Chapter III: Gravimetric prospecting

II.2 Gravimetric prospecting

Geophysical prospecting using the gravimetric method focuses on variations in gravity on the ground, in boreholes, on the sea or in the air (by plane or helicopter), as these variations in gravity make it possible to map density variations between rock formations. However, spatial variations in gravity are largely explained by the earth's shape, topography and rotation. Theoretical correction calculations can cancel out these effects. Interpretation of these data can then be used to deduce the nature and distribution of these rock formations.

II.2.1 The gravitational force

To calculate an approximate expression for gravity on earth. The basic law is, of course, Newton's law of universal gravitation: The force of mutual attraction F between two masses mA and m_B whose characteristic dimensions are small compared to the distance r between them is:

$$\boldsymbol{F} = \frac{\boldsymbol{G}\,\mathrm{mA}\,\mathrm{mB}}{\boldsymbol{r}^2}$$

G: the universal gravitational constant. Its value is $G=6.673*10^{-11} m^3.kg^{-1}.s^{-2}$
r: the distance separating masses mA and m_B

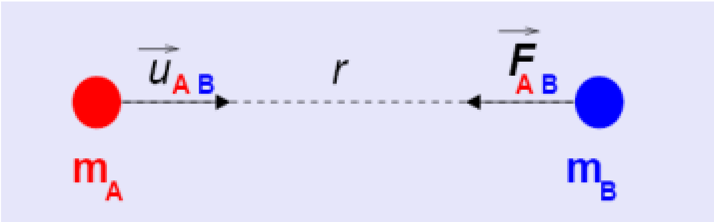


FIG. 1.1: Two point masses mA and mB at a distance r from each other.

II.2.2 Units

The average value of gravity at the earth's surface is of the order of $9.81 m.s^{-2}$. As we shall see, the spatial or temporal variations of interest to us vary between 10^{-8} and 10^{-3} $m.s^{-2}$, so it is not very convenient to use the unit of the international system ($m.s^{-2}$). Geophysicists use a more practical unit, the milligal or migrogal.
The abbreviation for the milligal is mGal, that for the microgal is µGal. So we end up with:

$$1\mathrm{mGal} = 10^{-5} m.s^{-2} \text{ et } 1\ \mu\mathrm{Gal} = 10^{-8}\ m.s^{-2}$$

The average value of gravity at the earth's surface is therefore 981000 mGal.

II.2.3 Geoid and reference ellipsoid

On Earth, the mean surface of the oceans merges with an equipotential surface of the gravity field. This is due to the properties of the fluids in equilibrium. This equipotential surface is called the geoid, and it defines the shape of the Earth. The shape of the geoid depends on the distribution of masses within the globe. The reference ellipsoid is defined by several elements:

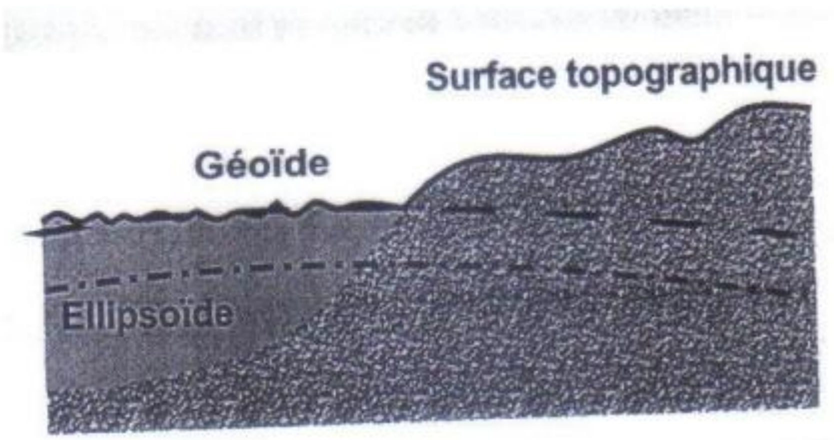
- Its equatorial radius noted a ;

c being the polar radius;

- Its flatness noted f

$$f = \frac{a-c}{c}$$

The Earth is not homogeneous, and the geoid will show undulations in relation to the ellipsoid, reflecting density heterogeneities.



