**Chapter 04: Conductor**

**and superconductor materials**

Part 1: Conductor materials

**4.1.1. Introduction**

 An electrical conductor is a medium in which electrical charges are free to move. These charges are electrons or ions. Metals, electrolytes, and plasmas (ionized gases) are conductive media.

**4.1.2. Conductor in a static electric field**

 Let us place a piece of metal in a static electric field. Inside the metal, the conduction electrons, which are free to move throughout the volume, are subjected to a force that sets them in motion. The electrons are stopped when they reach the walls of the metal and accumulate there. Their accumulation creates an electric field that adds to the external field. After this transient phase, a state of equilibrium is reached.

 At equilibrium, the electrons inside the conductor are immobile. This means that the electric field to which they are subjected is zero. The electric field is zero inside a conducting medium at equilibrium. We immediately deduce from Gauss's theorem that the total charge density is zero: the volumetric charge density is zero inside a conducting medium. In a metal for example, the negative charge density due to the electrons therefore exactly compensates for the positive charge density due to the nuclei.

 Since the electric field outside the conductor is not zero, there is a discontinuity in the electric field at the surface of the conductor. Some of the charges have accumulated on the surface. The field created by this surface charge density inside the conductor exactly compensates for the external electric field.

When the external electric field is changed, the charges move so that the electric field remains zero inside. If the change is slow, the electric currents are surface currents.

**4.1.3. Conductors in a changing electric field**

 When the electric field changes, the equilibrium cannot be instantaneous because the electric charges must start moving. Two phenomena then intervene: the inertia of the charges causes a delay in the response, the collisions of the carriers cause dissipation. Before studying real conductors, we will consider a model situation where these two phenomena are absent.

In this idealized situation, we will consider that there is no dissipation and that the response is instantaneous. We will then speak of a perfect conductor or an ideal conductor.

**4.1.4. From the perfect driver to real drivers**

 The perfect conductor is an idealization of real conductors. The study of real conductors will make it possible to determine the parameter domains in which they can be considered ideal. Superconducting media where dissipation is perfectly zero are also a very good example of what an ideal conductor can be (it should be noted, however, that only dissipation is absent from these media: the electrons retain their inertia).

**4.1.4. 1. The perfect driver**

 A perfect conductor behaves in dynamic mode in the same way as a conductor in static mode. For a perfect conductor, the internal electric field is zero:$\vec{E}\_{int} \vec{E}\_{int }\left(\vec{r }, t\right)=0$

We deduce from the Maxwell-Gauss equation that the volume charge density is zero:

$$ρ\_{int }\left(\vec{r }, t\right)= ε\_{0 }div \vec{E}\_{int }=0$$

 Therefore, only the surface charge density can be different from zero. The Maxwell-Faraday equation allows us to conclude that inside a perfect conductor the magnetic field cannot depend on time:

$$\frac{∂\vec{B }}{∂t }= - \vec{rot } \vec{E }=0 $$

 In a perfect conductor the magnetic field is necessarily static. Note that in superconductors, the magnetic field is zero (Meissner effect: when a conductor passes from the normal state to the superconducting state, the magnetic field lines are expelled so that the magnetic field becomes zero inside the superconductor).

We then deduce from the Maxwell-Ampère equation that electric currents are necessarily stationary, that is to say independent of time:

$$\vec{J }= \frac{1}{u\_{0}} \vec{rot}\vec{B} - ε\_{0} \frac{∂\vec{E}}{∂t}= \frac{1}{u\_{0}} \vec{rot}\vec{B}$$

The only currents that can depend on time are surface currents.

**4.1.4. 2. Reflection on a perfect conductor**

 What happens when an electromagnetic wave hits a perfect conductor? This wave sets the charges on the surface of the conductor in motion. Inside the conductor, the electric field and the magnetic field remain zero. The electromagnetic field emitted by the charges moving on the surface of the conductor exactly compensates for the incident field inside the conductor: the surface emits a wave of the same amplitude as the incident field and in phase opposition. If the surface is a plane, we deduce by symmetry that the field emitted by these charges moving towards the outside of the conductor is the symmetrical of the field it emits towards the inside. We find what we expect from a mirror, with the additional fact that the reflected field undergoes a phase shift of π relative to the incident field.

