Series no. 3 solution

Exercise 1

1.

A	B	$\overline{A}$	$\overline{B}$	$A . B$	$\overline{A} . \overline{B}$	$A + B$	$\overline{A} + \overline{B}$	$\overline{A . B}$	$\overline{A + B}$
0	0	1	1	0	**1**	0	**1**	**1**	**1**
0	1	1	0	0	**0**	1	**1**	**1**	**0**
1	0	0	1	0	**0**	1	**1**	**1**	**0**
1	1	0	0	1	**0**	1	**0**	**0**	**0**

2.

- $A + \overline{A} . B = (A + \overline{A}) . (A + B)$ (Distributivity)
  $= 1 . (A + B)$ (Complementarity)
  $= A + B$ (Neutral element)
- $A . (\overline{A} + B) = A . \overline{A} + A . B$ (Distributivity)
  $= 0 + A . B$ (Complementarity)
  $= A . B$ (Neutral element)

3.

a) $\overline{A} . B + A . B = (\overline{A} + A) . B$
$= 1 . B$
$= B$

b) $(A + B) . (A + \overline{B}) = A + (B . \overline{B})$
$= A + 0$
$= A$

c) $A + A . B = (A . 1) + (A . B)$
$= A . (1 + B)$
$= A . 1$ (Absorbing element)
$= A$

d) $A . (A + B) = (A + 0) . (A + B)$
$= A + 0 . B$
$= A + 0$ (Absorbing element)
$= A$

e) $\overline{A}.\overline{B} + \overline{A + B + C + D} = \overline{(\overline{\overline{A}.\overline{B}}).(A + B + C + D)}$ (De Morgan's theorem)

$= \overline{(\overline{\overline{A}} + \overline{\overline{B}}).(A + B + C + D)}$

$= \overline{(A + B).(A + B + C + D)}$

$= \overline{(A + B).((A + B) + (C + D))}$

From (d): $A.(A + B) = A$

=> $(A + B).((A + B) + (C + D)) = (A + B)$

Then:

$\overline{A}.\overline{B} + \overline{A + B + C + D} = \overline{(A + B)}$

f) $A + B.\overline{C} + \overline{A}.(\overline{B.\overline{C}}).(A.D + B) = (A + B.\overline{C}) + (\overline{A + B.\overline{C}}).(A.D + B)$

From question (2): $A + \overline{A}.B = A + B$

=> $(A + B.\overline{C}) + (\overline{A + B.\overline{C}}).(A.D + B) = (A + B.\overline{C}) + (A.D + B)$

Then:

$A + B.\overline{C} + \overline{A}.(\overline{B.\overline{C}}).(A.D + B) = (A + B.\overline{C}) + (A.D + B)$

$= (A + A.D) + (B + B.\overline{C})$ (Commutativity and Associativity)

From (c): $A + A.B = A$

=> $(A + A.D) = A$ and $(B + B.\overline{C}) = B$

Then:

$A + B.\overline{C} + \overline{A}.(\overline{B.\overline{C}}).(A.D + B) = A + B$

g) $(A \oplus B).B + A.B = (\overline{A}.B + A.\overline{B}).B + A.B$

$= \overline{A}.B.B + A.\overline{B}.B + A.B$

$= \overline{A}.B + A.0 + A.B$ (Idempotence and Complementarity)

$= \overline{A}.B + A.B$

From (a): $\overline{A}.B + A.B = B$

Then:

$(A \oplus B).B + A.B = B$

Exercise 2

Truth table:

A	B	F (A, B)
0	0	1
0	1	1
1	0	1
1	1	0

$F(A, B) = \overline{A.B} = A \uparrow B = A$ **NAND** B

=> $\overline{A}.\overline{B} + \overline{A}.B + A.\overline{B} = A$ **NAND** B

Exercise 3

1.

