Chapter 04
Part 02:
Simplification using: the Karnaugh map

Simplification using: the Karnaugh map

Karnaugh Map Simplification Method

This method is often employed to address challenges faced in the algebraic method.

It is particularly useful when the number of variables in the function does not exceed 6. Beyond six variables, its application becomes challenging.

It relies on the visual inspection of tables arranged in such a way that two adjacent cells in both rows and columns only differ in the state of a single variable.

Simplification using: the Karnaugh map

Méthode de simplification de Karnaugh

Cette méthode est souvent utilisée pour remédier aux difficultés que l'on rencontre dans la méthode algébrique. Elle est très intéressante lorsque le nombre de variables de la fonction ne dépasse pas 6.

Au-delà de six variables, elle est difficile à utiliser.

Elle est basée sur **l'inspection visuelle** de tableaux disposés de façon que deux cases adjacentes en ligne et en colonne ne diffèrent que par l'état d'une variable et une seule.

Simplification using: the Karnaugh map

- Adjacent Terms

Let's consider the following expression: $A.B + A.\overline{B}$

- The two terms have the same set of variables.
- The only difference is the state of variable B, which changes.

If we apply simplification rules, we get:

- These terms are called adjacent.

$$AB + A\overline{B} = A(B + \overline{B}) = A$$

Simplification using: the Karnaugh map

- Example of Adjacent Terms
- The following terms are adjacent:

$$\bar{A}.B + A.B = B$$

$$A.\bar{B}.C + A.B.C = A.C$$

$$A.B.C.D + A.B.\bar{C}.D = A.B.D$$

- The following terms are not adjacent:

$$A.B + \bar{A}.\bar{B}$$

$$A.B.C + A.\bar{B}.\bar{C}$$

$$A.B.C.D + \bar{A}.\bar{B}.C.D$$

Simplification using: the Karnaugh map

- Description of the Karnaugh Map
- The Karnaugh method is based on the said rule.
- The method involves emphasizing, through a graphical approach (a table), all terms that are adjacent (differing only in the state of a single variable).
- The method can be applied to logical functions with 2, 3, 4, 5, and 6 variables.
- A Karnaugh map consists of 2^n cells (where N is the number of variables).

Simplification using: the Karnaugh map

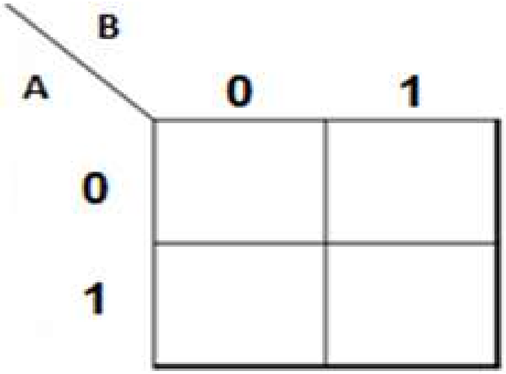


Table with 2 variables

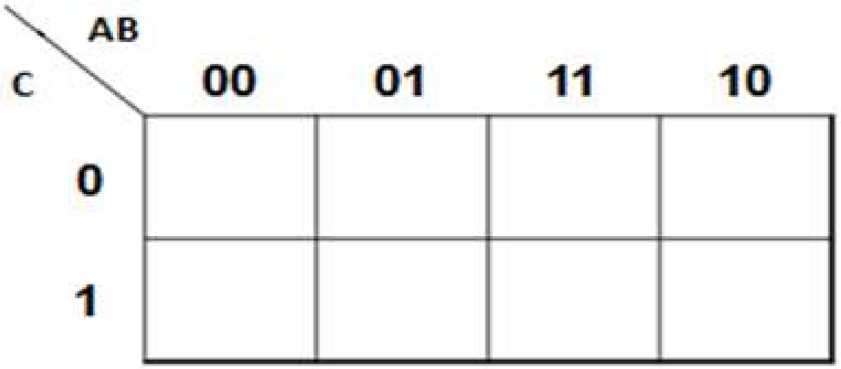


Table with 3 variables

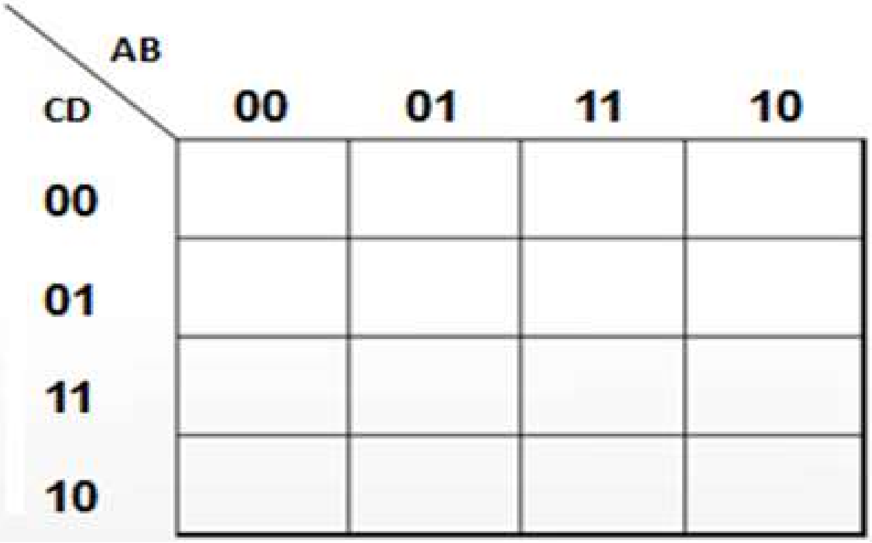


Table with 4 variables

Simplification using: the Karnaugh map

For example, the <u>shaded cell</u> in the following table corresponds to minterm m1 representing:

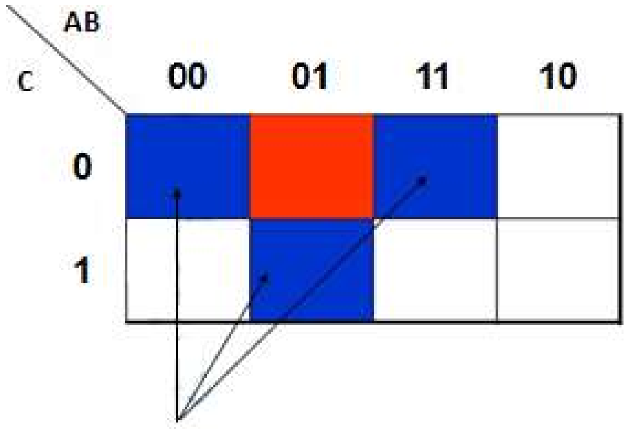
(x, y, z, t) = (0, 0, 0, 1)

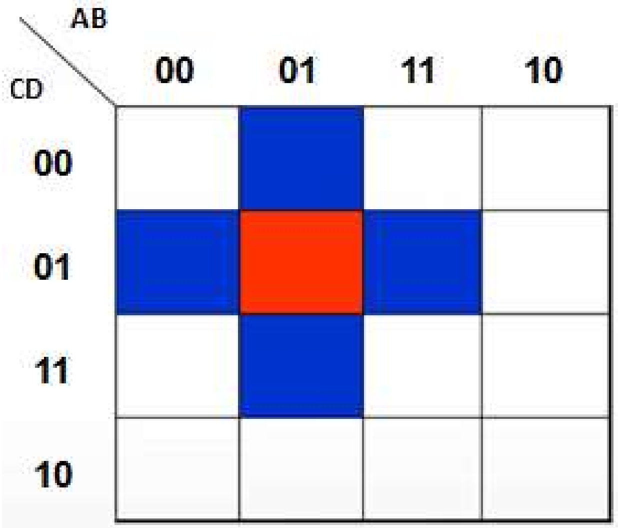
$\boldsymbol{m1 = \bar{x}\,\bar{y}\,\bar{z}\,t}$

zt \ xy	00	01	11	10
00	m_0	m_4	m_{12}	m_8
01	m_1	m_5	m_{13}	m_9
11	m_3	m_7	m_{15}	m_{11}
10	m_2	m_6	m_{14}	m_{10}

Simplification using: the Karnaugh map

- In a Karnaugh map, each cell has a certain number of adjacent cells.





- The three blue cells are adjacent to the red cell.

Simplification using: the Karnaugh map

- Transition from Truth Table to Karnaugh Map
- For each combination representing a minterm, it corresponds to a cell in the table that should be set to 1.
- For each combination representing a maxterm, it corresponds to a cell in the table that should be set to 0.
- When filling out the table, one must individually consider the minterms or the maxterms.