Representation of the geoid and reference ellipsoid

$$\mathbf{g = g_0\ (1+k_1 \sin^2\varphi - k_2 \sin^2 2\ \varphi)}$$

Where:

$\mathbf{g_0}$ is gravity at the equator

$\mathbf{K_1}$ and $\mathbf{K_2}$ are constants that depend on the shape and rotation speed of the Earth.

II.2.3 Gravimetric corrections and anomalies

To study a set of measurements, it is necessary to bring them back to the same reference level (geoid) so that they can be compared with each other. The variations in gravity (or anomaly) observed on the corrected map are essentially due to localized heterogeneities in the Earth's crust, but also in the upper mantle. The corrections we need to make to the raw gravity measurements, in addition to the drift and lunisolar corrections, are as follows:

II.2.3.1 Drift correction

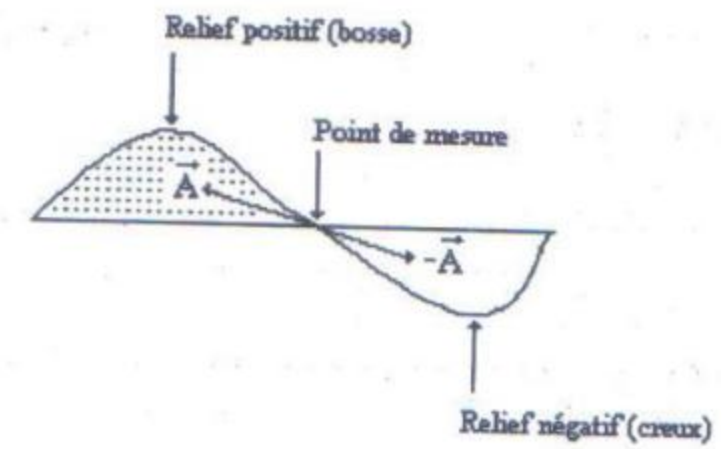
Since gravimeter drift is the variation in measurement due to the influence of pressure and temperature changes, it can be quantified as the difference between two successive measurements taken at the same point. Drift correction eliminates the influence of pressure and temperature variations on the gravimeter measurement.

II.2.3.2 Lunar-solar correction

This can be done easily, since we know in advance the position of the moon and sun in relation to any point on the earth's surface. In practice, the tables produced by the EAEG (European Association of Exploration Geophysics) allow us to make lunisolar corrections.

II.2.3.3 Terrain correction (T)

Tends to correct the influence of topographical masses located in the vicinity of the station. Correcting the influence of landforms above and below the horizontal plane passing through the measurement point allows us to consider a homogeneous terrain of constant thickness. Relief correction is always positive. Excess mass above the horizontal plane passing through the station, and deficit mass below it, have a negative effect on the measurement.

