University Center of Mila
Department of ST

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L1-S02

Mathematics 2
Work Sheet 01

Exercise 1. We define the matrices :

$$A = \begin{pmatrix} 0 & 2 \\ 4 & 1 \\ 3 & -2 \\ 5 & 0 \\ 1 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 & -1 & 0 & 7 \\ 0 & 1 & -2 & 1 & 2 \\ 0 & 4 & 1 & -1 & 1 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 2 & -1 & 3 \\ 2 & 0 & 3 & -1 \end{pmatrix}.$$

Is it possible to calculate the products ABC, CBA, BAC ? If yes, find them with tow methods (verify the associativity of the product).

Exercise 2. Let A be the matrix

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

In each case, find the matrix A such that :

(a) $A^2 = A$.

(b) $A^2 = I_2$.

(c) $AB = BA$, and $B = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}$.

Exercise 3. Let a be a non zero real number, and let

$$A = \begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix}$$

be a matrix of order 2. Calculate A^n the power of A ($n \in \mathbb{Z}$).

Exercise 4.

(a) Find the inverse matrix of the next matrices.

$$A_1 = \begin{pmatrix} 2 & -3 \\ 4 & 5 \end{pmatrix} \quad A_2 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$A_3 = \begin{pmatrix} 1 & 5 & -3 \\ 2 & 11 & 1 \\ 2 & 9 & -11 \end{pmatrix} \quad A_4 = \begin{pmatrix} 1 & 5 & -3 \\ 2 & 11 & 1 \\ 1 & 4 & -10 \end{pmatrix}.$$

(b) Suppose

$$A = \begin{pmatrix} 2 & 5 & -3 \\ 2 & 1 & 1 \\ 2 & 0 & -1 \end{pmatrix}.$$

Prove that A verify the relation $A^3 - 2A^2 - 5A - 24I_3 = 0$. Deduce the inverse matrix of A.

Exercise 5. Let $A = (a_{ij})$ be a skew- symmetric matrix of order n ($A = (a_{ij})$ is a square matrix of order n such that $A = A^T$.)

(a) Calculate $|A|$ for n = 2,3,4.

(b) Prove that $|A| = 0$ if n is an odd number.

Exercise 6. Let A be a matrix of order n. Using $|A|$, write $|adj(A)|$.