# **I.1 Introduction**

Fluid mechanics is a branch of engineering which is concerned with the study of fluids at rest (i.e. fluid statics) as well as the study of fluids in motion or the study of fluid flow (i.e. kinematics and fluid dynamics).



Fluid mechanics applications are diverse:

Urban hydraulics (water distribution network), air conditioning, heating, hydroelectricity, urban aerodynamics, the study of vibrations, aeronautics, the chemical and food industries, meteorology….. etc

## **I.2 Physical definition of a fluid**

## **I.2.1 States of matter**

Matter can exist in three different states:

- **Exercise** Solids: low temperature material
	- $\checkmark$  Ordered molecules, very close together, linked.
	- $\checkmark$  Invariable form
	- $\checkmark$  Constant volume



**↓** Liquid: materials at medium temperature and sufficiently high pressure.

- $\checkmark$  molecules disordered, close, together, linked.
- $\checkmark$  Variable form.
- $\checkmark$  Constant volume.

Gaz: material at sufficiently high temperature at low pressure

- $\checkmark$  disordered, spaced out, very agitated molecules
- $\checkmark$  variable form
- $\checkmark$  variable volume

The term fluid includes gases and liquids, because the latter have similarities.

#### *The main differences between liquids and gases are:*

- $\checkmark$  Liquids are practically incompressible while gases are compressible and must often be treated as such.
- $\checkmark$  liquids occupy well-defined volumes and have free surfaces while a given mass of gas expands until it occupies all parts of the container which contains it. the gas does not form a free surface.



**Figure I.1: the free surface of a liquid**

## **Divided material**

Not all fluids are pure liquids or gases. we encounter fluids where two phases in thermodynamic equilibrium coexist. compared to liquids pure, the presence of particles (gas bubbles, solid particles, droplets) induces the presence of a multitude of interfaces between the liquid (continuous phase) and the particles (dispersed phase), which can radically change the nature of the mixture. We distinguish:

*The dispersions*:



**Dispersions that do not sediment (colloids)**



**suspensions:** coarse particles in a liquid (saturated soil, etc.).

**Emulsions:** these are mixtures of fine droplets of a liquid in another (vinaigrette, etc.).

**foams:** gas bubbles in a liquid (foam, etc.).

## **I.2.2 what is a fluid?**

## **I.2.2.1 definition**

A fluid can be constituted as being made up of a large number of material particles, very small and free to move relative to each other. A fluid is therefore a continuous material medium, deformable, without rigidity and which can flow. The fluid is an isotropic material.

- $\triangleright$  Continuous: its properties vary continuously.
- $\triangleright$  Deformable: it has no proper shape; the molecules can easily slide over each other, this mobility causes the fluid to take the shape of the container that contains it.
- $\triangleright$  Which can flow: but any fluid can flow more or less easily from one container to another or in a pipe: friction forces which oppose the sliding of the fluid particles against each other can appear because every real fluid has a viscosity.
- $\triangleright$  Isotropy: at a point in the fluid, all the properties of the fluid are the same in all directions in space.

**The fluid particle**: it is a portion of fluid to which correspond, at an instant, a pressure, a temperature, a density…. etc.

Fluids can be classified as follows:

- **Perfect fluid**: is a fluid inside which the tangential forces of internal friction are zero, there is no friction (without viscosity  $\mu = 0$ ), the contact forces are perpendicular to the surface elements on which they are exerted.
- **Real fluid**: In real fluids, the tangential forces of internal friction, which oppose the relative sliding of the fluid layers are taken into consideration ( $\mu \neq 0$ ). This highly viscous phenomenon appears during the movement of the fluid. The statics of real fluids merge with the statics of perfect fluids.



- **Incompressible fluid**: a fluid is said to be incompressible when the volume occupied by a given mass does not vary as a function of external pressure  $(\rho = cste)$ . liquids can be considered as incompressible fluids (water, oil, etc.)
- **Compressible fluid**: a fluid is said to be compressible when the volume occupied by a given mass varies as a function of external pressure  $(\rho \neq \text{cste})$ . gases are compressible fluids.