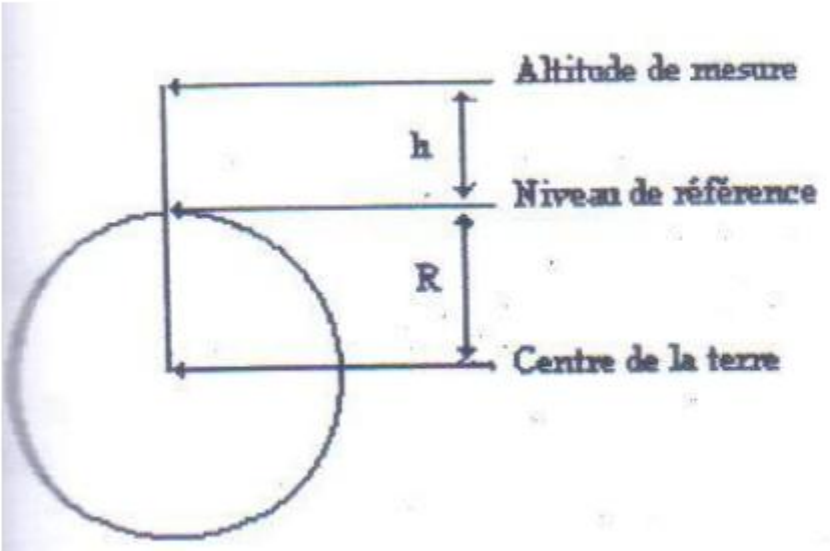


Attraction of a relief near a measuring point

II.2.3.4 Open-air correction (Faye, Ar)

This eliminates the influence of the station's altitude (h), without taking into account the masses located between it and the reference level. It depends solely on this altitude. It is given by the relation: A_F=0.3086 h

Where: [A_F] = mgals and [h] = meter

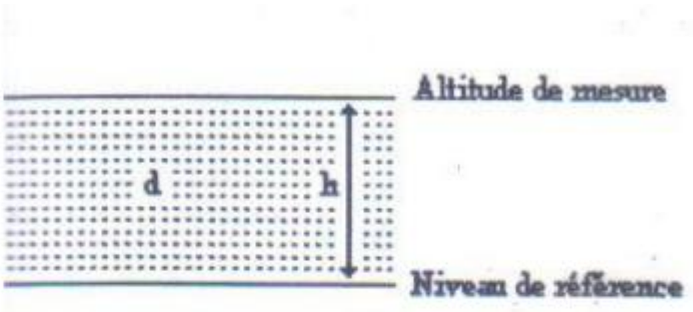


Correction à l'air libre

II.2.3.5 Plateau correction (P)

This is used to correct the effect of the flat slice (terrain) located between the horizontal plane passing through the station and the reference level. It depends on the altitude (h) of the station and the density (d) of the corrected terrain. It is given by the relationship: P=0.0419 d h

where: [P] = mgals, [d] = g/cm^3 and [h] = meters.



Flat slice of density (d) and thickness (h)

II.2.3.6 Normal or latitude correction (gth)

This corrects the measurement for the influence of the earth's flattening. It depends only on the latitude of the station. It is given by the following formula:

$$g_{th} = 978031.85\,(1+0.0053024\sin^2\varphi - 0.0000059\,\sin^2 2\,\varphi)$$

Where: [φ] = degree, represents the latitude of the station

978031.85 mgals represents the value at the equator

The earth's flattening coefficient corresponding to this formula is:

E = 1/298.257

II.2.3.7 Calculating the Bouguer anomaly

By definition, the simple anomaly is the difference between the value measured at a given altitude h and the theoretical value corrected for free-air and plateau correction. This is:

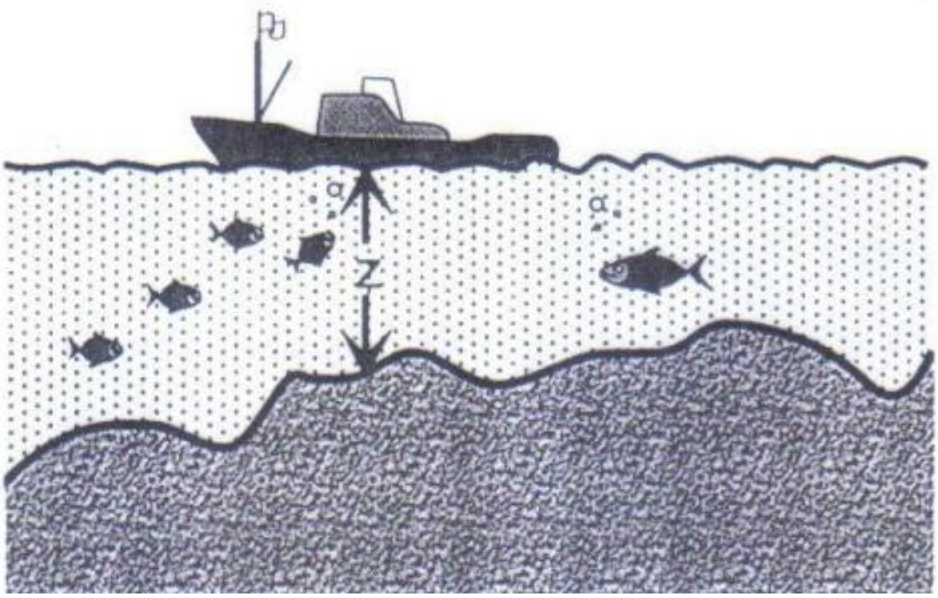
$$A_{BS} = g_m - (g_0 - 0.3086\ h + 0.0419\rho\ h)$$