**4.1.5. Real Driver Models**

 The study of media is not a "principles" theory like electromagnetism in a vacuum. For electromagnetism in a vacuum, it is sufficient to take as postulates the four Maxwell equations, the expression of the Lorentz force and the fundamental relation of dynamics. Everything else is constructed from these equations and deduced by logical reasoning.

For media, we do not have a system of equations that could be considered as postulates. The most precise theories available are extremely complex and use quantum theory. Our goal here is rather to study large classes of generic behavior, particularly in limiting cases. For this, materials will be described on the one hand at the macroscopic level by “equations of state” (also called constitutive relations), i.e. coefficients such as electrical conductivity, permittivity, etc. We also have microscopic models that we call phenomenological because certain aspects are not deduced from first principles but added “by hand” so that the behavior obtained mimics as closely as possible the behavior observed in real materials. In addition to their predictive aspect, these models have the great interest of feeding physical intuition. However, we must remain vigilant and not necessarily take them literally. It should also be noted that while some of the justifications sometimes given for these models seem simplistic, there are very often very profound reasons for their effectiveness.

**4.1.5.1. The damped electron**

 In the proposed model, electrons are considered to be responsible for the conduction of the medium. A free electron of mass me and electric charge q= -e obeys the following evolution equation:$m\_{e} \frac{d\vec{v}}{dt}= \vec{F}\_{L}- T\_{\vec{v}}$

The first term, is the Lorentz force:$\vec{F}\_{L}$

$$\vec{F}\_{L}=q(\vec{E}+ \vec{v}\*\vec{B })$$

 In the following, when the electric field and the magnetic field both come from the same electromagnetic wave, we will generally neglect the term due to the magnetic field, which is lower than that of the electric field by a factor , which is very small as long as the speeds are not relativistic. Be careful, when we are in the presence of an electromagnetic wave and a static magnetic field, only the magnetic field coming from the wave can be neglected, because it alone is proportional to the electric field. The static field can lead to a force comparable to that of the electric field of the wave even if the speeds are not relativistic.$ \frac{v}{c} $

 The second term is a viscous friction force added for phenomenological reasons. It accounts for the dissipative mechanisms present in the medium.$T\_{\vec{v}}$

The friction coefficient cannot generally be calculated from first principles (Maxwell's equations, quantum mechanics, etc.); its value is generally obtained by relating it to the macroscopic parameters of the medium. In a plasma, friction is due to collisions of electrons with ions and with molecules that have remained neutral. In a metal, it is the interaction between electrons and mechanical vibrations of the crystal lattice.

In a static electric field the equation of evolution of the electron has the solution:$\vec{E}\_{0} $

$$\vec{v} \left(t\right)= \vec{v}\_{0} e^{^{t}/\_{τ}}+ \left(1- e^{^{t}/\_{τ}}\right)\frac{q}{T}\vec{E}\_{0}$$

Where is the velocity of the electron at the initial time t =0. The characteristic damping time is τ.$\vec{v}\_{0}$

$$τ= \frac{m\_{e}}{T}$$

the initial speed is damped while the speed of the electron tends towards a limiting speed:$\vec{v}\_{L} $

$$\vec{v}\_{L}= \frac{q}{T} \vec{E}\_{0}$$

**4.1.5.2. Electrical conductivity**

When the volume electron density is Ne, the steady-state current density is:$\vec{J}$

$$\vec{J}=q N\_{e} \vec{v}\_{L}= \frac{N\_{e} e^{2}}{T} \vec{E}\_{0}$$

This current density is proportional to the electric field: we thus find an ohmic behavior:

$$\vec{J}= σ\_{0} \vec{E}\_{0}$$

corresponding to a conductivity:$σ\_{0}σ\_{0}= \frac{N\_{e} e^{2}}{T}$

For a given medium, we can therefore reexpress the phenomenological friction coefficient Γ using fundamental constants or measured macroscopic quantities:

$$T= \frac{N\_{e} e^{2}}{σ\_{0}}$$

We also deduce the characteristic amortization time:

$$τ^{-1}= \frac{N\_{e}e^{2}}{σ\_{0}m\_{e}}$$

 If the electric field is no longer static but depends on time, as long as the characteristic time of evolution of the electric field is large compared to this damping time, the electrons are permanently at their limiting speed and the conductor is ohmic.

Part 2: Superconductor

**4.2.1. Introduction**

 The Thalys network is about to invade the whole of Europe. It allows you to reach any destination at the dizzying speed of 300 km/h. Did you say dizzying? ... For years, on the other side of the planet, the Japanese have been experimenting with a new railway technology, allowing them to reach speeds of over 600 km/h! These speeds are achieved thanks to the absence of contact between the train and the rails; the train literally flies above the track. The principle behind this technology and many others is called: SUPERCONDUCTIVITY

**4.2.2. Electricity: a river of electrons**

 The universe and its structure are governed by four forces: gravitation, electrostatic forces, and weak and strong interactions. These four forces all arose from a single force some time after the birth of the universe. Electrostatic forces, responsible for the structure of the objects around us, and in particular for the very cohesion of the human body, are at the origin of electricity. Transposed to our daily experience, electricity is an energy vector, that is to say, it carries the energy produced in power plants to the various users.

Electricity, or more precisely electric current, is characterized by two quantities. On the one hand, voltage, which measures the potential difference, represents the electrical imbalance between two points. On the other hand, amperage, which measures the current intensity, refers to the electrical flow induced by the voltage.

**4.2.2.1. Resistance: obstacle to electrons**

 What links voltage (V) and amperage (I) is none other than resistance (R). This link is given by the relation: .$R= \frac{V}{I}$

Resistance indicates how difficult it will be for current to flow through an electrical wire, and more generally through any conductive material. The higher the resistance, the less current there is. Conversely, the lower the resistance, the easier it will flow, and therefore the more current there is.

 How does a material prevent current from flowing? Simply by placing many nuclei in the path of the electrons, which then have difficulty moving because they bump into the nuclei. For this reason, it turns out that all materials are at least somewhat resistant (there are always nuclei in the way).

What happens when current passes through a high-resistance material? The electrons have a hard time moving. They keep bumping into the nuclei.

Each of these shocks heats the nucleus and the electron; combined, these shocks cause the material to heat up to more than a thousand degrees. This heat production is in fact energy lost by the electrons.

 Ultimately, a toaster or a light bulb is just that: passing current through highly resistant wires. As a result, the wires heat up so much that they start to glow, and the toast is toasted or the room is lit.

When a material is heated, its resistance increases. Conversely, when a material is cooled, its resistance decreases. Why? As we have seen, resistance occurs when nuclei are placed in the path of electrons. On our scale, called the macroscopic scale, what we feel as high temperature (heat) is evidence, on the microscopic scale, of the quantity of movement of atoms, of their vibration.

At high temperatures, the nuclei move strongly relative to each other. This general agitation makes the passage of electrons even more difficult. Conversely, at low temperatures, the nuclei vibrate much less, they are calmer. Electrons carrying electric current therefore have less difficulty passing through the nuclei. In the extreme, if the nuclei are immobile (at a temperature called absolute zero), the resistance is very low (note that it is not zero) and is only linked to the very presence of the nuclei, not to their agitation.

**4.2.3. The phenomenon of superconductivity throughout the 20th century**

 Since electricity has no – or no longer has – any secrets for you, let's get to the heart of the matter. The first episode of this scientific adventure took place in 1911…

**4.2.3.1. A little leap into the past**

 Theory has always been ahead of practice. Even today, many theories cannot be verified experimentally due to the lack of adequate techniques. Thus, at the beginning of the century, the hypothesis that resistance varies with the temperature of the material was present in the minds of researchers. But how can this phenomenon be verified for very low temperatures when the cooling processes were not efficient?

