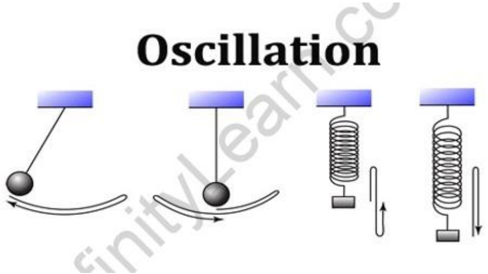
Chapter I: Generalities on Vibrations

I.1 Definitions

I.1.1 Definition of an oscillation (Vibration)

✓ **Oscillation** is defined as the process of repeating variations of any quantity or measure about its equilibrium value in time



I.1.2 Periodic motion:

The usual physics terminology for motion that repeats itself over and over is ***periodic motion***

- the time required for one repetition is called ***the period,*** often expressed as the letter *T*, expressed in seconds (s).

Mathematically, the periodic motion of period T is defined by: x (t + T) = x(t).

- The number of repetitions per second is called **frequency** (noted *f*, measured in (Hertz) or (s-1)). It is related to the period by:

$$f = \frac{1}{T}$$

- **Angular frequency ω (omega)** refers to the angular displacement per unit time (e.g., in rotation) or the rate of change of the phase of a sinusoidal waveform (e.g., in oscillations and waves), or as the rate of change of the argument of the sine function.

The number of revolutions per second is called pulsation (denoted ω, measured in rad/s.)

$$\omega = 2\pi\frac{1}{T}$$

I.1.3 Sinusoidal motion:

A vibrational motion is sinusoidal if the elongation x (y or z) of a vibrating point

is a simple sinusoidal function of time of the type:

$$x(t) = A \sin(\omega t+\varphi) \quad \text{or} \quad x(t) = A \cos(\omega t+\varphi)$$

x(t) is called the elongation (or position) at time t.

A: the amplitude of a motion or the maximum elongation.

ω: The pulsation of the motion and expressed in (rad/s).

φ: the initial phase, corresponds to the phase at time $t = 0$

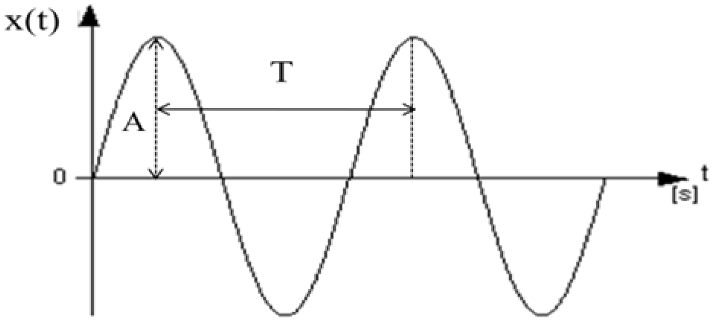


Figure 1 Example of a sinusoidal periodic motion

Example: In Figure 1, consider a sinusoidal vibratory motion of the form:

x(t) = Acos (ωt +φ) , with period T= 1s and amplitude 1cm.

- calculate the pulsation ω and determine A and φ?

  $\omega = \frac{2\pi}{T} = 6.28/1 = 6.28\ rad.s^{-1} = 2\pi$

we have : A =1so x(t) = cos(2πt+φ).

At t = 0⇒x (0) = 0 so φ = π/2

We get : $x(t) = \cos(2\pi t + \frac{\pi}{2})$ (cm)

I.1.4 Simple Harmonic Motion (SHM)

A simple form of oscillatory motion is Simple Harmonic Motion (SHM). In this motion, the restoring force is directly proportional to its displacement from its equilibrium position. This is Hooke's Law

$$\mathbf{F_r = -k\,x}$$

the negative sign shows that the force is acting against the displacement x.

Examples of harmonic motion:

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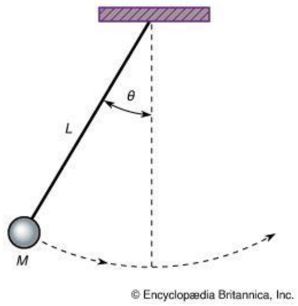
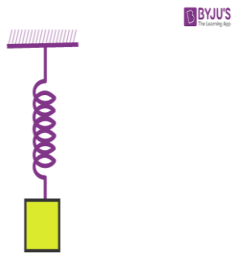
1. Simple pendulum

A simple pendulum is composed of a mass attached to a wire that is moved away from its equilibrium position and then released. It performs a back-and-forth movement repeated over time.