Simplification using: the Karnaugh map

A	B	C		S
0	0	0		**0**
0	0	1		**0**
0	1	0		**0**
0	1	1		1
1	0	0		**0**
1	0	1		1
1	1	0		1
1	1	1		1

Simplification using: the Karnaugh map

A	B	C	S
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

C \ AB	00	01	11	10
0				
1				

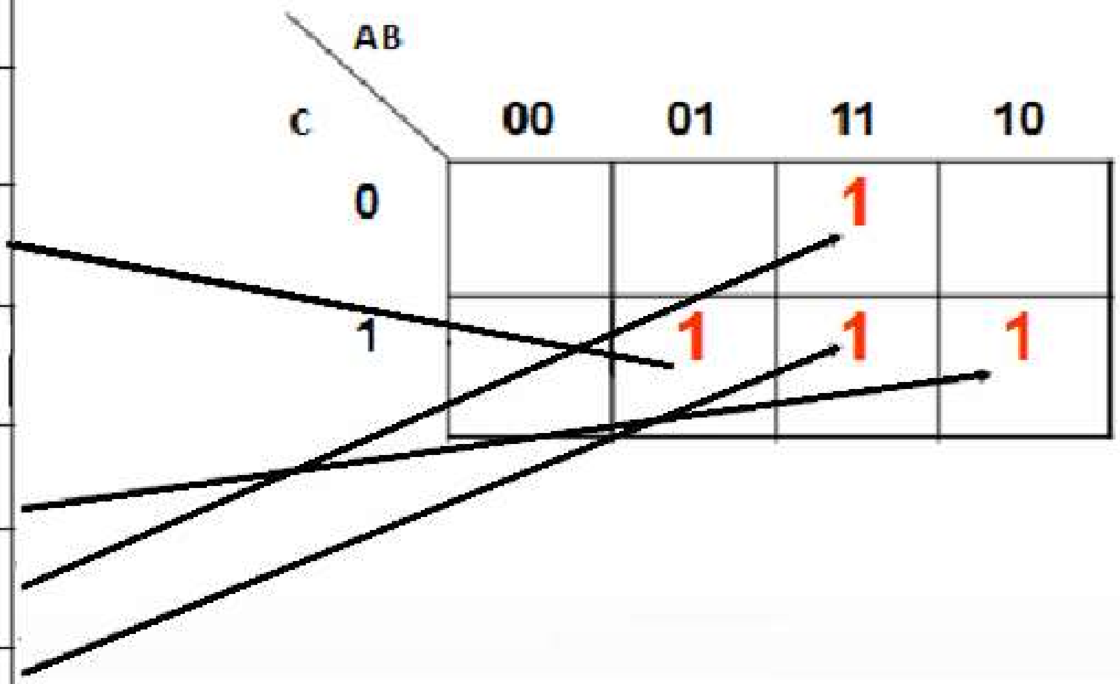
Simplification using: the Karnaugh map

A	B	C	S
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

C \ AB	00	01	11	10
0			1	
1		1	1	1

Simplification using: the Karnaugh map

A	B	C		S
0	0	0		0
0	0	1		0
0	1	0		0
0	1	1		1
1	0	0		0
1	0	1		1
1	1	0		1
1	1	1		1



Simplification using: the Karnaugh map

- Transition from Canonical Form to Karnaugh Map
- If the logical function is given in the first canonical form (disjunctive), then its representation is direct: each term corresponds to a single cell that should be set to 1.
- If the logical function is given in the second canonical form (conjunctive), then its representation is direct: each term corresponds to a single cell that should be set to 0.

Simplification using: the Karnaugh map

- **Exemple**

- F1(A, B, C) = $\sum(1,2,5,7)$

C \ AB	00	01	11	10
0		1		
1	1		1	1

- F2(A, B, C) = $\prod$ (0,2,3,6)

C \ AB	00	01	11	10
0	0	0	0	
1		0		

Simplification using: the Karnaugh map

- Simplification Method (Example: 3 variables)
- Basic idea:

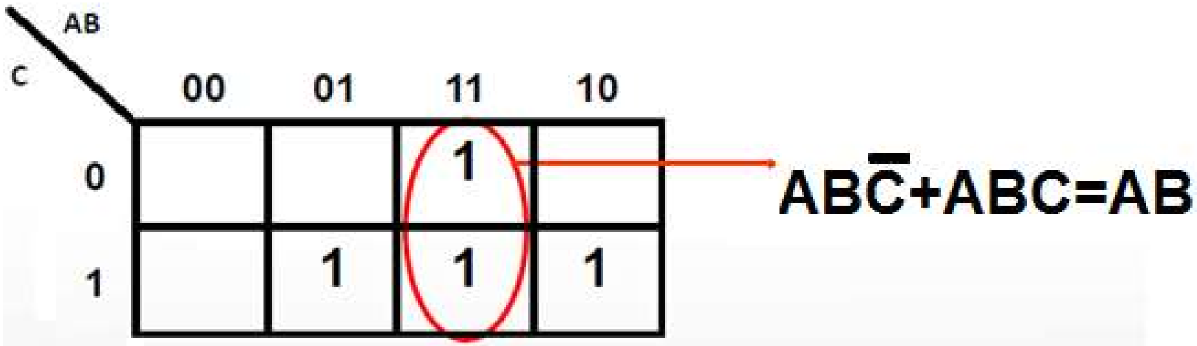
  Try to group

  adjacent cells

  containing 1s

  Try to create

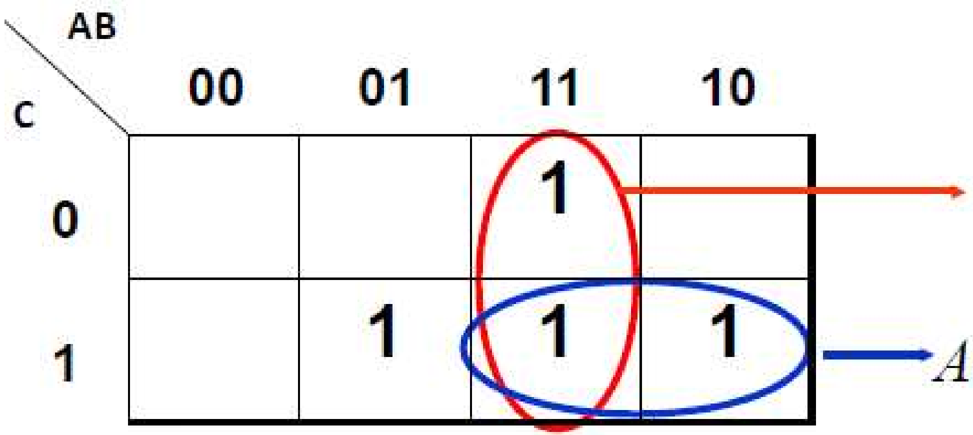
  groups with the maximum number of cells (16, 8, 4, or 2)



- In our example, we can only form groups of 2 cells.

Simplification using: the Karnaugh map

Since there are still cells outside of a group,
we repeat the same procedure: forming groups.
A cell can belong to multiple groups.