Before 1900, the lowest temperatures ever reached were still too high to answer this question. It was then that a sophisticated technique was developed that allowed helium to be liquefied. With liquid helium, it was now possible to bring any material to a temperature of –269 °C, very close to absolute zero! Liquid helium would be able to provide an answer to questions about changes in resistance at very low temperatures.

**4.2.3.2. The discovery of superconductivity**

 Very quickly, physicists were experimenting with various materials cooled with liquid helium. In 1911, a Dutch researcher, Heike Kamerlingh Onnes, measured the resistance of mercury at increasingly lower temperatures. He observed that as the temperature dropped, the resistance declined. Up until then, everything was going according to plan. Suddenly, at a certain temperature called the critical temperature, the resistance abruptly dropped to zero, the material no longer offering any resistance to the passage of electric current.



***4.1: Historical figure highlighting superconductivity***

 This unexpected phenomenon was named superconductivity. It indicates that below a critical temperature, electrons move without collisions with nuclei, and therefore without loss of energy.

Very quickly, the same phenomenon was observed for other materials. For each of these, the researchers found a specific critical temperature. Unfortunately, the first materials tested had very low critical temperatures, which made applications difficult to design for several decades. Superconductivity was gradually forgotten by the general public.

In 1987, a discovery made by a team of researchers revived the scientific community's interest in the phenomenon: a superconducting material at -183°C. This temperature can be reached using liquid nitrogen, ten times cheaper and much easier to use than liquid helium. This discovery shook up the scientific and industrial world, which believed that superconductivity was doomed to remain an obscure laboratory subject. It was talked about as a revolution of the same magnitude as that caused by the invention of the transistor. Just a few years later, the critical temperature record was set at -140 degrees. This temperature remains unbeaten to this day.

 Today, the search for the highest critical temperature has given way to understanding the phenomenon and the synthesis of new types of superconductors. Progress is being made in a slower and more directed manner. Because while critical temperatures are now easy to reach, superconducting materials are among the most complex materials ever made.

**4.2.4. The Bardeen–Cooper Schrieffer theory (BCS)**

 In the 1950s, three scientists, Bardeen, Cooper and Schrieffer, proposed an explanatory model of superconductivity to which they would give their names. The superconducting materials discovered after the work of the three scientists seemed to be outside the scope of their theory, but it continued to serve as the basis for the explanation of the superconductivity of these new complex materials.

To understand this theory, you just need to remember the two components of an atom, both actors of the electric current: the nuclei and the electrons. The first, the nuclei, are heavy and not very mobile; they are positively charged. We can imagine them as being fixed on a sort of infinite monkey cage, called a network; at each crossing, a nucleus vibrates around its equilibrium position, according to the temperature. The second, the electrons, are much smaller and much more mobile; they are negatively charged. We can imagine them as tiny little balls (1000 times smaller than the nuclei) which move between the nuclei.

 When no current passes through the material in question, the electrons move little. There is no overall movement. When current passes, the electrons all start moving in the same direction. There is concerted movement. These electrons, by moving, cause an incalculable number of collisions with the nuclei. This implies a significant loss of energy in the form of heating. However, as we have seen, if the critical temperature of the material is reached, this loss of energy suddenly becomes zero. Why?

Let's imagine a superconducting material subjected to an electric current. Let's carefully observe a given electron. The electron, like all its companions, flies through the meshes of the network. As a reminder, the electron is negatively charged and the network is positively charged; but opposite charges attract each other!

 The reasoning is as follows. Our electron passes between four nuclei; these are attracted by the negative charge of the electron. Due to this attraction, the network undergoes a local contraction. This contraction causes the appearance of a local excess of positive charge. This excess creates a small positive zone which attracts another electron. This one joins the first electron and now forms a Cooper pair with it. The first leads the way and the second only has to follow, and so on.