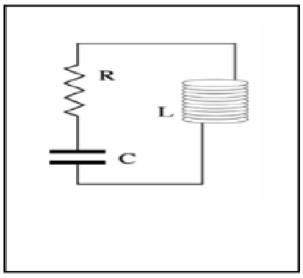
2. Spring Mass System

Composed of a mass attached to a spring, moved away from its equilibrium position, and then released, it performs a movement and return that is repeated over time.



3. Oscillator Circuit

A linear circuit contains an electrical resistance, a capacitor (a capacitance), and a coil (an inductance). It is capable of making electrical oscillations.



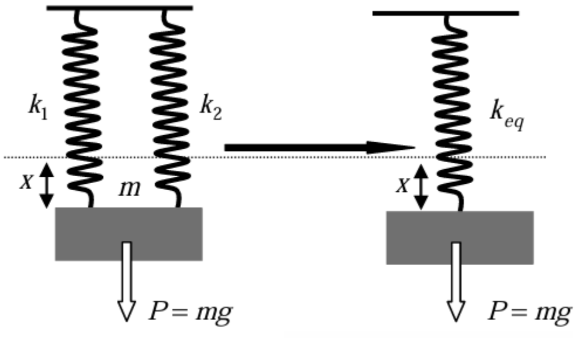
I.2: Equivalent spring

In practice we find springs in series and others in parallel.

I.2.1Parallel springs

When two massless springs following Hooke's Law, are connected via a thin, vertical rod as shown in the figure, these are said to be connected in parallel. Spring 1 and 2 have spring constants k1 and k2 respectively. A constant force F is exerted on the rod so that it remains perpendicular to the direction of the force. So that the same amount extends to the springs. Alternatively, the direction of force could be reversed so that the springs are compressed.

his system of two parallel springs is equivalent to a single Hookean spring, of spring constant k. The value of k can be found from the formula that applies to capacitors connected in parallel in an electrical circuit.



The equations of the system at equilibrium are written as follows:

For the real system:

$$P = k_1 x + k_2 x$$

$$P = (k_1 + k_2)\, x$$

From model mathematics we can write

$$P = k_{eq}\, x$$

we can deduce the stiffness constant of the equivalent spring by the following relation:

$$k_{eq} = (k_1 + k_2)$$

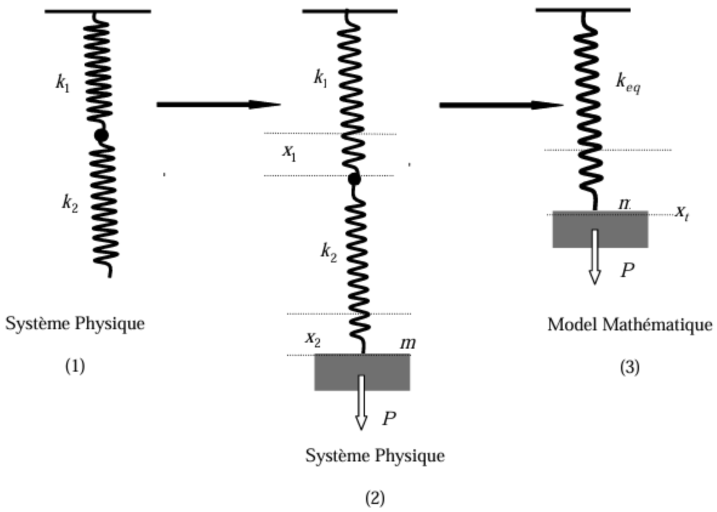
If the system consists of several springs in series, then the constant of the stiffness of the equivalent spring is given by:

$$k_{eq} = \sum_i k_i$$

I.2.2 Springs in series

The suspension of the mass m at the free end of the two springs k_1 and k_1 causes the elongations x_1 and x_2 in k_1 and k_1 respectively. The total elongation is

$$x_t = x_1 + x_2$$



In mechanical equilibrium: If we consider the real system, we can write the following:

$$P = k_1 x_1,\ P = k_2 x_2$$

for the mathematic model $\qquad P = k_{eq}\, x_t$

From the preceding relations, we can write:

$$k_1 x_1 = k_{eq} x_t \quad \Rightarrow \quad x_1 = \frac{k_{eq}}{k_1} x_t$$

$$k_2 x_2 = k_{eq} x_t \quad \Rightarrow \quad x_2 = \frac{k_{eq}}{k_2} x_t$$

taking into account the relation $x_t = x_1 + x_2$, we obtain that :

$$x_1 + x_2 = \frac{k_{eq}}{k_1} x_t + \frac{k_{eq}}{k_2} x_t \quad \Rightarrow \quad x_t = k_{eq}\left(\frac{1}{k_1} + \frac{1}{k_2}\right) x_t$$

Finally, we find the relation which gives the stiffness constant of the equivalent spring.