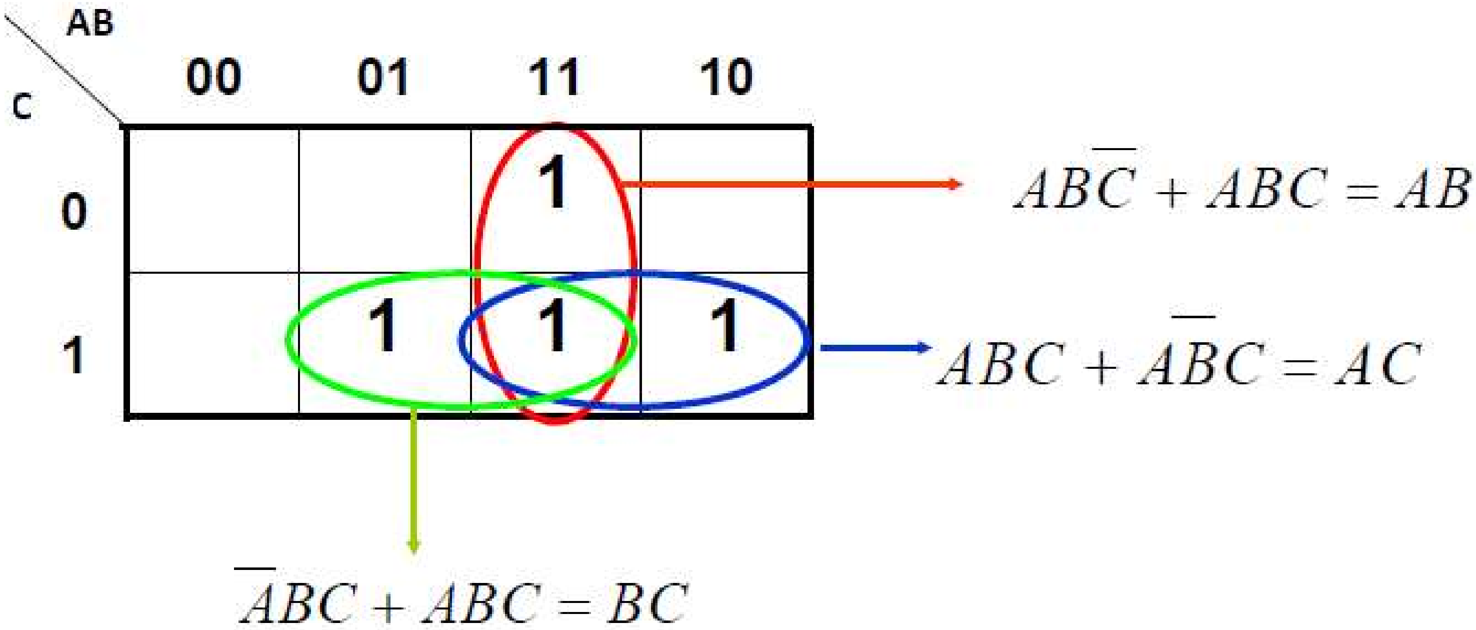


$$A B \overline{C} + A B C = A B$$

$$A B C + A \overline{B} C = A C$$

Simplification using: the Karnaugh map

- We stop when there are <u>no more **1's** outside</u> the groups.
- The final function is equal to the union (sum) of the terms after simplification.
- $F(A, B, C) = AB + AC + BC$



Simplification using: the Karnaugh map

- Summary
- For simplifying a function using the Karnaugh map, follow these steps:

1. Fill in the table from the truth table or canonical form.
2. Group cells in blocks of 16, 8, 4, 2, 1 (powers of 2), based on grouping adjacent **1s** into rectangular or square blocks. Each group should contain the maximum possible 1s, allowing the same terms to participate in multiple groups. Intersection between groups is allowed, but inclusion is not allowed.

Simplification using: the Karnaugh map

- 4. In a grouping:
  - If it contains a single term, we cannot eliminate any variables.
  - If it contains two terms, we can eliminate one variable (the one that changes state).
  - If it contains 4 terms, we can eliminate 2 variables.
  - If it contains 8 terms, we can eliminate 3 variables, and so on.

5. A cell with 1 must be appropriate to at minimum one grouping.
6. The final logical expression is the union (sum) of the groupings corresponding to the blocks obtained after simplification and elimination of variables that change state within the block.