The complete Bouguer anomaly will be the difference between the value measured at a given altitude h and the theoretical value corrected for open-air correction, plateau correction and terrain corrections. This is :

$$A_{BS} = g_m - (g_0 - 0.3086\ h + 0.0419\rho\ h - \rho\ T)$$

ρ T being field corrections (always positive)

It is this anomaly that needs to be calculated and analyzed to highlight mass heterogeneities beneath the topographic surface. Let's now consider the case of measurements taken at sea.



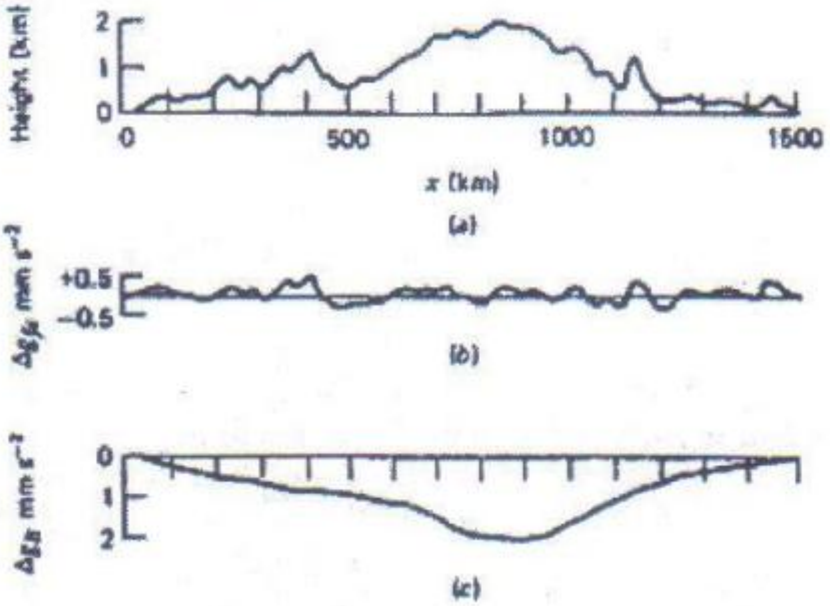
Gravimetric measurements made at the sea surface.

If we call ρe and ρr the densities of seawater and rock respectively (z positive downwards):

$$A_{BS} = gm - g0 + 0.0419\ (\rho_r - \rho_e)$$

II.2.3.8 Gravity anomalies and topography

Figure 1.7 shows curves of topography (a), free-air gravity anomaly (b), and Bouguer anomaly (c), on the same terrain. It is clear that the free-air anomaly is correlated with and dominated by the short-wavelength topography. The Bouguer anomaly, on the other hand, is anti-correlated with long-wave topography. This highlights the phenomenon of compensation: at depth, there is a density anomaly opposite to that at the surface. This deep anomaly acts as a float, keeping the excess mass generated by the topography at the surface.



Gravity anomalies and topography

Figure 1.8 clearly shows the presence of anomalously warm (sparsely populated) mantle at depth, visible in both the Bouguer anomaly and the seismic profile.