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

In the general case where the system consists of several springs in parallel, the constant of the stiffness of the equivalent spring can be given as follows:

$$\frac{1}{k_{eq}} = \sum_i \frac{1}{k_i}$$

1.3. Generalized coordinates:

We call **generalized coordinates** of a physical system a set of real variables allowing us to describe this system (position and movement).

$systm$ in motion $\begin{cases} Translation\ (x, y, z) \\ Rotation\ (Ox, Oy, Oz) \end{cases}$

1.4. Degree of freedom:

We call a system's degree of freedom (DOF) its capacity to perform translation and rotation with respect to the axes.

$$\boldsymbol{d = N - r}$$

d: Degree of freedom;

N: Number of generalized coordinates;

r: Number of relations linking these coordinates between them.

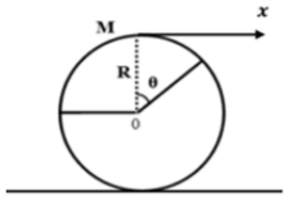
<u>Example 1</u>

A disk of mass m and radius r, rolls without slipping on a horizontal plane.

We have two generalized coordinates x and $\boldsymbol{\theta}$ so N=2

x and θ are linked with a relation: $\boldsymbol{x} = \boldsymbol{r\theta}\ \boldsymbol{donc}\ \boldsymbol{r} = \mathbf{1}$The number of degrees of freedom:

$\boldsymbol{d} = \boldsymbol{N}-\boldsymbol{r} = \mathbf{2}-\mathbf{1} = \mathbf{1}$



Example 2: Particle in free fall:

On the three coordinates x, y, z

Number of general coordinates: 03.

We have two constant coordinates: y and x.

d = 3-2 =1.

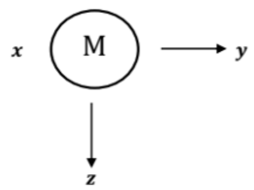
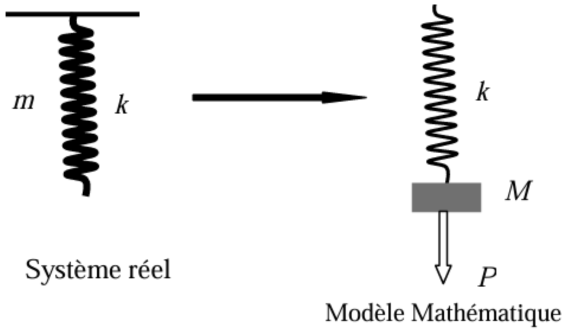


Figure I.3 : Particule

I.5: The equivalent mass

a. Spring with a non-negligible mass

The mass of an element of and length dl is given by $\frac{m}{L}\,dl$. The speed of the elements constituting the spring (points of the spring) are linear with the distance l. The speed of the element at the fixed end is zero while the speed of the element at the free end x is the maximum.



. The velocity of the element dl can be given as follows:

$$\dot{x}_s(t) = \frac{1}{L}\dot{x}(t).$$

The kinetic energy T of the spring is equal to the sum of all the energies of its elements and as the distribution of mass is continuous then the sum will be replaced by an integral:

$$T = \int_0^L \frac{1}{2}\left(\frac{m}{L}dl\right)\left(\frac{l}{L}\dot{x}\right)^2 = \frac{1}{2}\frac{m}{L^3}\dot{x}^2\int_0^L l^2 dl$$

$$T = \frac{1}{2}\left(\frac{m}{3}\right)\cdot \dot{x}^2$$

Knowing that the kinetic energy of the equivalent system (mathematical model) is written:

$$T = \frac{1}{2}m_{eq}\cdot \dot{x}^2$$

we deduce the following:

$$m_{eq} = \frac{m}{3}.$$