# **I.3 Physical quantities of a fluid**

## **I.3.1 Temperature**

• Relative temperature scale

*Celsius scale (°C)*: in the international system, temperature is divided into 100 degrees.

• absolute temperature scale

*Kelvin (K) scale:* in the international system  $T(K) = T({}^{\circ}C) + 273.15$ 

## **I.3.2 Density** or masse density  $(\rho)$

The density  $(\rho)$  of a body is the ratio of its mass  $(m)$  to the volume  $(V)$  it occupies.

$$
\rho=\frac{m}{V}
$$

 $\rho$  is expressed as kg.m<sup>-3</sup>

V: volume as  $m<sup>3</sup>$ 

m: masse as kg

for gases, density depends on temperature and pressure, for **a perfect gas**, density can be calculated from its equation of state.

 $P V = m R T \Rightarrow P = \rho R t \Rightarrow \rho = \frac{P}{R t}$  $\frac{P}{RT}$  Where :

- P : Absolute pressure in (Pa)
- T : absolute temperature in (K)
- R : gas constant in  $(J.kg^{-1}.K^{-1})$

$$
R = \frac{r}{M_{gas}}
$$

r: universal gas constant  $(r = 8.314 \text{ kJ.kmol}^{-1} \cdot \text{K}^{-1})$ 

M: molar mass of gas in  $(kg$ .mol<sup>-1</sup>)

for air R=287 J.kg $^{-1}$ .K $^{-1}$ 

#### **I.3.3 Specific volume**

The spécific volume  $(v)$  is the inverse of the density.

$$
\nu = \frac{1}{\rho} = \frac{V}{m}
$$

v is expressed in  $(m^3.kg^{-1})$ 

#### I.3.4 **Specific weight or weight density**

The specific weight  $(\varpi)$  is the weight of the unit of volume.

$$
\varpi = \rho \, g = \frac{m \, g}{v}
$$

 $\overline{\omega}$  is expressed in  $(N.m^{-3})$ 

m: mass in (kg)

g: acceleration of gravity in  $(m. s^{-2})$ 

V: volume in  $(m^3)$ .

## **I.3.5 Specific gravity (** $\delta$ **)**

The Specific gravity  $(\delta)$  of a fluid is the ratio between the density of the fluid considered and the density of the fluid reference taken from the same temperature and pressure conditions.

$$
\delta = \frac{density \ of \ fluid}{density \ of \ a \ standard \ fluid} = \frac{\rho}{\rho_{ref}} \ (undimensional)
$$

Generally, the reference fluid is :

$$
✓ Water for liquids  $\left( \delta = \frac{\rho_{\text{liquid}}}{\rho_{\text{water}}} \right)$   
✓ Air for gases  $\left( \delta = \frac{\rho_{\text{gas}}}{\rho_{\text{air}}} \right)$ .
$$

## **I.4 Viscosity of a fluid**

Viscosity is a property of a fluid due to Cohesion and the interaction between molecules that resist deformation. This property comes into play whenever the fluid is in motion.

## **I.4.1 Dynamic viscosity**

To obtain a relationship for viscosity, consider a volume of fluid placed between two infinite parallel and horizontal flat plates, separated by a distance h. The lower plate is held stationary while the upper plate is moved. is mobile. In order for the upper plate to maintain a constant velocity **U**, it is necessary to a constant tangential force **F** must be applied to it. There is therefore a viscous interaction between the plate and the fluid, which manifests itself in the form of a drag on the former and a shearing force on the latter. Shear force on the fluid. The fluid in contact with the upper plate will the fluid in contact with the upper plate will adhere to it and move at speed **U**, while the fluid in contact with the lower plate will have a speed U.

Fixed plate will have zero velocity (non-slip condition).

The shear stress is:

$$
\tau = \frac{F}{S}
$$

Where:

- $\tau$ : The shear stress in  $(N, m^{-2})$
- S: the area of the plate in  $(m^2)$ .