Simplification using: the Karnaugh map

- **Exemple 1 :**
- 3 variables

C \ AB	00	01	11	10
0			1	
1	1	1	1	1

Simplification using: the Karnaugh map

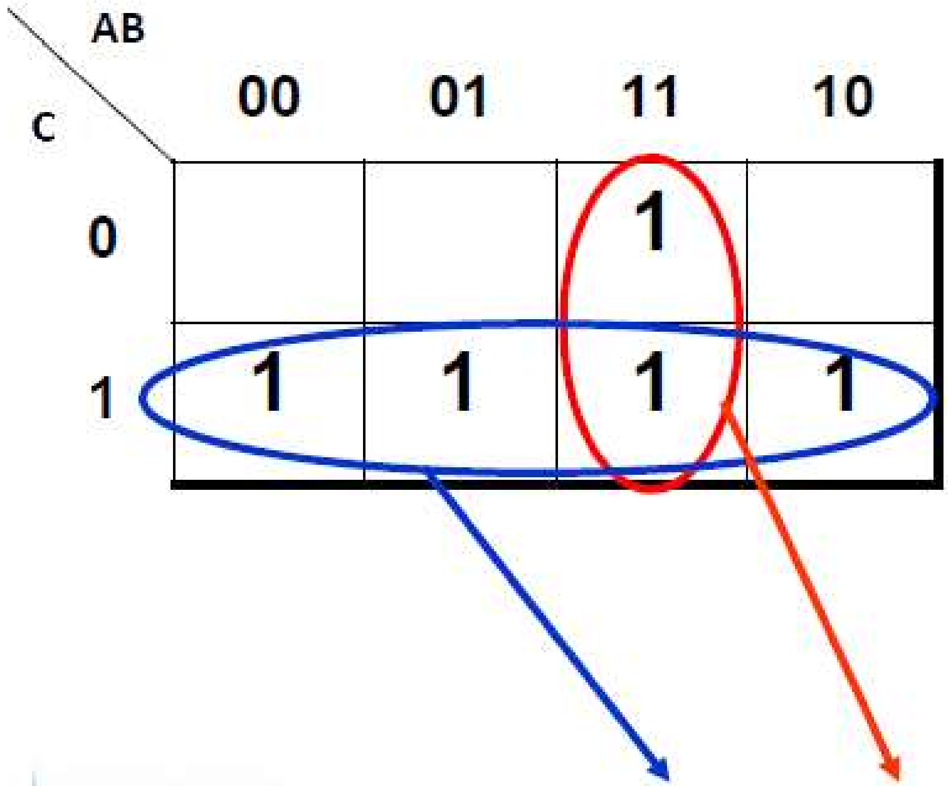
- **Exemple 1 :**
- 3 variables

C \ AB	00	01	11	10
0			1	
1	1	1	1	1

$$F(A,B,C) = C +$$

Simplification using: the Karnaugh map

- **Exemple 1 :**
- 3 variables



$$F(A,B,C) = C + AB$$

Simplification using: the Karnaugh map

- **Exemple 2:**
  4 variables

CD \ AB	00	01	11	10
00				1
01	1	1	1	1
11				
10		1		

$$F(A, B, C, D) =$$

Simplification using: the Karnaugh map

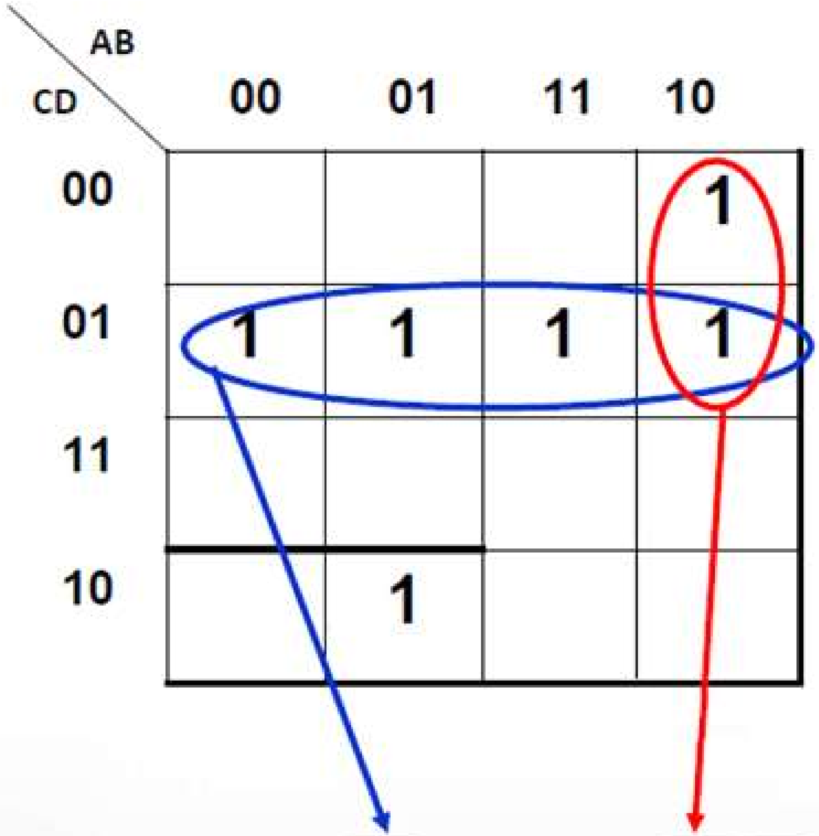
- **Exemple 2:**
  4 variables

CD \ AB	00	01	11	10
00				1
01	1	1	1	1
11				
10		1		

$$F(A, B, C, D) = \overline{C}.D +$$

Simplification using: the Karnaugh map

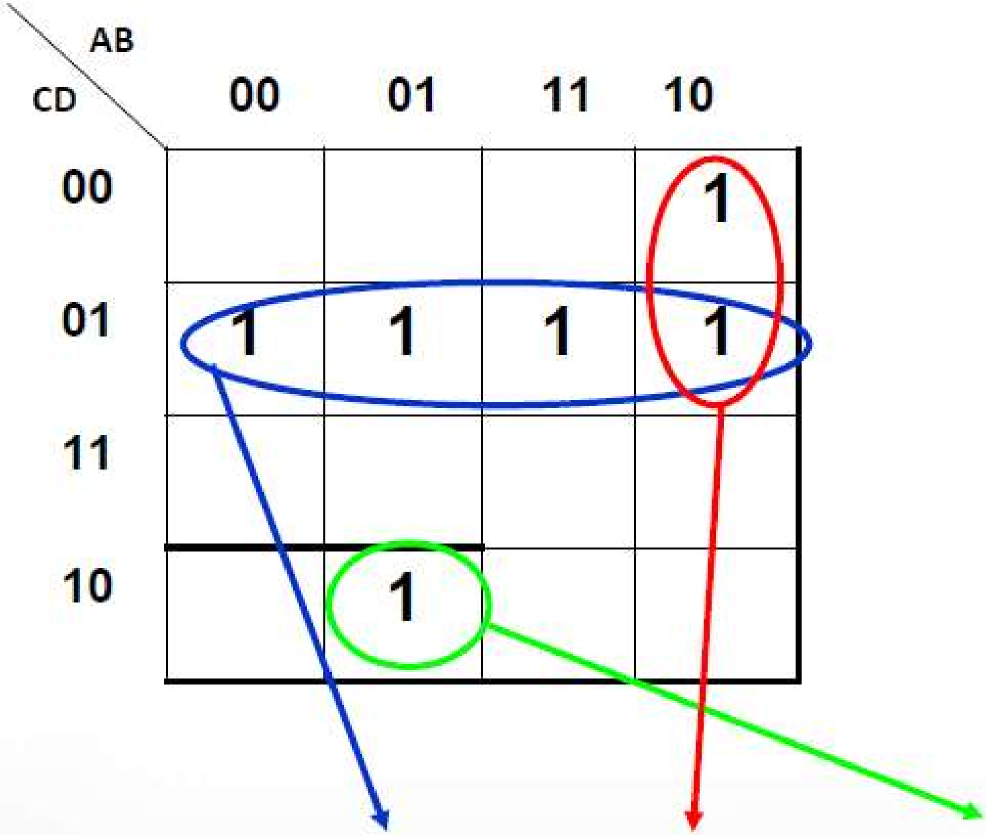
- **Exemple 2:**
  4 variables



$$F(A, B, C, D) = \overline{C}.D + A.\overline{B}.\overline{C} +$$

Simplification using: the Karnaugh map

- **Exemple 2:**
  4 variables



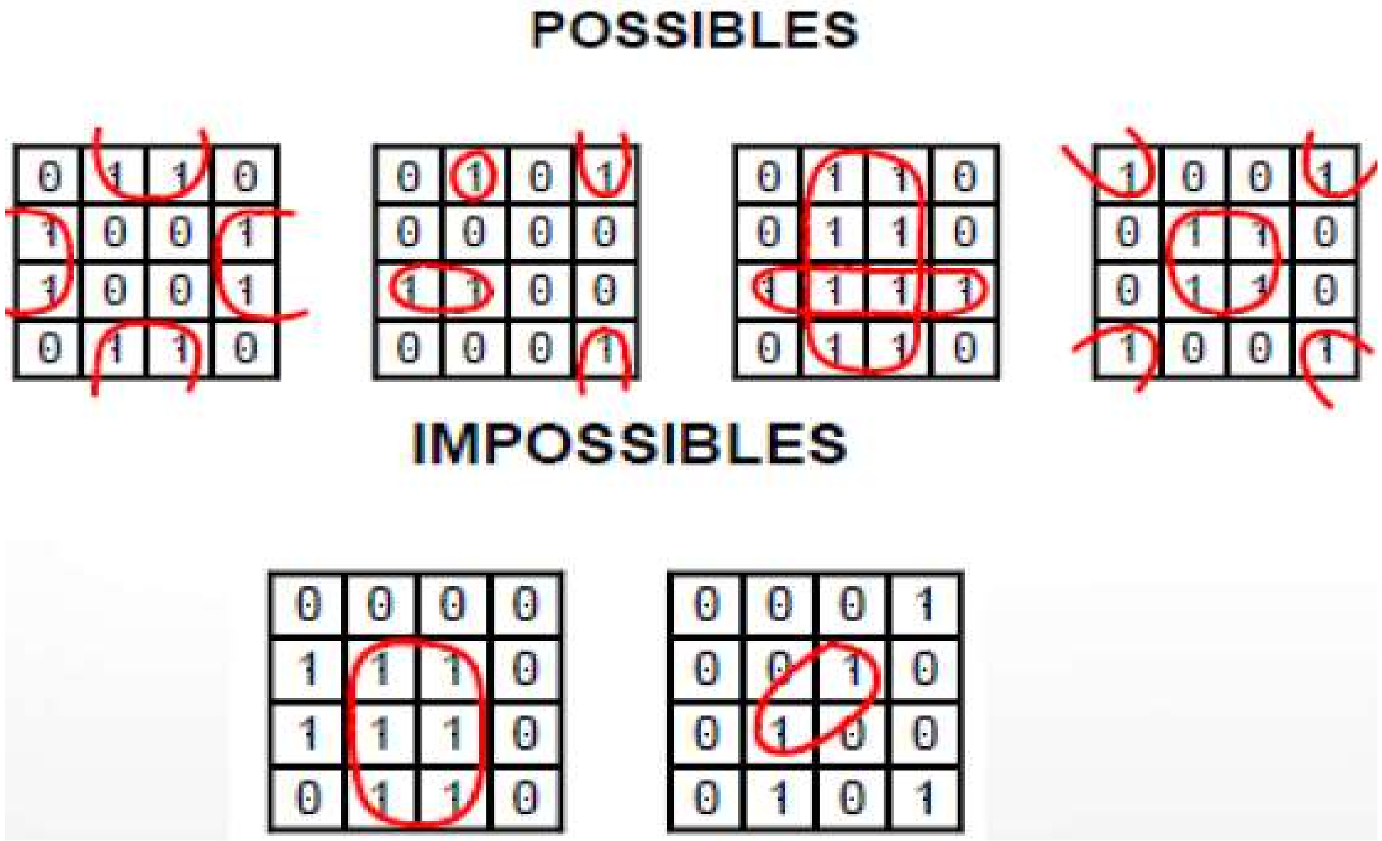
$$F(A,B,C,D) = \overline{C}.D + A.\overline{B}.\overline{C} + \overline{A}.B.C.\overline{D}$$

Simplification using: the Karnaugh map

- Remarks: With the Karnaugh method, the goal is to minimize the number of groupings while maximizing the number of cells within each grouping. The corner cells are Consider:

Adjacent cells.

- **Exemples**



Simplification using: the Karnaugh map

- **Exemple 3 :**

4 variables

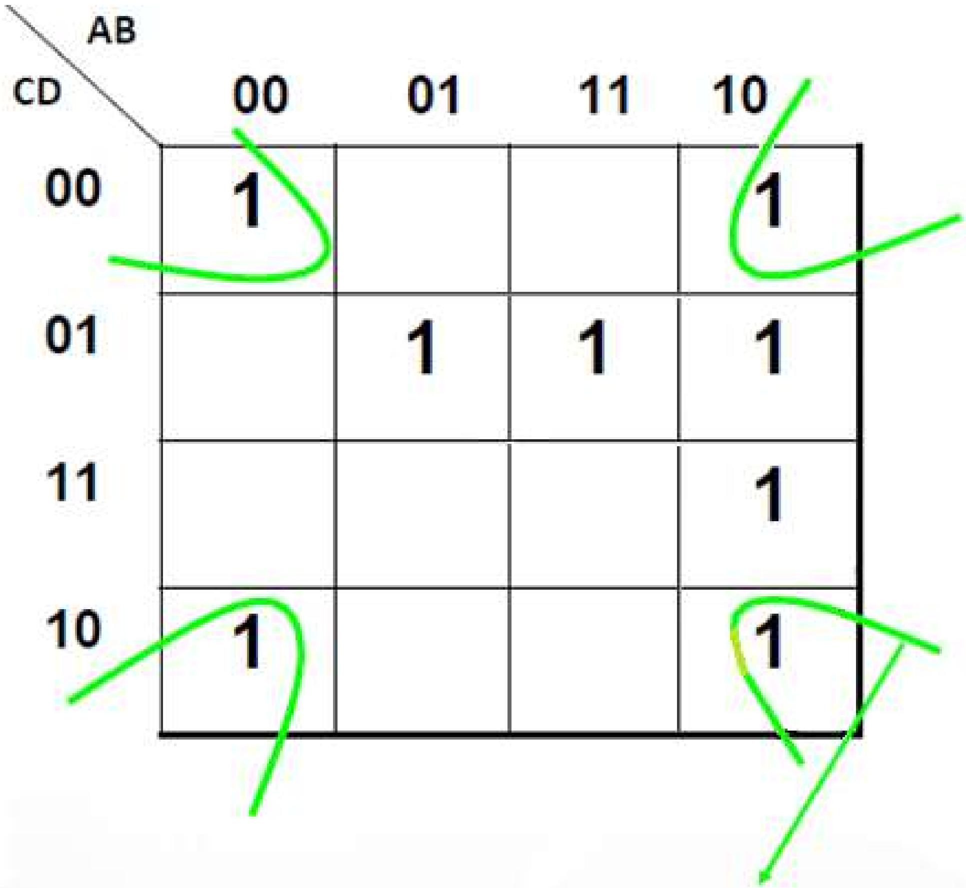
CD \ AB	00	01	11	10
00	1			1
01		1	1	1
11				1
10	1			1

