

Series N 2

Exercice 1 :

Demonstrate the convergence and calculate the value of the integrals:

$$I_1 = \int_0^{+\infty} t^3 e^{-t} dt; \quad I_2 = \int_1^{+\infty} \frac{1}{t\sqrt{t^2+1}} dt; \quad I_3 = \int_0^{+\infty} \frac{t \ln(t)}{(t^2+1)^2} dt$$

Exercice 2 :

Are the following improper integrals convergent or divergent?

$$I_1 = \int_2^{+\infty} \ln(t) dt; \quad I_2 = \int_0^2 \ln(t) dt; \quad I_3 = \int_0^{+\infty} e^{-4t} dt$$

Exercice 3 :

Let F be the function defined by:

$$F(x) = \int_1^x \frac{\ln(1+t^2)}{t^2} dt$$

- Calculate $F(x)$.
- Deduce that the integral is convergent and determine its value.

$$I = \int_1^{+\infty} \frac{\ln(1+t^2)}{t^2} dt$$