When the fluid moves in parallel layers, the flow is said to be laminar, and so the velocity of the fluid changes linearly between 0 and U, and so the velocity profile and the velocity gradient are:

$$
u(y) = \frac{y}{h} U \quad \text{and} \quad \frac{du}{dy} = \frac{U}{h}
$$

The force F depends on the surface area s and the velocity gradient.

Newton's law states that its norm is proportional to these two quantities:

$$
F = \tau S = \mu S \frac{du}{dy}
$$



Figure I.3: Behaviour of a fluid in laminar flow between two parallel plates.

$$
\tau = \frac{F}{S} = \mu \frac{du}{dy}
$$

 $\mu$ : is the dynamic or absolute viscosity.

In the international system, the unit of dynamic iriscosity is the **(Pa.s)** or **poiseuille(pl)**.

knowing that: *1Pa.s=1N.s.m-2 = 1Pl= kg.m-1 .s-1* other units: *1Pl=10 poise*.

#### **I.4.2 Variation in viscosity**

Viscosity depends on:

- The nature of the fluid

- Temperature: Viscosity varies with temperature.  $\mu = f(T)$ 

For liquids, viscosity decreases with increasing temperature.

For gases, the viscosity increases when the temperature increases.



## **Figure 1.4: Variation of viscosity as a function of temperature for gases and liquids**

• Pressure: For liquids, dynamic viscosity varies with pressure.

According to the law  $\frac{\mu}{\mu_0} = a^{\left(\frac{P}{P_0}\right)}$  $\frac{1}{P_0}$  $^{-1}$ , where P<sub>0</sub> is the atmospheric pressure and  $\alpha$ depends on the nature of the fluid ( $\alpha = 1.003$  for mineral oils).

- Deformation: Depending on the relationship between the shear stress and the velocity gradient, there are different types of fluids:
- $\checkmark$  **Newtonian fluids**: A fluid is said to be Newtonian when the viscous stress tensor is a linear function of the deformation rate tensor. The proportionality factor is called viscosity $\mu$ , and is constant and independent of the rate of deflection. The equation describing the Newtonian behaviour is :

$$
\tau = \mu \frac{du}{dy}
$$

This is the case for gases, vapours, liquids with low molar mass.... etc.

- $\checkmark$  **Non-Newtonian fluids**: a fluid is said to be non-Newtonian when the viscous stress tensor is not a linear function of the strain rate tensor. There are several types of non-Newtonian fluid (**Figure I.5**):
	- *Rheofluidifying or pseudoplastic fluid* (fats, mayonnaises, etc.)
- *Rheo-thickening or dilatant fluid* (solutions with a high concentration of powders (water/sand), etc.).
- **Bingham fluid** (chocolate, paints, soups, soap, etc.)



## **Figure I.5: Variation of shear stress as a function of shear rate for Newtonian and non-Neuwtonian fluids**

## **I.4.3 Kinematic viscosity**

Kinematic viscosity is the ratio of the dynamic viscosity to the density of a fluid.

$$
\nu = \frac{\mu}{\rho}
$$

In the international system, *kinematic viscosity* ( $\nu$ ) is expressed in  $(m^2, s^{-1})$ .

Other units:  $1m^2$ .  $s^{-1} = 10^4$ stockes.

## **I.5 Surface tension of a fluid**

#### **I.5.1 A few comments**

A number of observations can be made:

- Droplets of rainwater or dew on tree leaves.
- Water dripping from a leaky tap falls in the form of almost spherical droplets.
- A soap bubble released into the air has an almost spherical shape.
- Small insects walk on the water-free surface.
- The free surface of the water in a tube takes on a concave shape.

In these and other observations, the free surface of the liquid behaves like an elastic membrane stretched under tension, and the droplets behave like small balloons filled with a liquid.



Liquids have two forms of molecular attraction, cohesion and adhesion. Cohesion allows allows the liquid to resist stretching stresses, while adhesion allows it to adhere to a body.