$$F(A,B,C,D) =$$

Simplification using: the Karnaugh map

- **Exemple 3 :**

4 variables

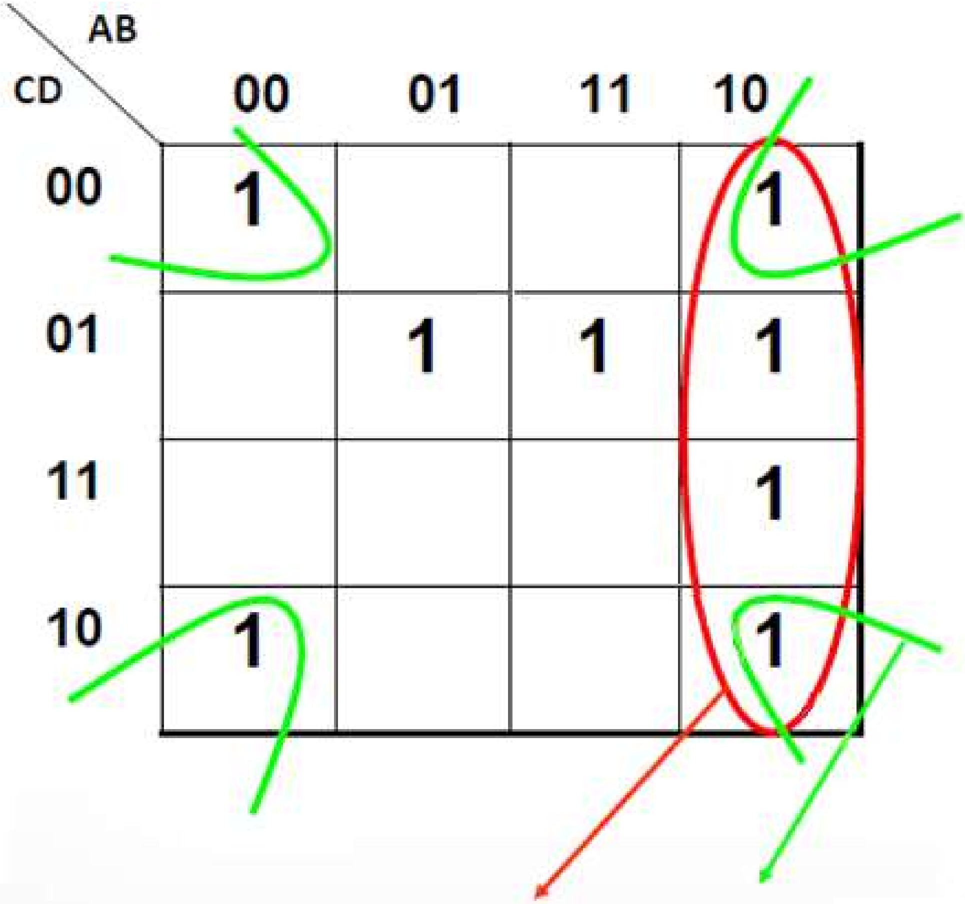


$$F(A, B, C, D) = \overline{B}\,\overline{D}$$

Simplification using: the Karnaugh map

- **Exemple 3 :**

4 variables

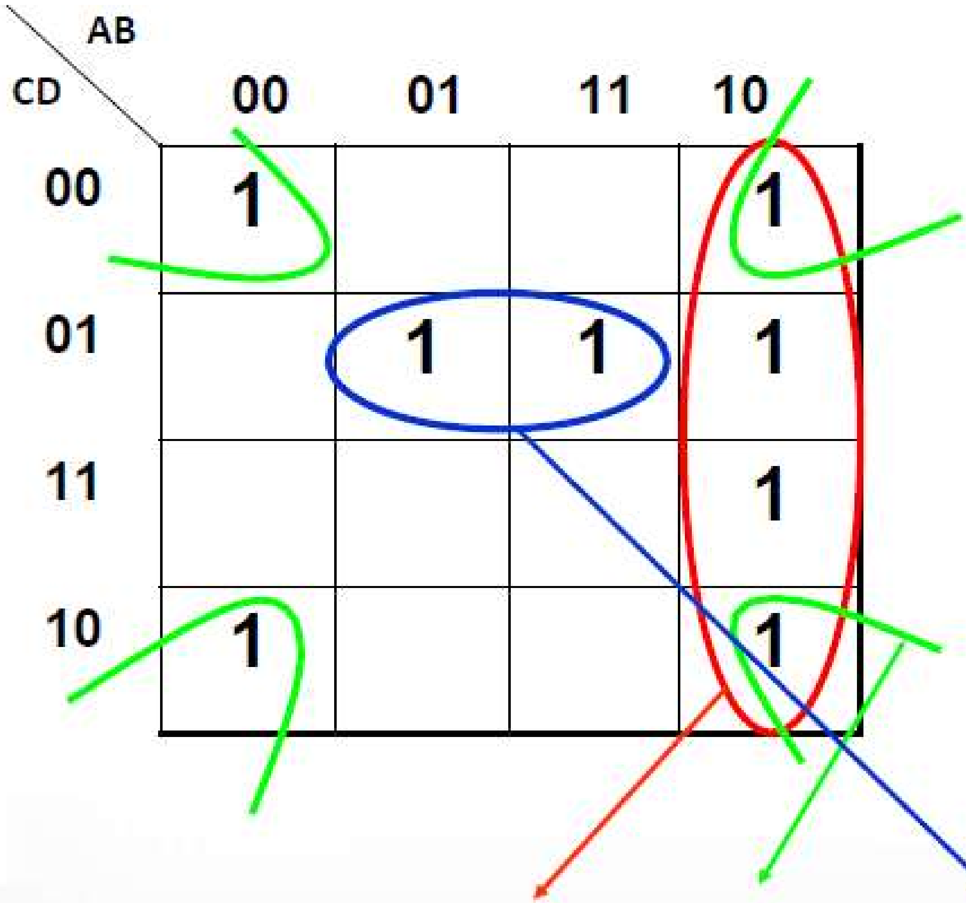


$$F(A,B,C,D) = A\overline{B} + \overline{B}\,\overline{D}$$

Simplification using: the Karnaugh map

- **Exemple 3 :**

4 variables



$$F(A,B,C,D) = A\overline{B} + \overline{B}\,\overline{D} + B\overline{C}D$$

Simplification using: the Karnaugh map

- Simplification of the following expressions:
- **Exemple1:** $F(A,B,C,D)=\Sigma(3,5,6,7,8,9,10,11,12,13,14,15)$

C.D \ A.B	00	01	11	10
00				
01				
11				
10				

Simplification using: the Karnaugh map

- Simplification of the following expressions:
- **Exemple1:** $F(A,B,C,D)=\Sigma(3,5,6,7,8,9,10,11,12,13,14,15)$

C.D \ A.B	00	01	11	10
00	0	0	1	1
01	0	1	1	1
11	1	1	1	1
10	0	1	1	1

Simplification using: the Karnaugh map

- Simplification of the following expressions:
- **Exemple1:** $F(A,B,C,D)=\Sigma(3,5,6,7,8,9,10,11,12,13,14,15)$

$$F(A,B,C,D) = A$$

C.D \ A.B	00	01	11	10
00	0	0	1	1
01	0	1	1	1
11	1	1	1	1
10	0	1	1	1

Simplification using: the Karnaugh map

- Simplification of the following expressions:
- **Exemple1:** $F(A,B,C,D)=\Sigma(3,5,6,7,8,9,10,11,12,13,14,15)$

$$F(A,B,C,D) = A + C.D$$

C.D \ A.B	00	01	11	10
00	0	0	1	1
01	0	1	1	1
11	1	1	1	1
10	0	1	1	1

Simplification using: the Karnaugh map

- Simplification of the following expressions:
- **Exemple1:** $F(A,B,C,D)=\Sigma(3,5,6,7,8,9,10,11,12,13,14,15)$