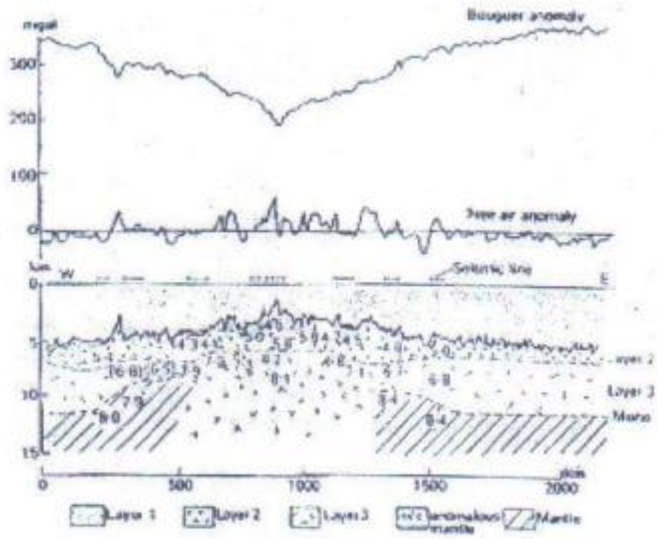


FIG. 1.9: Open-air anomaly and Bouguer anomaly generated on a terrain whose density anomalies are given by a seismic profile

To summarize

- Open-air anomaly: mainly short-wave signal
- Topography $\lambda > 100$ km no correlation with free-air anomaly.
- Topography $\lambda < 250$ km good correlation with free-air anomaly, dominated by mass excess due to peaks.

- Bouguer anomalies: signal at all wavelengths.
- Bouguer anomaly $\lambda > 1000$ km anti-correlated with long-wavelength topography.
- Bouguer anomaly $\lambda < 250$ km no correlation with topography. Allows study of structures buried beneath sediments.

II.2.4 Choice of density for Bouguer anomaly calculations

The marker of the gravimetric method is the density, which is involved in the calculation of certain corrections and in the quantitative interpretation. In all cases, the value of this parameter is chosen, and this choice is crucial to the quality and accuracy of the gravimetric results. There are several methods for selecting the density:

- Outcrop measurements.
- Indirect measurements using a gravimeter.
- Nettelton profile method.
- Triplet method.

Here are some density values for rocks

Sel	**Séd. non cons.**	**Arg. et marnes**	**Grés**	**Calcaire**	**Granite**	**Basalte**
2.2	1.8 à 2.2	2 à 2.3	2 à 2.5	2.4 à 2.8	2.6	3

Et de quelques minerais.

Magnétite	**Pyrite**	**Galène**	**Charbon**
5.1	5	7.5	1.2 à 1.8

II.2.5 Isostasy

Examination of large-scale Bouguer anomaly maps (e.g. for France: figure 1.10) shows that mountain ranges are systematically associated with negative anomalies. In other words, the excess mass due to topography is “compensated” by a mass defect at depth. In fact, Pierre Bouguer was the first to point out that the attraction of mountains is weaker than can be calculated from their apparent mass alone. In his book La figure de la Terre (The Figure of the Earth), published in 1749, he wrote about an analysis of measurements taken in the vicinity of Mount Chimborazo:

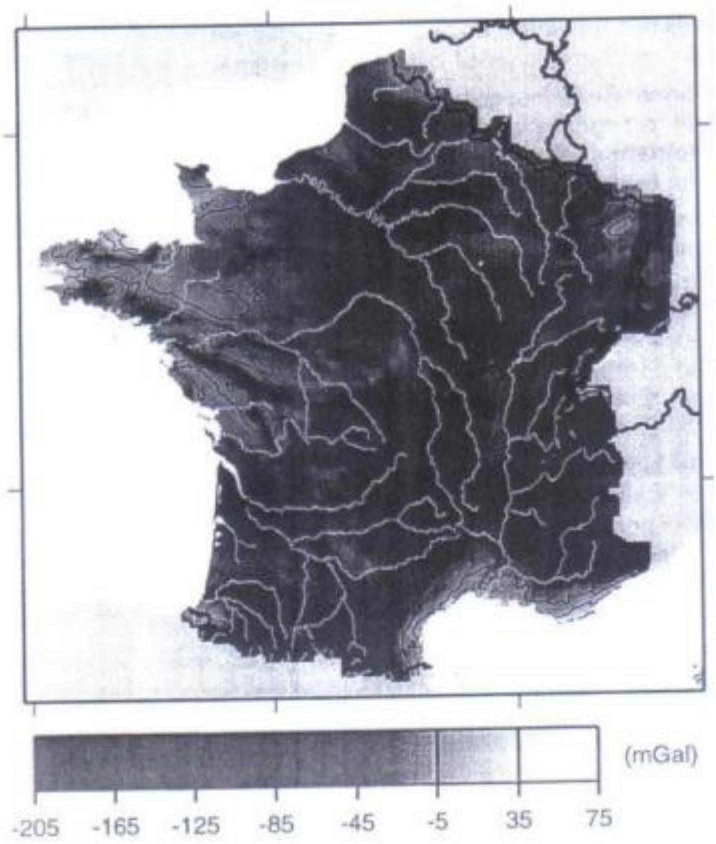


FIG. 1.10: Bouguer anomalies over France

This idea, later taken up by HaYford in 1910, led to a model of the outer part of the globe in which densities vary laterally in the columns according to their elevation relative to the geoid. The higher the column, the less dense it is, and vice versa. These density variations occur up to a certain depth called the compensation depth, of the order of 100km in Pratt's model.

On January 25, 1855, again before the Royal Society of England, astronomer Airy put forward another theory. For him, mountains are so heavy that the Earth's crust cannot support them, and mountains of constant density float in the underlying medium. According to Archimedes' principle, the higher the mountains, the greater their root. In this model, reliefs are compensated by a crustal root and depressions by an anti-root.

We can easily calculate the height h' of the root as a function of altitude h in this model. Equalizing the weights of the various columns, we find :

$$\boldsymbol{h'} = \frac{\rho_c}{\rho_{m-}\rho_c} \text{ à terre et } \boldsymbol{h'}_0 = \frac{\rho_c - \rho_e}{\rho_{m-}\rho_c} \boldsymbol{h_0} \text{ en mer.}$$

With ρc, ρm and ρe the densities of crust, mantle and seawater respectively.

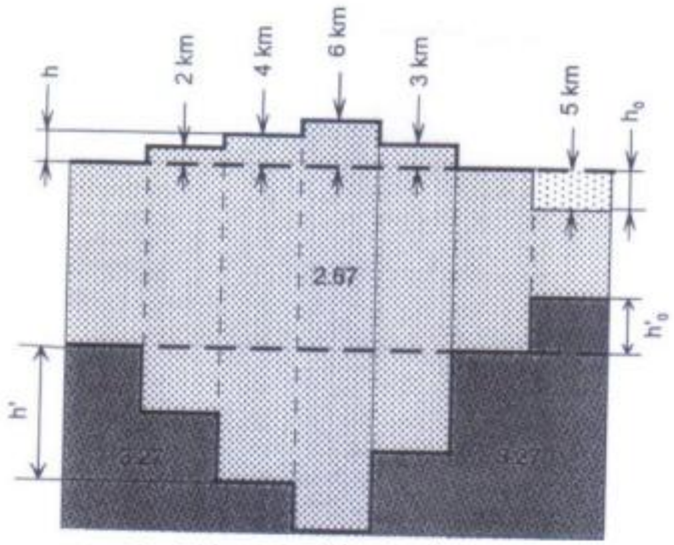


FIG 1.12 : Modèle d'Airy

II.2.5 Interpretation of results

The aim of interpreting gravity anomalies is to find the distribution of sources: density contrasts and geometries that create the anomalies observed on the surface. It can be shown theoretically that gravity data alone are not sufficient to uniquely determine a mass distribution at depth. Figure 1.13 illustrates this point: several very different geometries can create the same gravity anomalies.