The attraction between the molecules forms an imaginary film capable of resisting the tensions at the interface between two immiscible liquids or the interface between a liquid and a gas. The property of the liquid which creates this ability is called **"surface tension"** and designated by  $\sigma = \frac{N}{m}$ 

 At the interface between the liquid and the gas, the intermolecular force of attraction causes this tension. This force per unit length is called **surface tension**.

To explain the phenomenon of surface tension, a microscopic view is shown in **Figure (I.6).**

Each molecule located inside a liquid is under the influence of attractive forces in all directions, and the resultant of these forces is zero, whereas a molecule located at the separation surface (liquid-gas interface) is subject to a non-zero resultant of intermolecular attractive forces. This resultant directed towards the interior of the liquid and normal to the free surface.

Consequently, the movement of the molecules on the surface requires work against this force, and so the energy of the molecules on the surface is greater than that of the molecules inside. This is why liquid droplets take on a spherical shape, which has the minimum surface area for a given volume.



**Figure I.6: Intermolecular forces**

#### **I.5.2 Expression of the surface tension coefficient**

Consider a liquid film (such as the film of a soap bubble) trapped on a metal frame with a movable side (AB) (Figure I.7). The liquid film pulls the movable rod inward to reduce its surface area.

To balance the mobile rod, a force F must be applied to it in the opposite direction.

When the moving rod moves a distance **dx**, the surface area of the liquid film increases by  $dS = 2 L dx$  and the work  $(\delta W)$  provided during this stretching process is:

$$
\delta W = F dx = \frac{F}{2L} 2L dx = \frac{F}{2L} dS \implies \frac{\delta W}{dS} = \frac{F}{2L}
$$

The quantity  $\frac{F}{2L}$  or  $\frac{\delta W}{dS}$  is called surface tension coefficient or surface tension:  $\sigma = \frac{\delta W}{dS} = \frac{F}{2L}$ 

In the international system, the unit of surface tension  $(\sigma)$  is: **N.m<sup>-1</sup>** or **J.m**<sup>-2</sup>.

The surface tension of the liquid is the work necessary to bring the molecules from the interior of the liquid towards the free surface to form a new surface unit.



Figure I.7 of surface tension.

#### Some surface tension coefficient values  $\sigma$



# **Table I.1: surface tension coefficient of some fluids placed in air at 1 atmosphere and 20°C**

## **I.5.3 Exemples of surface tensions**

## **I.5.3.1 Curved Surfaces - Young Laplace's Law**

To calculate the pressure in a drop of fluid, let us take the diagram of a half-spherical drop and illustrate the forces acting on it. The force developed on the periphery is due to the surface tension  $\mathbf{F}_{\text{surface tension}}(\mathbf{F}_{\sigma})$ . This force must be balanced by the force generated by the pressure difference  $\Delta P$  between the internal pressure  $P_i$  and the external pressure  $P_e$ , acting on the circular surface

The pressure inside a curved surface (drop, bubble, meniscus, etc.) of fluid can be calculated from Young Laplace's law.

## **Case of a droplet**

Consider for example, a droplet of liquid in the air. This droplet is divided into two halfspheres (**Figure I.8**).



**Figure I.8: Half of a droplet**

The equilibrium of half the droplet translates as:  $\sum F = 0 \Rightarrow F_{surpression}(F_P) = F_{surface \ tension}(F_{\sigma})$ 



The pressure difference force is given by:

$$
F_{surpression} = F_P = \Delta P \underbrace{\pi R^2}_{\text{surface}} = (P_i - P_e) \pi R^2
$$

The surface tension force is given by:

$$
F_{\text{surface tension}} = F_{\sigma} = \underbrace{2\pi R}_{\text{power tension de surface}}
$$

We therefore obtain the expression for the overpressure in a drop:

$$
\Delta P = (P_i - P_e) = \frac{2\sigma}{R}
$$

Where:

 $P_i$ : the pressure inside the drop.

 $P_e$ : the pressure outside the drop.