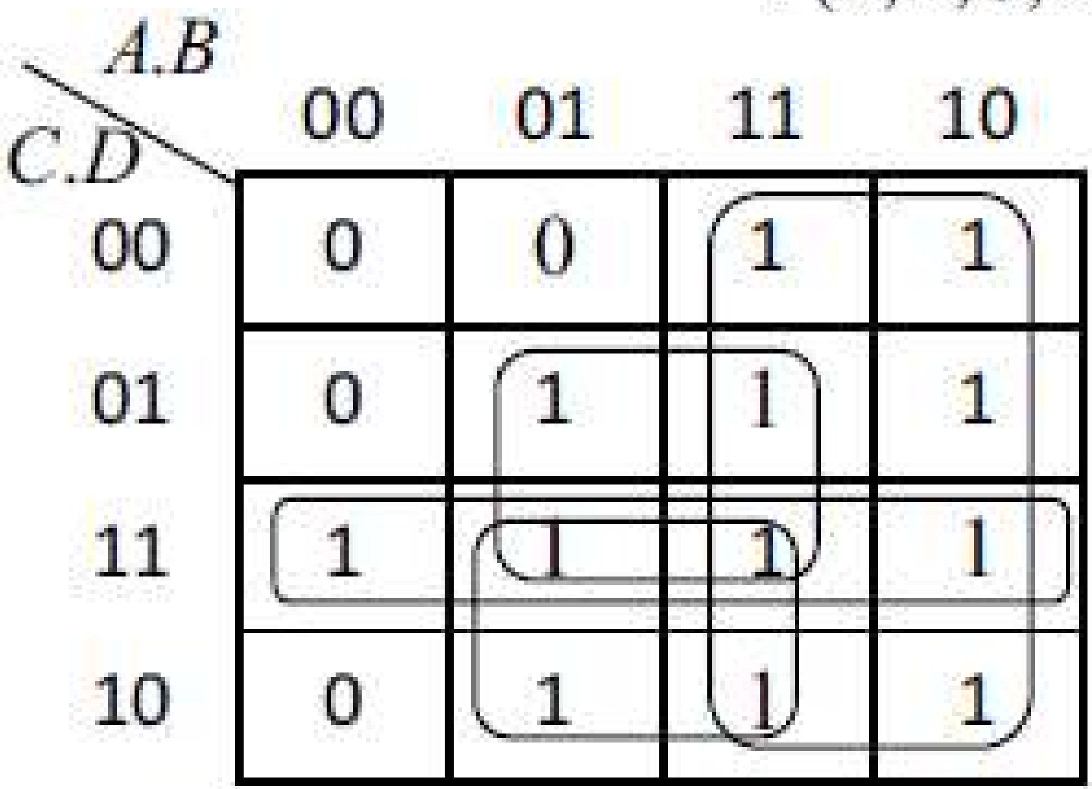
$$F(A,B,C,D) = A + C.D + B.C$$

C.D \ A.B	00	01	11	10
00	0	0	1	1
01	0	1	1	1
11	1	1	1	1
10	0	1	1	1

Simplification using: the Karnaugh map

- Simplification of the following expressions:
- **Exemple1:** $F(A,B,C,D)=\Sigma(3,5,6,7,8,9,10,11,12,13,14,15)$

$$F(A,B,C,D) = A + C.D + B.C + B.D$$



Simplification using: the Karnaugh map

- Simplification of 5-variable Karnaugh maps:
- **Exemple1:**

$e = 0$

cd \ ab	00	01	11	10
00				
01				
11				
10				

$e = 1$

cd \ ab	00	01	11	10
00				
01				
11				
10				

e	d	c	b	a	F
0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	1	0	1
0	0	0	1	1	1
0	0	1	0	0	0
0	0	1	0	1	1
0	0	1	1	0	1
0	0	1	1	1	1
0	1	0	0	0	0
0	1	0	0	1	1
0	1	0	1	0	1
0	1	0	1	1	1
0	1	1	0	0	0
0	1	1	0	1	0
0	1	1	1	0	0
0	1	1	1	1	1

e	d	c	b	a	F
1	0	0	0	0	1
1	0	0	0	1	1
1	0	0	1	0	1
1	0	0	1	1	1
1	0	1	0	0	0
1	0	1	0	1	0
1	0	1	1	0	0
1	0	1	1	1	0
1	1	0	0	0	1
1	1	0	0	1	0
1	1	0	1	0	0
1	1	0	1	1	0
1	1	1	0	0	1
1	1	1	0	1	0
1	1	1	1	0	1
1	1	1	1	1	0

Simplification using: the Karnaugh map

- Simplification of 5-variable Karnaugh maps:
- **Exemple1:**

$e = 0$

cd \ ab	00	01	11	10
00	0	2	3	1
01	8	10	11	9
11	12	14	15	13
10	4	6	7	5

$e = 1$

cd \ ab	00	01	11	10
00	16	18	19	17
01	24	26	27	25
11	28	30	31	29
10	20	22	23	21

Simplification using: the Karnaugh map

- Simplification of 5-variable Karnaugh maps:
- **Exemple1:**

$e = 0$

cd \ ab	00	01	11	10
00	0 (0)	1 (2)	1 (3)	0 (1)
01	0 (8)	1 (10)	1 (11)	0 (9)
11	0 (12)	0 (14)	0 (15)	0 (13)
10	1 (4)	1 (6)	1 (7)	1

$e = 1$

cd \ ab	00	01	11	10
00	1 (16)	0 (18)	0 (19)	1 (17)
01	1 (24)	0 (26)	0 (27)	1 (25)
11	0 (28)	0 (30)	0 (31)	1 (29)
10	0 (20)	0 (22)	0 (23)	0 (21)

Simplification using: the Karnaugh map

- Simplification of 5-variable Karnaugh maps:
- **Exemple1:**

$e = 0$

cd \ ab	00	01	11	10
00	0 (0)	1 (2)	1 (3)	0 (1)
01	0 (8)	1 (10)	1 (11)	1 (9)
11	0 (12)	0 (14)	0 (15)	1 (13)
10	1 (4)	1 (6)	1 (7)	1 (5)

$e = 1$

cd \ ab	00	01	11	10
00	0 (16)	1 (18)	0 (19)	0 (17)
01	0 (24)	1 (26)	0 (27)	1 (25)
11	0 (28)	0 (30)	1 (31)	1 (29)
10	0 (20)	1 (22)	1 (23)	0 (21)

Simplification using: the Karnaugh map

- Simplification of 5-variable Karnaugh maps:
- <u>**Exemple1:**</u>

$e = 0$

cd \ ab	00	01	11	10
00	0_{0}	1_{2}	1_{3}	0_{1}
01	0_{8}	1_{10}	1_{11}	1_{9}
11	0_{12}	0_{14}	0_{15}	1_{13}
10	1_{4}	1_{6}	1_{7}	1_{5}

$F = c\bar{d}\bar{e}$

$e = 1$

cd \ ab	00	01	11	10
00	0_{16}	1_{18}	0_{19}	0_{17}
01	0_{24}	1_{26}	0_{27}	1_{25}
11	0_{28}	0_{30}	1_{31}	1_{29}
10	0_{20}	1_{22}	1_{23}	0_{21}

Simplification using: the Karnaugh map

- Simplification of 5-variable Karnaugh maps:
- **Exemple1:**

$e = 0$

cd \ ab	00	01	11	10
00	0_{0}	1_{2}	1_{3}	0_{1}
01	0_{8}	1_{10}	1_{11}	1_{9}
11	0_{12}	0_{14}	0_{15}	1_{13}
10	1_{4}	1_{6}	1_{7}	1_{5}

$e = 1$

cd \ ab	00	01	11	10
00	0_{16}	1_{18}	0_{19}	0_{17}
01	0_{24}	1_{26}	0_{27}	1_{25}
11	0_{28}	0_{30}	1_{31}	1_{29}
10	0_{20}	1_{22}	1_{23}	0_{21}

$$F = c\bar{d}\bar{e} + b\bar{c}\bar{e} +$$

Simplification using: the Karnaugh map

- Simplification of 5-variable Karnaugh maps:
- **Exemple1:**

$e = 0$

cd \ ab	00	01	11	10
00	0 (0)	1 (2)	1 (3)	0 (1)
01	0 (8)	1 (10)	1 (11)	1 (9)
11	0 (12)	0 (14)	0 (15)	1 (13)
10	1 (4)	1 (6)	1 (7)	1 (5)