 The result is no longer an anarchic flow of electrons advancing blindly, but an ordered current of electrons that literally slip through the meshes of the net, without ever bumping into the nuclei. No shocks, therefore no loss of energy, or even no resistance.

Why does the phenomenon only occur at a very low critical temperature? For electrons to be able to slide between the meshes of the network, the nuclei must still be relatively stable. It is clear that the critical temperature is a threshold: as soon as the nuclei have reached a certain degree of stability, the electrons suddenly switch from a chaotic movement to a perfectly ordered and concerted movement.

**4.2.5. The experience of levitation or a challenge to gravity**

 The levitation experiment always amuses those who attend it and is also relatively simple to perform… It is undoubtedly thanks to this experiment that superconductivity has enjoyed a certain success with the general public. Indeed, it allows one to visualize at the macroscopic level one of the effects of superconductivity. Let's see that…

Let's take a material that is superconducting (i.e. it will be so below its critical temperature) at room temperature.

 Next, let's bring this material below its critical temperature (Tc). It is now superconducting; that is to say, its resistance is annihilated, or the current can pass without hindrance.

Let's bring a small magnet above the superconductor. The magnet begins to levitate, that is, to float a few millimeters above the superconductor.



How does this experiment help us understand superconductivity? Let's start with the situation where the material is at room temperature; it is therefore still only a classical conductor. The magnet, by definition, continuously emits a magnetic field. As it approaches the material, its magnetic field generates a weak electric current within it, in accordance with Lenz's theorem. This electric current is not induced by the magnetic field as such but by the movement of this field. In other words, as soon as the magnet stops, the induced current disappears. Indeed, this current is of much too low intensity to be able to counter the resistance of the material.

Now let's bring the material below its critical temperature (Tc) so that it becomes superconducting. When the magnet stops, it no longer induces current in the material; however, the flow of electrons (or current) induced during the movement of the magnet continues to flow in it since the resistance is reduced to zero.

So far, these phenomena do not yet explain why the magnet levitates. There is only one element missing.

We have seen that a moving magnetic field induces an electric current. The reverse is also true. Indeed, an electric current also induces a magnetic field (you only have to hold a compass close to an electric wire to see this). Therefore, the current induced in the superconductor in turn induces a magnetic field. This is oriented upwards, therefore against the magnet. The opposition of the two magnetic fields (that of the superconductor and that of the magnet) will cause the magnet to levitate.

**4.2.6. Superconductivity at the service of man**

 The applications of superconductivity are increasingly numerous. Medicine has already found several applications for this property. Particle accelerators and energy accumulators in the form of magnetic fields use superconductivity. The future potential of this phenomenon is infinite. Let us think, for example, of computing. The creation of microprocessors whose components would be built from superconducting material would increase the power of computers tenfold. Indeed, without resistance, there is no heating, and without heating, miniaturization – increasingly problematic – could resume with renewed vigor.

 Let us recall the trains on magnetic cushions, prototypes of which are already running. The principle is simple: replace the wheels with magnets, and the rails with superconductors. No contact between the train and the rails, therefore no friction. This is why speeds of around 600 km/h can be reached.

Finally, let's not forget the primary advantage of the superconductor: the transport of electricity without energy loss. Today, despite the use of high voltage (100,000 volts) in high-voltage lines, several tens of percent of the energy transported is lost. This loss could be eliminated entirely with superconducting cables. The low critical temperature still remains the major obstacle to the realization of these projects. The day when a material that is superconducting at room temperature is discovered, the technology will undoubtedly experience a new golden age.

**4.2.7. Other Important Applications of Superconductor**

**In Medicine**

 In the field of medical diagnostics, nuclear magnetic resonance imaging (MRI) consists of using the small magnets that the nuclei of atoms in the human body have to visualize what surrounds them (the brain, muscles, etc.). To do this, these magnets must first be aligned using a magnetic field in which the patient is placed. The larger the field, the better the image.