R: radius of the drop.

 $\sigma$ : surface tension coefficient.

**Case of a bubble**



**Figure I.9: Half of a bubble**

A bubble is made up of two liquid-air interfaces.

The excess pressure in a bubble is given by this expression:

$$
\Delta P = (P_i - P_e) = \frac{4\,\sigma}{R}
$$

Where:

 $P_i$ : the pressure inside the bubble.

 $P_e$ : the pressure outside the bubble.

R: radius of the bubble.

 $\sigma$ : surface tension coefficient.

#### **I.5.4 Wettability phenomenon and contact angle**

A drop of liquid is placed on a flat horizontal plate. Depending on the liquid-solid contact angle, several situations can be distinguished.

• The contact angle  $\theta = 0^{\degree}$ , the liquid completely wets the solid (wetting

Perfect).

• The contact angle  $\theta$  < 90°, the liquid incompletely wets the solid (wetting

imperfect)

• The contact angle  $\theta > 90^\circ$ , the liquid does not wet the solid.



**Fig I.10: Different shapes of liquid on a solid flat plate**

The equilibrium of the drop deposited on a solid horizontal flat plate (**figure I.11**) translated by the balance of surface tension forces between the three phases.



**Figure I.11: Equilibrium of three, phases in contact.**

By projecting these forces onto the horizontal plane of the solid, we obtain:

$$
\sigma_{SG} = \sigma_{SL} + \sigma_{LG} \cos \theta \implies \cos \theta = \frac{\sigma_{SG} - \sigma_{SL}}{\sigma_{LG}}
$$

Where:

 $\sigma_{SL}$ : the surface tension between the solid and the liquid  $\sigma_{LG}$ : the surface tension between the liquid and the gas.  $\sigma_{SC}$ : the surface tension between the solid and the gas.  $\theta$ : liquid – solid contact angle.

## **I.5.5 Capillary effect: Jurin's law**

Capillarity is due to cohesion and adhesion at the same time. When the strength of cohesion is less than that of adhesion, the liquid wets the solid surface with which it is in contact contact and goes back to the point of contact. On the other hand, if cohesion is important, the surface of the liquid descends to the point of contact. For example, capillarity causes water to rise in a glass tube, while that the mercury drops below the actual level.



If we plant a capillary tube of radius (**R**) in a wetting liquid ( $\theta$  < 90°), the liquid rises in the tube at a distance (**h**) (**figure I.12a**). Whereas if the liquid does not wet the solid ( $\theta > 90^{\circ}$ ), the liquid descends in the tube (**figure I.12b**). The free surface of the liquid in the tube forms a concave or a convex.





#### **in a small diameter glass tube**

## **Jurin's law**

We can consider a liquid wetting the solid as shown in **figure (I.13)**. Calculate the height (**h**) to which the liquid rises in the capillary tube.



**Figure I.13: Forces acting on the ascending column of liquid in the capillary tube.**

The equilibrium of the column of liquid in the tube translates by:

 $F_{\text{weight of liquid column}} = F_{\text{surface tension}}$ 

By projection along the vertical axis, we obtain the expressions of the forces:

 $F_{\rm weight \, of \, liquid \, column} = mg = \, \pi R^2 \rho g h$ 

 $F_{surface \ tension} = F = 2\pi R\sigma \cos\theta$ 

Hence:  $\pi R^2 \rho gh = 2\pi R\sigma \cos\theta$ 

Finally, we obtain the expression of h:  $h = \frac{2\sigma cos\theta}{2\sigma^2}$  $\frac{\partial \cos \theta}{\partial gR}$ ; With:

R: inner radius of the tube

- $\rho$ : density of the liquid.
- g: acceleration of gravity.
- $\sigma$ : surface tension of the liquid.
- $\theta$ : liquid-solid contact angle.

## *Noticed :*

For liquids that do not wet the solid  $(\theta > 90^\circ)$  Jurin's law gives negative **h** (the sign **(-)** reflects *capillary depression*).