$e = 1$

cd \ ab	00	01	11	10
00	0 (16)	1 (18)	0 (19)	0 (17)
01	0 (24)	1 (26)	0 (27)	1 (25)
11	0 (28)	0 (30)	1 (31)	1 (29)
10	0 (20)	1 (22)	1 (23)	0 (21)

$$F = c\bar{d}\bar{e} + b\bar{c}\bar{e} + acde$$

Simplification using: the Karnaugh map

- Simplification of 5-variable Karnaugh maps:
- **Exemple1:**

$e = 0$

cd \ ab	00	01	11	10
00	0 (0)	1 (2)	1 (3)	0 (1)
01	0 (8)	1 (10)	1 (11)	1 (9)
11	0 (12)	0 (14)	0 (15)	1 (13)
10	1 (4)	1 (6)	1 (7)	1 (5)

$e = 1$

cd \ ab	00	01	11	10
00	0 (16)	1 (18)	0 (19)	0 (17)
01	0 (24)	1 (26)	0 (27)	1 (25)
11	0 (28)	0 (30)	1 (31)	1 (29)
10	0 (20)	1 (22)	1 (23)	0 (21)

même position Alors: pas de e

$F = c\bar{d}\bar{e} + b\bar{c}\bar{e} + acde + a\bar{b}d$

Simplification using: the Karnaugh map

- Simplification of 5-variable Karnaugh maps:
- **Exemple1:**

$e = 0$

cd \ ab	00	01	11	10
00	0 (0)	1 (2)	1 (3)	0 (1)
01	0 (8)	1 (10)	1 (11)	1 (9)
11	0 (12)	0 (14)	0 (15)	1 (13)
10	1 (4)	1 (6)	1 (7)	1 (5)

$e = 1$

cd \ ab	00	01	11	10
00	0 (16)	1 (18)	0 (19)	0 (17)
01	0 (24)	1 (26)	0 (27)	1 (25)
11	0 (28)	0 (30)	1 (31)	1 (29)
10	0 (20)	1 (22)	1 (23)	0 (21)

$$F = c\bar{d}\bar{e} + b\bar{c}\bar{e} + acde + a\bar{b}d + bc\bar{d}$$

Simplification using: the Karnaugh map

- Simplification of 5-variable Karnaugh maps:
- **Exemple1:**

$e = 0$

cd \ ab	00	01	11	10
00	0 (0)	1 (2)	1 (3)	0 (1)
01	0 (8)	1 (10)	1 (11)	1 (9)
11	0 (12)	0 (14)	0 (15)	1 (13)
10	1 (4)	1 (6)	1 (7)	1 (5)

$e = 1$

cd \ ab	00	01	11	10
00	0 (16)	1 (18)	0 (19)	0 (17)
01	0 (24)	1 (26)	0 (27)	1 (25)
11	0 (28)	0 (30)	1 (31)	1 (29)
10	0 (20)	1 (22)	1 (23)	0 (21)

$$F = c\bar{d}\bar{e} + b\bar{c}\bar{e} + acde + a\bar{b}d + bc\bar{d} + \bar{a}b\bar{c}$$

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10	1 (4)	1 (6)	1 (7)	1 (5)

$e = 1$

cd \ ab	00	01	11	10
00	0 (16)	1 (18)	0 (19)	0 (17)
01	0 (24)	1 (26)	0 (27)	1 (25)
11	0 (28)	0 (30)	1 (31)	1 (29)
10	0 (20)	1 (22)	1 (23)	0 (21)

$$F = c\bar{d}\bar{e} + b\bar{c}\bar{e} + acde + a\bar{b}d + bc\bar{d} + \bar{a}b\bar{c}$$

Simplification using: the Karnaugh map

- Simplification of 5-variable Karnaugh maps:
- **Exemple1:**

F(A, B,C,D, E) =Σ(0,1,2,4,6,8,9,10,14,16,17,18,19,22,24,25,26,27,30)

D.E \ B.C	00	01	11	10
00				
01				
11				
10				

$A = 0$

D.E \ B.C	00	01	11	10
00				
01				
11				
10				

$A = 1$

$F(A,B,C,D,E) =$

Simplification using: the Karnaugh map

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- **Exemple1:**

F(A, B,C,D, E) =Σ(0,1,2,4,6,8,9,10,14,16,17,18,19,22,24,25,26,27,30)

D.E \ B.C	00	01	11	10
00	1	1	0	1
01	1	0	0	1
11	0	0	0	0
10	1	1	1	1

$A=0$

D.E \ B.C	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	0	0	1
10	1	1	1	1

$A=1$

$$F(A,B,C,D,E)=$$

Simplification using: the Karnaugh map

- Simplification of 5-variable Karnaugh maps:
- **Exemple1:**

F(A, B,C,D, E) =Σ(0,1,2,4,6,8,9,10,14,16,17,18,19,22,24,25,26,27,30)

D.E \ B.C	00	01	11	10
00	1	1	0	1
01	1	0	0	1
11	0	0	0	0
10	1	1	1	1

$A = 0$

D.E \ B.C	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	0	0	1
10	1	1	1	1

$A = 1$

$$F(A,B,C,D,E) = D.\overline{E}$$

Simplification using: the Karnaugh map

- Simplification of 5-variable Karnaugh maps:
- **Exemple1:**

F(A, B,C,D, E) =Σ(0,1,2,4,6,8,9,10,14,16,17,18,19,22,24,25,26,27,30)

D.E \ B.C	00	01	11	10
00	1	1	0	1
01	1	0	0	1
11	0	0	0	0
10	1	1	1	1

$A = 0$

D.E \ B.C	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	0	0	1
10	1	1	1	1

$A = 1$

$$F(A,B,C,D,E) = D.\overline{E} + \overline{C}.\overline{D}$$

Simplification using: the Karnaugh map

- Simplification of 5-variable Karnaugh maps:
- **Exemple1:**

F(A, B,C,D, E) =Σ(0,1,2,4,6,8,9,10,14,16,17,18,19,22,24,25,26,27,30)

D.E \ B.C	00	01	11	10
00	1	1	0	1
01	1	0	0	1
11	0	0	0	0
10	1	1	1	1

$A = 0$

D.E \ B.C	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	0	0	1
10	1	1	1	1

$A = 1$

$$F(A,B,C,D,E) = D.\overline{E} + \overline{C}.\overline{D} + A.\overline{C}$$

Simplification using: the Karnaugh map

- Simplification of 5-variable Karnaugh maps:
- **Exemple1:**

F(A, B,C,D, E) =Σ(0,1,2,4,6,8,9,10,14,16,17,18,19,22,24,25,26,27,30)

$A = 0$

D.E \ B.C	00	01	11	10
00	1	1	0	1
01	1	0	0	1
11	0	0	0	0
10	1	1	1	1

$A = 1$

D.E \ B.C	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	0	0	1
10	1	1	1	1

$$F(A,B,C,D,E) = D.\overline{E} + \overline{C}.\overline{D} + A.\overline{C} + \overline{A}.\overline{B}.\overline{E}$$

Exemple : $F(A,B,C,D,E,F) = \sum(1,5,8,9,12,13,16,17,18,21,24,25,26,29,$
$33,35,37,39,40,41,44,45,48,49,50,51,53,55,56,57,58,61).$

E.F \ C.D	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

$A.B = 00$

E.F \ C.D	00	01	11	10
00	16	20	28	24
01	17	21	29	25
11	19	23	31	27
10	18	22	30	26

$A.B = 01$

E.F \ C.D	00	01	11	10
00	32	36	44	40
01	33	37	45	41
11	35	39	47	43
10	34	38	46	42

$A.B = 10$

E.F \ C.D	00	01	11	10
00	48	52	60	56
01	49	53	61	57
11	51	55	63	59
10	50	54	62	58

$A.B = 11$

Exemple : $F(A, B, C, D, E, F) = \sum(1, 5, 8, 9, 12, 13, 16, 17, 18, 21, 24, 25, 26, 29,$
$33, 35, 37, 39, 40, 41, 44, 45, 48, 49, 50, 51, 53, 55, 56, 57, 58, 61).$

$E.F$ \ $C.D$	00	01	11	10
00				
01				
11				
10				

$A.B = 00$

$E.F$ \ $C.D$	00	01	11	10
00				
01				
11				
10				

$A.B = 01$

$E.F$ \ $C.D$	00	01	11	10
00				
01				
11				
10				

$A.B = 10$

$E.F$ \ $C.D$	00	01	11	10
00				
01				
11				
10				

$A.B = 11$

$F(A, B, C, D, E, F) =$

Exemple : $F(A,B,C,D,E,F) = \sum(1,5,8,9,12,13,16,17,18,21,24,25,26,29,$
$33,35,37,39,40,41,44,45,48,49,50,51,53,55,56,57,58,61).$

