Abdelhad Boussouf University Center - Mila
Institute of Nature and Life Sciences
Department of Earth and Universe Sciences

1st Year License
Geology
Mathematics 01

Serie of T.D N ° 4 (S1)

Exercice 2 : study of a sequence of the type $u_{n+1} = a \times u_n + b$

Let the sequence (u_n) be defined by $u_1 = 2$ and, for any integer n of $\mathbb{N}^{\times}$, $u_{n+1} = 3u_n - 2$.
We consider the sequence (v_n) defined on $\mathbb{N}^*$ by $v_n = u_n + \alpha$ (with $\alpha \in \mathbb{R}$).

1) Determine α so that (v_n) is a geometric, and specify the reason et le premier terme.
2) Write v_n, then u_n as funcion of n.

Exercice 3

Specify, if possible, the variations and the limit of the sequences (v_n) suivantes :

1) $v_n = (-1)^n + 1$
2) $v_n = -3 \times 2^n$
3) $v_n = 3 \times \left(\frac{-2}{3}\right)^n + 5$
4) $v_n = \left(\frac{-5}{3}\right)^{n+1}$

Exercice 4

Prove by recurrence the formula:

$$1 + 3 + 5 + \dots + (2n - 1) = n^2$$

Exercice 5

Prove that for any natural number n, the number $3n^2 + 3n + 6$ is a multiple of 6.

Exercice 6

Let x be a positive real number.

1) Prove by recurrence on n that, for all n natural numbers,, $1 + nx \leq (1 + x)^n$
2) Propose another demonstration of this result.