Abdelhafid Boussouf University Center, Mila
Institute of Mathematics and Computer Science
First year of Computer Science License 2025/2026

Algebra I, Worksheet 1

Exercise n°1

I. Express symbolically, using quantifiers, the following assertions :
1. The square of every real number is a positive real number.
2. There is a positive integer whose square is equal to itself.
3. There are two different real numbers that have the same square.
4. Every real number has a cube root.
5. For every real number x, there exists at least one natural number greater than or equal to x.

II. Let S be the set of subscribers in a library, and B the set of books. Consider the assertion P : "subscriber s likes book b," denoted by sLb. Translate each of the following symbolic assertions into an English sentence :

$$(a)\forall s \in S, \exists b \in B : sLb.$$
$$(b)\forall b \in B, \exists s \in S : sLb.$$
$$(c)\exists b \in B, \forall s \in S : sLb.$$
$$(d)\exists s \in S, \forall b \in B : sLb.$$

Exercise n°2 : Translate each of the following symbolic assertions into English sentences and indicate the truth value of each assertion. Then, write the negations of these assertions :

$$(a)\exists x \in \mathbb{R}, \forall y \in \mathbb{R} : x + y > 0.$$
$$(b)\forall x \in \mathbb{R}, \exists y \in \mathbb{R} : x + y > 0.$$
$$(c)\forall x \in \mathbb{R}, \forall y \in \mathbb{R} : x + y > 0.$$
$$(d)\exists x \in \mathbb{R}, \forall y \in \mathbb{R} : y^2 > x.$$

Exercise n°3 : Let P, Q, R, and S be four assertions. Consider the following assertion

$$V : (P \wedge Q) \Longrightarrow (R \vee S)$$

1. Find the negation $not(V)$ of V.
2. Write the contrapositive of V.

Exercise n°4 : Using a truth table, prove the following logical equivalences for any three assertions P, Q and R.

(a). Distributive Law
$$P \wedge (Q \vee R) \Longleftrightarrow (P \wedge Q) \vee (P \wedge R)$$
(b). Distributive Law
$$P \vee (Q \wedge R) \Longleftrightarrow (P \vee Q) \wedge (P \vee R)$$
(c). De Morgan's Laws.
$$\overline{(P \wedge Q)} \Longleftrightarrow \overline{P} \vee \overline{Q}\,, \overline{(P \vee Q)} \Longleftrightarrow \overline{P} \wedge \overline{Q}.$$

Exercise n°5 :Negate the following assertions for a function $f : \mathbb{R} \longrightarrow \mathbb{R}$.

$(a)\forall x \in \mathbb{R} : f(x) \neq 0.$
$(b)\forall M > 0, \exists A > 0, \forall x \geq A : f(x) > M.$
$(c)\forall x \in \mathbb{R} : f(x) > 0 \Longrightarrow x \leq 0.$
$(d)\forall \varepsilon > 0, \exists \delta > 0, \forall x, y \in I : \left|x - y\right| \leq \delta \Longrightarrow \left|f(x) - f(y)\right| \leq \varepsilon.$

Exercise n°6 : Let $n \in \mathbb{Z}$ be an integer. Prove the following assertion using a proof by contrapositive : " If n^2 is an even integer, then n is even".

Exercise n°7 :Show, using a proof by contradiction, that the square root of 2 is an irrational number ; that is, prove that $\sqrt{2} \notin \mathbb{Q}$.