A	B	C	F (A,B,C)	Mintermes	Maxtermes
0	0	0	0		$A + B + C$
0	0	1	1	$\bar{A} . \bar{B} . C$	
0	1	0	1	$\bar{A} . B . \bar{C}$	
0	1	1	0		$A + \bar{B} + \bar{C}$
1	0	0	1	$A . \bar{B} . \bar{C}$	
1	0	1	1	$A . \bar{B} . C$	
1	1	0	1	$A . B . \bar{C}$	
1	1	1	0		$\bar{A} + \bar{B} + \bar{C}$

- Disjunctive canonical form (1st canonical form):

  $F(A, B, C) = \bar{A} . \bar{B} . C + \bar{A} . B . \bar{C} + A . \bar{B} . \bar{C} + A . \bar{B} . C + A . B . \bar{C}$

- Conjunctive canonical form (2nd canonical form):

  $F(A, B, C) = (A + B + C) . (A + \bar{B} + \bar{C}) . (\bar{A} + \bar{B} + \bar{C})$

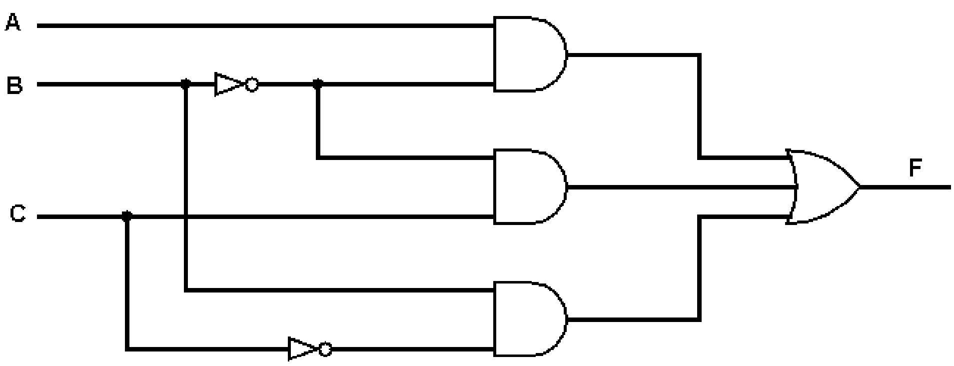
2.

- Karnaugh map :

BC / A	00	01	11	10
0	**0**	**1**	**0**	**1**
1	**1**	**1**	**0**	**1**

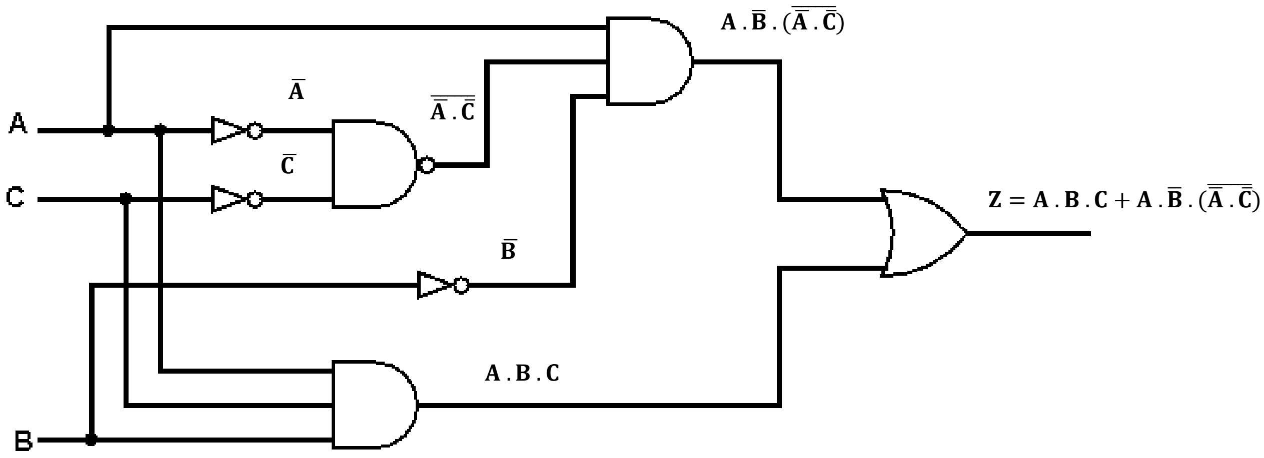
- The simplified sum of products (SOP) expression: $F (A, B, C) = A . \bar{B} + \bar{B} . C + B . \bar{C}$

3. Logic diagram:



Exercise 4

1.



