University center of Mila Abdelhafid Bousouf 2025/2026

Institute of Mathematics and Computer Science

probability and statistics

3^{rd} year computer science

Serie N 3

Note : questions marked $(*)$ left to the students

Exercise 1 :

Two events A and B are such that $\mathbb{P}(A) = 0.5$, $\mathbb{P}(B) = 0.3$, and $\mathbb{P}(A \cap B) = 0.1$. Calculate

1. $\mathbb{P}(A|B)$
2. $\mathbb{P}(B|A)$
3. $\mathbb{P}(A|A \cup B)$
4. $\mathbb{P}(A|A \cap B)$
5. $\mathbb{P}(A \cap B|A \cup B)$.

Exercise 2 : $(*)$

Let A, B and C be events. We pose $E_1 = A \cap B \cap C$ And $E_2 = A \cap (B \cup C)$.

1. Show that E_1 and E_2 are incompatible. 2. Determine the set $E_1 \cup E_2$.

3. We know that $\mathbb{P}(A) = 0,6$, $\mathbb{P}(B) = 0,4$, $\mathbb{P}(C) = 0,3$, $\mathbb{P}(B \bigcap C) = 0,1$, $\mathbb{P}(A \bigcap C) = 0,1$, $\mathbb{P}(A \cap B) = 0,2$ and $\mathbb{P}(A \cap B \cap C) = 0,05$.

Calculate $\mathbb{P}(E_1) + \mathbb{P}(E_2)$ and deduce $\mathbb{P}(E_1)$.

Exercise 3 :

Let A and B be two events such that $\mathbb{P}(A) = \frac{1}{5}$ and $\mathbb{P}(A \bigcup B) = \frac{1}{2}$.

1. Suppose A and B are incompatible. Calculate $\mathbb{P}(B)$.
2. Suppose A and B are independent. Calculate $\mathbb{P}(B)$.
3. Calculate $\mathbb{P}(B)$ assuming that event A can only occur if event B occurs.

Exercise 4 :

An urn contains r red balls and b blue balls, $r \geq 1$, $b \geq 3$. Three balls are selected, without replacement, from the urn. Using the notion of conditional probability to simplify the problem, find the probability of the sequence Blue, Red, Blue.

Exercise 5 : $(*)$

Students in a class are randomly selected one after the other to take an exam.

Calculate the probability p that we have alternately a boy and a girl knowing that :

1. The class is composed of 4 boys and 3 girls.
2. The class is made up of 3 boys and 3 girls.

Exercise 6 : $(*)$

In a high school 0.25 students fail math, 0.15 fail chemistry, and 0.1 fail both math and chemistry. A student is chosen at random.

1. If the student failed chemistry, what is the probability that he or she also failed mathematics ?

2. If the student failed mathematics, what is the probability that he also failed chemistry ?

3. What is the probability that he failed mathematics or chemistry ?

Exercise 7 : (Cours)

Three machines A, B and C produce respectively 0.6, 0.3 and 0.1 of the total number of parts manufactured in a factory. The percentages of defective results for these machines are 0.02, 0.03, and 0.04, respectively. We choose a part at random and find that it is defective.

Calculate the probability that this part was produced by machine C.

Exercise 8 :

We consider three urns U_1, U_2 and U_3. The first U_1 contains 9 white balls, 4 red and 2 black. The second contains 8 white balls, 5 red and 2 black. The third contains 6 white balls, 4 red and 5 black. One of three urns is chosen at random, then three balls are drawn simultaneously from this urn. 1. Calculate the probability of being : "1 white and 1 red and 1 black". 2. Suppose the balls drawn are 2 white and 1 red, calculate the probability that they come from urn U_1. This time we choose a ball from U_1, a ball from U_2 and a ball fromU_3. 3. Calculate the probability of being : "1 white and 1 red and 1 black".

Exercise 9 : $(*)$

One bag contains 50 balls, including : 20 red balls and 30 black balls. On 15 red balls and 9 black balls, we mark "Won" and on the rest we mark "Lost". A ball is drawn at random. Calculate the probabilities of the following events using a weighted probability tree :

R : "The ball drawn is red." W : "The ball drawn is marked Won".

$R \cap W$: "The drawn ball is red and marked Won".

$\overline{R} \cap \overline{W}$: "The drawn ball is black and marked Lost".

Exercise 10 :

An urn U_1 contains a_1 red balls and a_2 black balls, another urn U_2 contains b_1 red balls and b_2 black balls. We draw a ball from U_1 and put it in U_2, we designate by E the event "the ball drawn from U_1 is red" and by F the event "the ball drawn from U_2 is red" and by $\overline{E}$ and $\overline{F}$ the opposite events.

1. Calculate the probabilities $\mathbb{P}(E)$, $\mathbb{P}(\overline{E})$, $\mathbb{P}(F|E)$, $\mathbb{P}(F|\overline{E})$.

2. Calculate the probabilities $\mathbb{P}(F)$, $\mathbb{P}(E|F)$, $\mathbb{P}(\overline{E}|F)$.

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