To produce intense fields, a strong electric current must be passed through a coil of several thousand turns of wire. If metal wires, such as copper, are used, they will heat up so much that they will eventually melt. This is why in all nuclear magnetic resonance imaging (MRI) devices, the coils are made of superconducting wire immersed in a very cold liquid such as helium.

**Superconducting coils**

 Superconducting coils can produce magnetic fields of several teslas (1 tesla is about 10,000 times the Earth's field). They are obtained by using windings of several thousand turns of superconducting wire immersed in liquid helium, often based on alloys of niobium and titanium (NbTi) or niobium and tin (Nb3Sn). These coils are often called "superconducting magnets" by abuse of language. These high magnetic fields are used for Nuclear Magnetic Resonance in chemistry and physics, in physics laboratories to study the effect of fields on solids and thus explore the magnetic properties of materials.

**Electronics and telecommunications**

 Despite the disadvantage of the necessary low temperatures, the exceptional properties of superconductors make it possible to design high-performance and original electronic circuits. Electronics based on superconducting materials has therefore been developed, using conventional superconductors or superconductors with high critical temperatures. These systems are now reliable, and have left the laboratory setting for use by industrialists. Thus, the most efficient filters available for cell phone relay antennas use superconductors.

Other electronic systems protect circuits from overvoltages, or build ultra-sensitive magnetic field measuring devices. Other developments in superconducting electronics are currently being prepared in laboratories, and may lead to the next technological revolutions. For example, the Josephson junction may replace the silicon-based transistor in our current circuits, allowing computers to reach rates of 100 GHz?

Superconductivity could even be used to build a quantum computer, enabling massively parallel computing, but which currently only exists on paper. Superconducting electronics is therefore a reality today, even if its use remains limited by the temperatures required for its proper functioning.

**Transport and storage of electrical energy**

 However, current electrical cables can only carry limited currents, otherwise they will heat up too much and melt. A network of superconducting cables would solve this problem because 10,000 times more electric current can flow through them: smaller cables for more current. Such a network is not yet cost-effective because the cables must be cooled to become superconductors. However, prototypes of superconducting cuprate cables cooled with liquid nitrogen are beginning to appear, over short distances.

Superconductors are also used as current limiters in power plants, a kind of super-fuse. Finally, superconductors are used for electrical energy storage solutions in devices called SMES. An electric current is stored in a superconducting coil that is closed on itself.

The current remains trapped in the coil indefinitely because there is absolutely no loss of energy, and this current can be recovered at will and above all in a very short time, unlike usual batteries.

**Magnetic confinement**

 Fusion is the energy source of the Sun and other stars. A star begins to shine when the matter in its core reaches, under the effect of gravitational forces, sufficient densities and temperatures to trigger thermonuclear reactions that release energy. The tendency of the plasma to disperse, and therefore to cool, is counterbalanced by the gravitational force. On Earth, gravitational confinement is impossible.

Two ways are being studied to reproduce these reactions. Bring a small volume of matter to very high pressure and high temperature for an extremely short time. This is called inertial confinement. In this way, we seek to obtain the greatest possible number of fusion reactions before the plasma disperses.

Trap and maintain a plasma at very high temperature. This plasma is confined in an immaterial toroidal box created by magnetic fields; this is called magnetic confinement.

In order for the fuel, in the plasma state, to be able to produce sufficient thermonuclear reactions, it must be kept in a limited volume and kept away from any material wall in order to maintain its high temperature: this is confinement.

In a free-flowing plasma, the trajectory of the particles is random and the particles will escape. Since the plasma is formed of charged particles, intense magnetic fields can interact with them. If this same plasma is bathed in a rectilinear magnetic field, the particles wrap around the field lines and can no longer reach the side walls. In order to avoid losses at the ends, the magnetic box is closed by creating a torus. The magnetic field thus created by a series of superconducting magnets surrounding the plasma is called the toroidal magnetic field. The magnets generating this field are toroidal magnets.