E.F \ C.D	00	01	11	10
00	0	0	1	1
01	1	1	1	1
11	0	0	0	0
10	0	0	0	0

$A.B = 00$

E.F \ C.D	00	01	11	10
00	1	0	0	1
01	1	1	1	1
11	0	0	0	0
10	1	0	0	1

$A.B = 01$

E.F \ C.D	00	01	11	10
00	0	0	1	1
01	1	1	1	1
11	1	1	0	0
10	0	0	0	0

$A.B = 10$

E.F \ C.D	00	01	11	10
00	1	0	0	1
01	1	1	1	1
11	1	1	0	0
10	1	0	0	1

$A.B = 11$

$F(A,B,C,D,E,F) =$

Exemple : $F(A,B,C,D,E,F) = \sum(1,5,8,9,12,13,16,17,18,21,24,25,26,29,$
$33,35,37,39,40,41,44,45,48,49,50,51,53,55,56,57,58,61).$

E.F \ C.D	00	01	11	10
00	0	0	1	1
01	1	1	1	1
11	0	0	0	0
10	0	0	0	0

$A.B = 00$

E.F \ C.D	00	01	11	10
00	1	0	0	1
01	1	1	1	1
11	0	0	0	0
10	1	0	0	1

$A.B = 01$

E.F \ C.D	00	01	11	10
00	0	0	1	1
01	1	1	1	1
11	1	1	0	0
10	0	0	0	0

$A.B = 10$

E.F \ C.D	00	01	11	10
00	1	0	0	1
01	1	1	1	1
11	1	1	0	0
10	1	0	0	1

$A.B = 11$

$F(A,B,C,D,E,F) = \overline{E}.F$

Exemple : $F(A, B, C, D, E, F) = \sum(1, 5, 8, 9, 12, 13, 16, 17, 18, 21, 24, 25, 26, 29,$
$33, 35, 37, 39, 40, 41, 44, 45, 48, 49, 50, 51, 53, 55, 56, 57, 58, 61).$

E.F \ C.D	00	01	11	10
00	0	0	1	1
01	1	1	1	1
11	0	0	0	0
10	0	0	0	0

$A.B = 00$

E.F \ C.D	00	01	11	10
00	1	0	0	1
01	1	1	1	1
11	0	0	0	0
10	1	0	0	1

$A.B = 01$

E.F \ C.D	00	01	11	10
00	0	0	1	1
01	1	1	1	1
11	1	1	0	0
10	0	0	0	0

$A.B = 10$

E.F \ C.D	00	01	11	10
00	1	0	0	1
01	1	1	1	1
11	1	1	0	0
10	1	0	0	1

$A.B = 11$

$$F(A, B, C, D, E, F) = \overline{E}.F + \overline{B}.C.\overline{E}$$

Exemple : $F(A,B,C,D,E,F) = \sum(1,5,8,9,12,13,16,17,18,21,24,25,26,29,$
$33,35,37,39,40,41,44,45,48,49,50,51,53,55,56,57,58,61).$

E.F \ C.D	00	01	11	10
00	0	0	1	1
01	1	1	1	1
11	0	0	0	0
10	0	0	0	0

$A.B = 00$

E.F \ C.D	00	01	11	10
00	1	0	0	1
01	1	1	1	1
11	0	0	0	0
10	1	0	0	1

$A.B = 01$

E.F \ C.D	00	01	11	10
00	0	0	1	1
01	1	1	1	1
11	1	1	0	0
10	0	0	0	0

$A.B = 10$

E.F \ C.D	00	01	11	10
00	1	0	0	1
01	1	1	1	1
11	1	1	0	0
10	1	0	0	1

$A.B = 11$

$$F(A,B,C,D,E,F) = \overline{E}.F + \overline{B}.C.\overline{E} + B.\overline{D}.\overline{F}$$

Exemple : $F(A,B,C,D,E,F) = \sum(1,5,8,9,12,13,16,17,18,21,24,25,26,29,$
$33,35,37,39,40,41,44,45,48,49,50,51,53,55,56,57,58,61).$

E.F \ C.D	00	01	11	10
00	0	0	1	1
01	1	1	1	1
11	0	0	0	0
10	0	0	0	0

$A.B = 00$

E.F \ C.D	00	01	11	10
00	1	0	0	1
01	1	1	1	1
11	0	0	0	0
10	1	0	0	1

$A.B = 01$

E.F \ C.D	00	01	11	10
00	0	0	1	1
01	1	1	1	1
11	1	1	0	0
10	0	0	0	0

$A.B = 10$

E.F \ C.D	00	01	11	10
00	1	0	0	1
01	1	1	1	1
11	1	1	0	0
10	1	0	0	1

$A.B = 11$

$$F(A,B,C,D,E,F) = \overline{E}.F + \overline{B}.C.\overline{E} + B.\overline{D}.\overline{F} + A.\overline{C}.F$$

Synthesis of a Logic Circuit

It is essential to understand the system's operation.

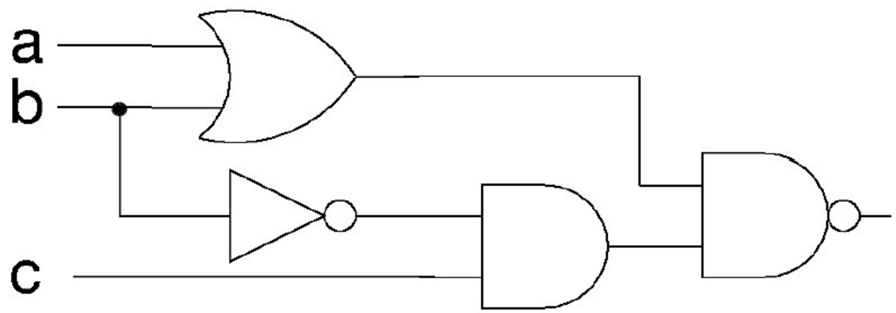
- Define the input variables.
- Define the output variables.
- Establish the truth table.
- Write algebraic equations for the outputs (based on the truth table).
- Perform simplifications (algebraic or using Karnaugh maps, etc.).
- Create the schematic with a minimal number of logic gates.

Analysis of a Logic Circuit

- Find its logical function
- Principle
- Provide the expression for the outputs of each gate/component based on the values of its inputs.
- Finally deduce the logical function(s) of the circuit.
- Later, one can
  - ✓ Determine the truth table of the circuit.
  - ✓ Simplify the logical function.

Analysis of a Logic Circuit

- Example of Logic Circuit Analysis
- 3 inputs,
- 1 output Composed solely of OR, AND, and NOT logic gates

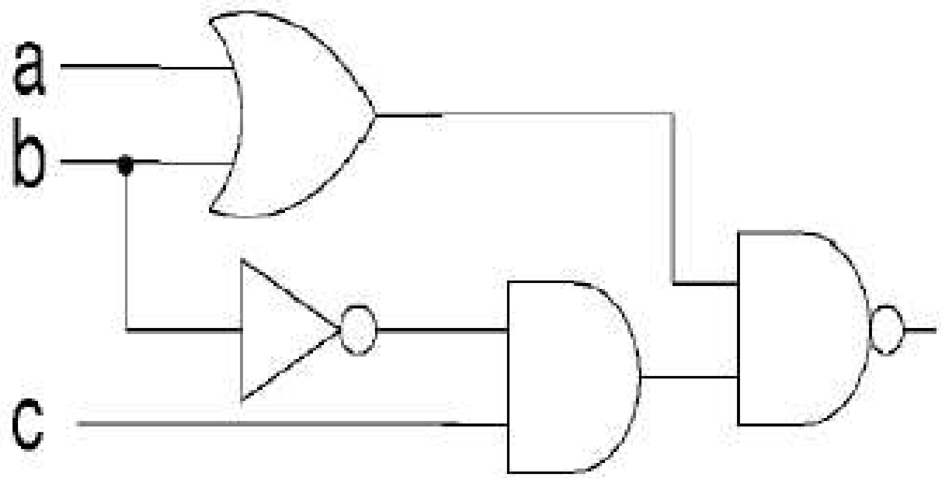


- From its logic diagram:

$$f(a,b,c) = \overline{(a+b)\bullet(\bar{b}\bullet c)}$$

Analysis of a Logic Circuit

$$f(a,b,c) = \overline{(a+b)\bullet(\bar{b}\bullet c)}$$



a	b	c	f
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1