$Z = A . B. C + A . \overline{B} . (\overline{\overline{A} . \overline{C}})$

2. Simplification:

$Z = A . B . C + A . \overline{B} . (\overline{\overline{A}.\overline{C}})$

$Z = A . B . C + A . \overline{B} . (\overline{\overline{A}} + \overline{\overline{C}})$

$Z = A. B. C + A . \overline{B} . (A + C)$

$Z = A. B. C + A . \overline{B} . A + A . \overline{B} . C$

$Z = A. B. C + A . \overline{B} \ + A . \overline{B} . C$

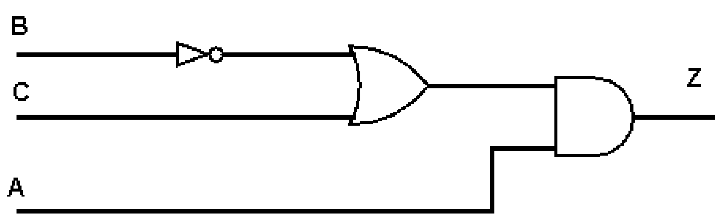
$Z = A . C . (B + \overline{B}) + A . \overline{B}$

$Z = A . C .1 + A . \overline{B}$

$Z = A . C + A \overline{. B}$

$Z = A . (\overline{B} \ + C)$

3.



Exercise 5

1. $F1\,(A, B, C) = A.\bar{B}.C + A.B.\bar{C} + A.B.C$

   - Karnaugh map:

BC \ A	BC	$\bar{B}C$	$\bar{B}\bar{C}$	$B\bar{C}$
A	1	1	0	1
$\bar{A}$	0	0	0	0

   - The simplified function:

     $F1\,(A, B, C) = A.B + A.C$

2. $F2\,(A, B, C) = \bar{A}.\bar{B}.\bar{C} + A.\bar{B} + A.B.C$

   $F2\,(A, B, C) = \bar{A}.\bar{B}.\bar{C} + A.\bar{B}.(C + \bar{C}) + A.B.C$

   $F2\,(A, B, C) = \bar{A}.\bar{B}.\bar{C} + A.\bar{B}.C + A.\bar{B}.\bar{C} + A.B.C$

   - Karnaugh map:

BC \ A	BC	$\bar{B}C$	$\bar{B}\bar{C}$	$B\bar{C}$
A	1	1	1	0
$\bar{A}$	0	0	1	0

   - The simplified function:

     $F2(A, B, C) = A.C + \bar{B}.\bar{C}$

3. $F3\,(A, B, C) = \bar{A}.\bar{B} + \bar{A}.B.\bar{C} + \bar{B}.\bar{C} + A.\bar{B}.C$

   $F3\,(A, B, C) = \bar{A}.\bar{B}.(C + \bar{C}) + \bar{A}.B.\bar{C} + \bar{B}.\bar{C}.(A + \bar{A}) + A.\bar{B}.C$

   $F3\,(A, B, C) = \bar{A}.\bar{B}.C + \bar{A}.\bar{B}.\bar{C} + \bar{A}.B.\bar{C} + A.\bar{B}.\bar{C} + \bar{A}.\bar{B}.\bar{C} + A.\bar{B}.C$

   $F3\,(A, B, C) = \bar{A}.\bar{B}.C + \bar{A}.\bar{B}.\bar{C} + \bar{A}.B.\bar{C} + A.\bar{B}.\bar{C} + A.\bar{B}.C$

- Karnaugh map:

A \ BC	BC	$\bar{B}C$	$\bar{B}\bar{C}$	$B\bar{C}$
A	0	1	1	0
$\bar{A}$	0	1	1	1

- The simplified function:

  $F3(A, B, C) = \bar{B} + \bar{A}.\bar{C}$

4. $F4\ (A, B, C, D) = B.\bar{C}.\bar{D} + \bar{A}.B.\bar{D} + A.B.C.\bar{D}$

   $F4\ (A, B, C, D) = B.\bar{C}.\bar{D}.(A + \bar{A}) + \bar{A}.B.\bar{D}.(C + \bar{C}) + A.B.C.\bar{D}$

   $F4\ (A, B, C, D) = A.B.\bar{C}.\bar{D} + \bar{A}.B.\bar{C}.\bar{D} + \bar{A}.B.C.\bar{D} + \bar{A}.B.\bar{C}.\bar{D} + A.B.C.\bar{D}$

   $F4\ (A, B, C, D) = A.B.\bar{C}.\bar{D} + \bar{A}.B.\bar{C}.\bar{D} + \bar{A}.B.C.\bar{D} + A.B.C.\bar{D}$

- Karnaugh map:

AB \ CD	CD	$\bar{C}D$	$\bar{C}\bar{D}$	$C\bar{D}$
AB	0	0	1	1
$\bar{A}B$	0	0	1	1
$\bar{A}\bar{B}$	0	0	0	0
$A\bar{B}$	0	0	0	0

