Tutorial Worksheet No.2

Exercise 1.

a) Let A and B be two sets in a universal set U such that
$A = \{1, 2, 3\} \qquad B = \{1, 2, 4\}$

- 1-a) Find the following :

$$A \cup B, \quad A \cap B, \quad A - B, \quad A \times B, \quad P(A),$$
$$A \cap \emptyset, \quad card(P(A)), \quad card(A \times B), \quad (A \cup B) - (A \cap B)$$

- 1-b) Represent graphically $A \times B$

b) Let E and F be two sets defined by

$$E = \{\{c\}, \{c, e\}\}, \quad F = \{\{a\}, \{a, b\}\}$$

- Prove that $(E = F) \Rightarrow (c = a) \wedge (e = b)$

Exercise 2.

The two sets A and B are defined by :
$A = \{set \quad of \quad odd \quad numbers\} = \{1, 3, 5, ...\}$
$B = \{set \quad of \quad even \quad numbers\} = \{0, 2, 4, ...\}$

- Find the following :

$$\mathbb{Z} - (A \cup B), \quad (A \cup B) - \mathbb{Z}, \quad (A \cup B) - \mathbb{N}, \quad \{0\} \cup \mathbb{N},$$
$$(A \cup B) - \{0\}, \quad (A \cup B) \cap \{0\}, \quad \mathbb{Z} \cap \mathbb{N}$$

Exercise 3.

Let A, B, C be subsets of U. Determine the following sets

- $D = (A \cap B) \cup \left(C_U^A \cap B\right) \quad E = \left(C_U^A \cup C_U^B\right) \cap \left(C_U^A \cup B\right), \quad F = \left(C_U^A \cap C_U^B\right) \cap C_U^{A \cap B}$

Exercise 4.

Let A and B be two sets. Prove that

- If $(A - B) \cup B = A$, then $B \subset A$.

Exercise 5.

Let S be a relation on $\mathbb{R}$defined by

$$\forall a, b \in \mathbb{R} \mid ab = 2a - 1$$

- Is the pair $(1, 1)$ an element of S ?
- Is S reflexive for $a = 1.$?
- What is the condition in the element (a, b) to satisfy the symmetric property of S ?
- What is the condition on the elements $a, b\ and\ c$ to satisfy the transitive property of S ?

Exercise 6.

Let $a \geq 0$ and let $\Re$ be a relation from E to F on $I = [0, +\infty)$ defined by

$$\forall (x, y) \in E \times F \mid \frac{ae^{x-2}}{y} - \frac{e^y}{x} = 0$$

- Determine a such that the relation $\Re$ is reflexive.
- By replacing the found value of a in $\Re$. Is $\Re$ an equivalence relation ?

Exercise 7.

Let $\mathcal{R}$be a relation on $\mathbb{R}$ defined by : $\forall (x, y) \in \mathbb{R}^2,\ xRy \Rightarrow cos^2(x) + sin^2(y) = 1$

1. Show that the relation $\mathcal{R}$ is an equivalence relation.
2. Find the class of equivalence $[x]$.

Home work exercise.

Let $\mathcal{S}$ be a relation defined on $\mathbb{N}$ by : $\forall k_1,\ k_2 \in \mathbb{N},\ k_1 \mathcal{S} k_2 \Rightarrow k_1\ divise\ k_2$

- Prove that $\mathcal{S}$ is a partial order relation on $\mathbb{N}$.

Exercise 8.

Let E and F be two subsets on $\mathbb{R}$ where $E = \{1, 2, -1\}$, and let f be a function defined by

$$f:\ E \longrightarrow F$$
$$x \longmapsto f(x) = x^2 + 2$$

1. Determine F and deduce $f(1),\ f^{-1}(6)$.
2. Determine the image of 3 and the pre-image of 5.
3. Is there a pre-image of 1 under f.

Exercise 9

Let f be a function defined by

$$f:\ \mathbb{R} \longrightarrow \mathbb{R}$$
$$x \longmapsto f(x) = x^2 + 2$$

1. Is f injective and surjective ?
2. Specify the case in which the function f is bejective.
3. Find $f^{-1}([2, 6])$.

Exercise 10

Let f and g be two functions defined by

$$f:\ \mathbb{R} - \{\tfrac{1}{2}\} \longrightarrow \mathbb{R} - \{2\} \qquad g:\ \mathbb{R} - \{2\} \longrightarrow \mathbb{R} - \{1\}$$
$$x \longmapsto f(x) = 2x + 1 \qquad x \longmapsto g(x) = \tfrac{x}{x-2}$$

1. Determine $g \circ f$
2. Show that the function $g \circ f$ is bijective.
3. Determine $(g \circ f)^{-1}$

Exercise 11

Let $g : \mathbb{R} \longrightarrow \mathbb{R}$ be a function defined by $g(x) = x^2 - 4x + 4$.

1. Check that $\forall a \in \mathbb{R}, g(2 + a) = g(2 - a)$. Deduce that $g(x)$ is not injective.
2. Show that $\forall x \in \mathbb{R}, g(x) \geq 0$. Is $g(x)$ surjective ?