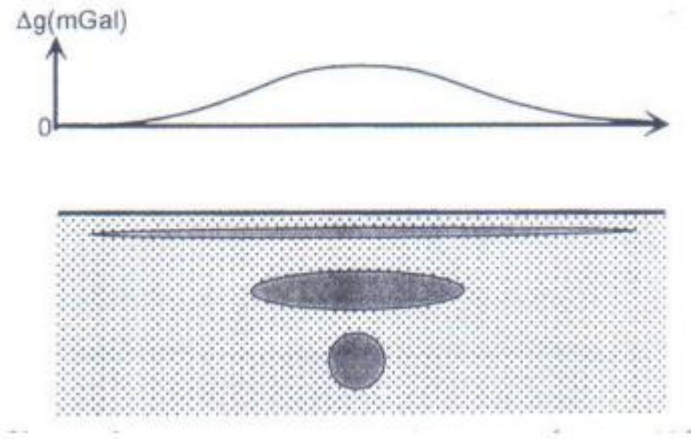


FIG.1.13: Each of the three bodies shown in this figure creates the same surface effects.

This is known as the non-uniqueness of inversion. This non-uniqueness is also true for other geophysical methods, such as geomagnetism. Consequently, interpretations of gravimetric measurements, in terms of mass distribution at depth, can only be unique with the contribution of other geological, geophysical, drilling and other parameters. It should be noted, however, that while a given anomaly theoretically corresponds to an infinite number of models, the number of reasonable models from a geological and geophysical point of view is relatively limited.

Examination of an anomaly map shows that it is possible to determine several first-order classes of anomalies based on their shape. There are linear anomalies and circular anomalies.

II.2.5.1 Effect of simple structures

Gravimetric effects created by simple geometric structures can be obtained analytically by direct calculation. Let's focus here on the vertical component, which can be measured with gravimeters.

Sphere

The vertical component of the attraction created by a sphere of mass M, assumed to be concentrated at its center and located at a depth h - this corresponds to the effect of a source point of mass M at depth h - at a point x on a horizontal axis passing vertically through the center of the sphere is:

$$g(x) = \frac{GMh}{(x^2 + h^2)^{3/2}}$$

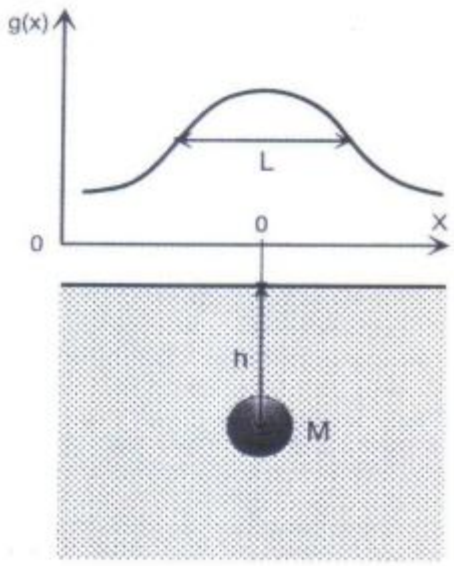


FIG. 1.14: Gravimetric effect created by a spherical source M whose mass is concentrated in its center at a depth h.

We can see that the maximum of this effect, at x =0, is proportional to the inverse of the square of the source depth h, and that the width at half-height of the signal is proportional to h. Indeed, the width at half-height is defined by L = 2x, with x such that g(x) = 0.5 gmax. Let:

$$\frac{GMh}{(x^2+h^2)^{\frac{3}{2}}} = 0.5\,\frac{GM}{h^2}\text{, donc L = 1.53 h, ou h = 0.65 L.}$$

The deeper the source, the lower the amplitude of the associated signal and the longer its wavelength.

Applications:

- Geological mapping (tectonics)
- Detection of metal deposits
- Archaeological research
- Public works (cavity detection)