- The simplified function:

  $F4(A, B, C, D) = B.\bar{D}$

5. $F5\ (A, B, C, D) = \bar{A} + A.B + A.\bar{B}.C + A.\bar{B}.C.D$

   $= \bar{A}.(B + \bar{B}).(C + \bar{C}).(D + \bar{D}) + A.B.(C + \bar{C}).(D + \bar{D}) + A.\bar{B}.C.(D + \bar{D})$
   $+ A.\bar{B}.C.D$

   $= \bar{A}.B.C.D + \bar{A}.B.C.\bar{D} + \bar{A}.B.\bar{C}.D + \bar{A}.B.\bar{C}.\bar{D} + \bar{A}.\bar{B}.C.D + \bar{A}.\bar{B}.C.\bar{D} +$
   $\bar{A}.\bar{B}.\bar{C}.D + \bar{A}.\bar{B}.\bar{C}.\bar{D} + A.B.C.D + A.B.C.\bar{D} + A.B.\bar{C}.D + A.B.\bar{C}.\bar{D}$
   $+ A.\bar{B}.C.D + A.\bar{B}.C.\bar{D} + A.\bar{B}.C.D$

$= \bar{A}.B.C.D + \bar{A}.B.C.\bar{D} + \bar{A}.B.\bar{C}.D + \bar{A}.B.\bar{C}.\bar{D} + \bar{A}.\bar{B}.C.D + \bar{A}.\bar{B}.C.\bar{D}$
$+ \bar{A}.\bar{B}.\bar{C}.D + \bar{A}.\bar{B}.\bar{C}.\bar{D} + A.B.C.D + A.B.C.\bar{D} + A.B.\bar{C}.D + A.B.\bar{C}.\bar{D}$
$+ A.\bar{B}.C.\bar{D} + A.\bar{B}.C.D$

- Karnaugh map:

AB \ CD	CD	$\bar{C}D$	$\bar{C}\bar{D}$	$C\bar{D}$
AB	1	1	1	1
$\bar{A}B$	1	1	1	1
$\bar{A}\bar{B}$	1	1	1	1
$A\bar{B}$	1	0	0	1

- The simplified function:

$F5(A, B, C) = B + \bar{A} + C$

6. $F6\ (A, B, C, D) = \bar{A}.\bar{B}.\bar{D} + \bar{A}.\bar{C}.\bar{D} + \bar{A}.B.C.\bar{D} + A.B.D + \bar{B}.\bar{C}.\bar{D} + A.\bar{B}.C.\bar{D}$

$F6\ (A, B, C, D) = \bar{A}.\bar{B}.\bar{D}.(C + \bar{C}) + \bar{A}.\bar{C}.\bar{D}.(B + \bar{B}) + \bar{A}.B.C.\bar{D} + A.B.D.(C + \bar{C})$
$+ \bar{B}.\bar{C}.\bar{D}.(A + \bar{A}) + A.\bar{B}.C.\bar{D}$

$F6\ (A, B, C, D) = \bar{A}.\bar{B}.\bar{D}.C + \bar{A}.\bar{B}.\bar{D}.\bar{C} + \bar{A}.B.\bar{C}.\bar{D} + \bar{A}.\bar{B}.\bar{C}.\bar{D} + \bar{A}.B.C.\bar{D} + A.B.C.D$
$+ A.B.\bar{C}.D + A.\bar{B}.\bar{C}.\bar{D} + \bar{A}.\bar{B}.\bar{C}.\bar{D} + A.\bar{B}.C.\bar{D}$

$F6\ (A, B, C, D) = \bar{A}.\bar{B}.C.\bar{D} + \bar{A}.\bar{B}.\bar{C}.\bar{D} + \bar{A}.B.\bar{C}.\bar{D} + \bar{A}.B.C.\bar{D} + A.B.C.D$
$+ A.B.\bar{C}.D + A.\bar{B}.\bar{C}.\bar{D} + A.\bar{B}.C.\bar{D}$

- Karnaugh map:

AB \ CD	CD	$\bar{C}D$	$\bar{C}\bar{D}$	$C\bar{D}$
AB	1	1	0	0
$\bar{A}B$	0	0	1	1
$\bar{A}\bar{B}$	0	0	1	1
$A\bar{B}$	0	0	1	1

- The simplified function:

$F6\ (A, B, C, D) = \bar{A}.\bar{D} + A.B.D + \bar{B}.\bar{D}$