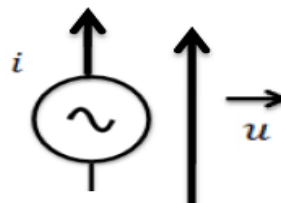


Figure 3: Generator convention.

In direct current (DC) conditions, dipoles are classified into two (02) categories:

- **Passive dipole:** A dipole that consumes electrical energy and does not contain any source of energy, (resistor, inductor, and light bulb).

- **Active dipole:** A dipole that contains an energy source, (battery or direct current (DC) electric motor).

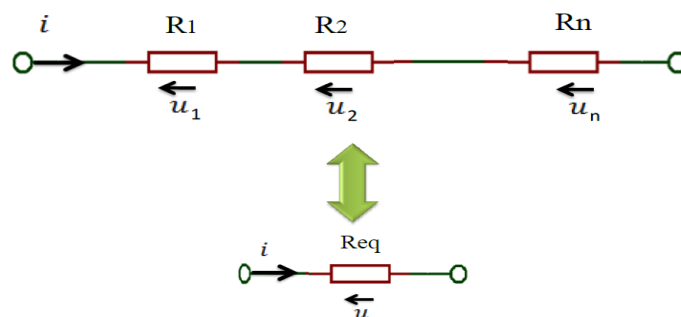
1.2. Combination of Dipoles

An association (or combination) of dipoles refers to the way in which several electrical components are connected together in a circuit.

Depending on how their terminals are connected, dipoles can be arranged in:

- Series connection (The current i is common to all dipoles. The voltage u is the sum of the voltages across each dipole).
- Parallel connection (The total current i is the sum of the currents across each dipole. The voltage u is common to all dipoles).
- Mixed (series-parallel) connection.

1.2.1. Series Connection of Resistors (R)

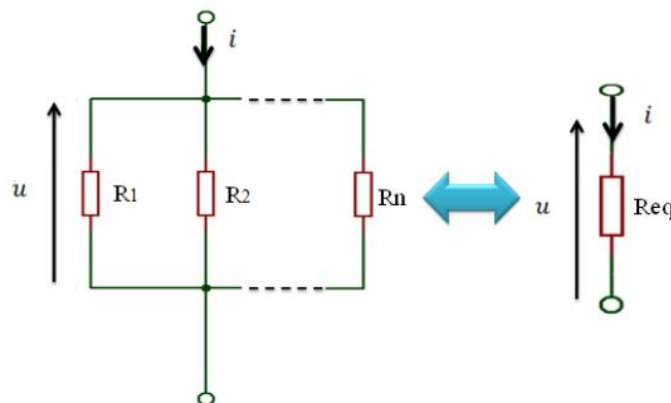


$$u = u_1 + u_2 + u_3 + \dots + u_n$$

$$= (R_1 + R_2 + R_3 + \dots + R_n)i = R_{eq} i$$

$$R_1 + R_2 + R_3 + \dots + R_n = \sum_1^n R_n$$

1.2.2. Parallel Connection of Resistors (R)



$$\begin{aligned}
 i &= i_1 + i_2 + i_3 + \dots + i_n \\
 &= \frac{u}{R_1} + \frac{u}{R_2} + \frac{u}{R_2} + \dots + \frac{u}{R_n} \\
 &= \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n} \right) u \\
 &= \frac{1}{R_{eq}} \cdot u
 \end{aligned}$$

$$Y_{eq} = \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n} = \sum_1^n \frac{1}{R_n}$$

- Case of two resistors connected in parallel:

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_1}$$

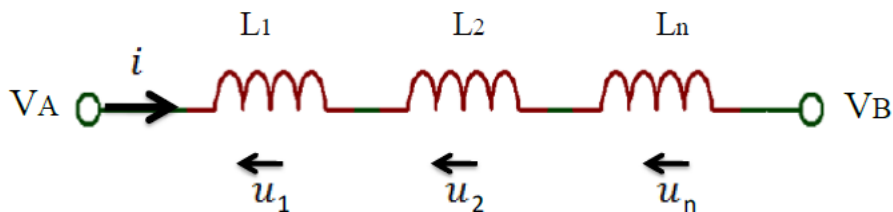
- Case of nnn identical resistors:

$$R_{eq} = \frac{R}{n}$$

1.2.3. Series Connection of Inductors (L)

Connecting inductors in series increases the total number of turns. The voltage across an inductor carrying a time-varying current is given by:

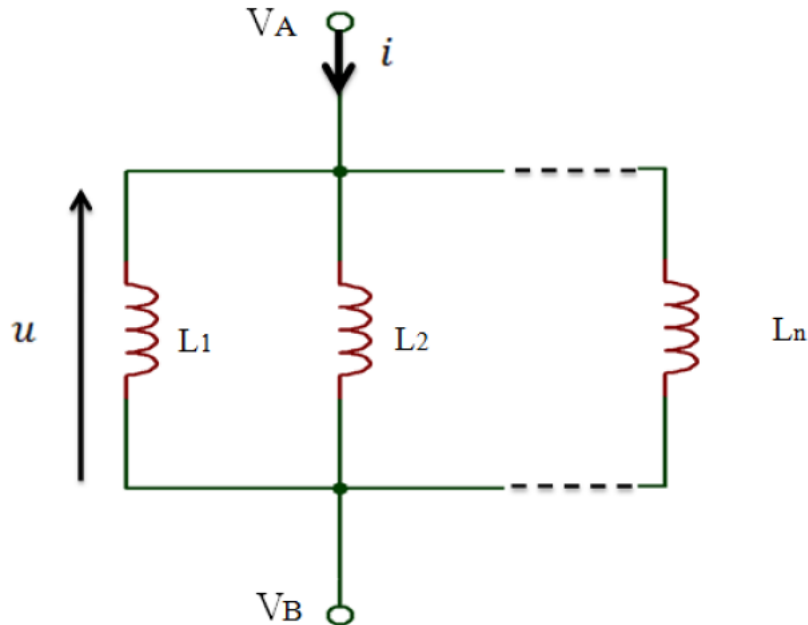
$$u_L = L \frac{di}{dt}$$



$$\begin{aligned}
 V_A - V_B &= u_1 + u_2 + u_3 + \dots + u_n \\
 &= L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt} + \dots + L_n \frac{di}{dt} \\
 &= (L_1 + L_2 + L_3 + \dots + L_n) \frac{di}{dt} = L_{eq} \frac{di}{dt}
 \end{aligned}$$

$$L_1 + L_2 + L_3 + \cdots + L_n = \sum_1^n L_n$$

1.2.4. Parallel Connection of Inductors (L)



$$i = i_1 + i_2 + i_3 + \cdots + i_n$$

$$\begin{aligned} \frac{di}{dt} &= \frac{di_1}{dt} + \frac{di_2}{dt} + \frac{di_3}{dt} + \cdots + \frac{di_n}{dt} \\ &= \frac{V_A - V_B}{L_1} + \frac{V_A - V_B}{L_2} + \frac{V_A - V_B}{L_3} + \cdots + \frac{V_A - V_B}{L_n} \\ &= (V_A - V_B) \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_n} \right) \end{aligned}$$

$$= (V_A - V_B) \frac{1}{L_{eq}}$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_n} = \sum_1^n \frac{1}{L_n}$$

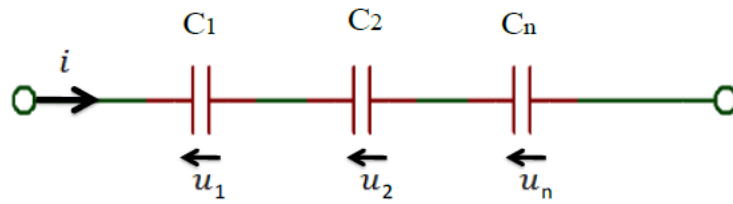
We therefore obtain the same formula as for resistors.

1.2.5. Series Connection of Capacitors (C)

A capacitor is characterized by its capacitance, denoted by C and expressed in farads (symbol **F**).

The voltage across a capacitor carrying a time-varying current is given by:

$$u_c = \frac{1}{c} \int i dt$$

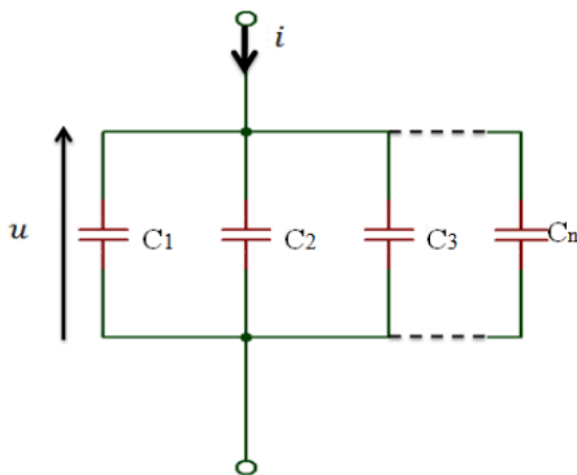


$$\begin{aligned} u &= u_1 + u_2 + u_3 + \dots + u_n \\ &= \frac{1}{c_1} \int i. dt + \frac{1}{c_2} \int i. dt + \frac{1}{c_3} \int i. dt + \dots + \frac{1}{c_n} \int i. dt \\ &= \left(\frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3} + \dots + \frac{1}{c_n} \right) \cdot \int i. dt \end{aligned}$$

$$\frac{1}{c_{eq}} = \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3} + \dots + \frac{1}{c_n} = \sum_1^n \frac{1}{c_n}$$

1.2.6. Parallel Connection of Capacitors (C)

$$\begin{aligned} i &= i_1 + i_2 + i_3 + \dots + i_n \\ &= C_1 \frac{du}{dt} + C_2 \frac{du}{dt} + C_3 \frac{du}{dt} + \dots + C_n \frac{du}{dt} \end{aligned}$$



$$\begin{aligned} &= (C_1 + C_2 + C_3 + \dots + C_n) \frac{du}{dt} \\ C_1 + C_2 + C_3 + \dots + C_n &= \sum_1^n C_n \end{aligned}$$