

Serie N 1 + 2

Note : questions marked (*) left to the students

Exercise 1 :

A keypad allows you to enter a building code using a letter followed by a 3-digit number, whether or not they are distinct.

1. How many different codes can be formed ?
2. How many codes are there without the digit 1 ?
3. How many codes are there containing at least one digit 1 ?
4. How many codes are there containing distinct digits ?

Exercise 2 :

We take 3 bulbs out of 15 simultaneously (at the same time), 5 of which are defective.

1. How many different ways can this draw be made ?
2. How many different ways can at least one defective bulb be obtained ?

Exercise 3 : (*)

A coat rack has 5 coat hangers in a row. How many distinct layouts (dispositions) and without stacking coats can be hung on it :

1. three coats ?
2. five coats ?
3. six coats ?

Exercise 4 : (*)

Four mathematicians and two physicists sit on a six-seater bench.

1. How many layouts are possible ?
2. Same question if mathematicians are on one side and physicists on the other ?
3. Same question if each physicist sits between two mathematicians ?
4. Same question if the physicists want to stay next to each other ?

Exercise 5 :

Let E be a set with n elements.

1. What is the number of parts of E with p elements ? ($1 \leq p \leq n$).
2. Deduce the cardinality of $\mathcal{P}(E)$
3. Let $a \in E$. Determine the number of subsets of E with cardinal p :
-which contain a ;

-which do not contain a.

Deduce that : $C_n^p = C_{n-1}^{p-1} + C_{n-1}^p$.

Exercise 6 :

Describe the sample space and all 16 events for a trial in which two coins are thrown

Exercise 7 :

A bag contains fifteen balls distinguishable only by their colours ; ten are blue and five are red. I reach into the bag with both hands and pull out two balls (one with each hand) and record their colours.

1. What is the random phenomenon ?
2. What is the sample space ?
3. Express the event that the ball in my left hand is red as a subset of the sample space.

Exercise 8 :

M sweets are of varying colours and the different colours occur in different proportions. The table below gives the probability that a randomly chosen M has each colour, but the value for tan candies is missing.

Colour	Brown	Red	Yellow	Green	Orange	Tan
Probability	0.3	0.2	0.2	0.1	0.1	?

1. What value must the missing probability be ?
2. You draw an M at random from a packet. What is the probability of each of the following events ?
 - i. You get a brown one or a red one.
 - ii. You don't get a yellow one.
 - iii. You don't get either an orange one or a tan one.
 - iv. You get one that is brown or red or yellow or green or orange or tan.

Exercise 9 :

An urn contains 10 balls, 4 of which are white and 6 are black. We draw 4 balls from it.

1. What is the number of possible draws ?
2. How many ways can you draw :
 - 4 white balls.
 - 2 white balls and 2 black balls.

Exercise 10 :

Show that

$$1. \sum_{i=1}^n C_n^i = 2^n$$

$$2. \sum_{i=1}^n (-1)^i C_n^i = 0$$

Exercise 11 :

Let A, B be events. Show that

$$P(A) = P(A \cap B) + P(A \cap B^c).$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Exercise 12 :

A fair coin is tossed, and a fair die is thrown at the same time (simultaneously). Write down sample space for this experiment.

Let A be the event that a head is tossed, and B be the event that an odd number is thrown. Directly from the sample space, calculate $\mathbb{P}(A \cap B)$ and $P(A \cup B)$.

Exercise 13 :

An urn U contains 6 white balls, 4 red balls and 5 green balls. We consider the following three methods :

Method 1 : one after the other with the ball drawn being returned.

Method 2 : one after the other without replacing the drawn ball.

Méthode 3 : simultanée.

Calculate with these methods the probability of obtaining :

1. Three red balls (Drawing three balls).
2. Two white balls and two green balls (Drawing four balls).

Exercise 14 :

We roll a die until the first appearance of the (6) Six. Note

$$A_i = \{\text{The } i\text{-th throw displays the number 6}\}, i \in \mathbb{N}^*.$$

1. Define with expressions each of the following events :

$$-E_1 = \bar{A}_1 \cap \bar{A}_2 \cap A_3.$$

$$-E_2 = \bigcap_{i=1}^{+\infty} \bar{A}_i.$$

1. Write using the events A_i and $\bar{A}_i$ the events :

$$-E_1 = \{\text{The first appearance of the 6 occurs in the sixth roll}\}$$

$$-E_2 = \{6 \text{ does not appear in the first 3 throws}\}$$

$$-E_3 = \{\text{We get at least one 6 in the first 10 throws}\}.$$

Exercise 15 :

An urn contains 2 white balls and 3 black balls. Two balls are drawn successively without replacement. Calculate and compare the probabilities of the following two events :

A : « Draw two balls of the same color ».

B : « Shoot two balls of different colors ».

