

Série of T.D N ° 2 (S1)

Exercise 1

1. Give the algebraic writing of the complex numbers below :

a. $z_1 = \frac{1+i}{i}$ b. $z_2 = \frac{1}{1-i}$ c. $z_3 = \frac{-2+i}{2+i}$

2. We consider the two complex numbers z_1 and z_2 defined by :

$z_1 = 1+i$ et $z_2 = 5-2i$

Determine the algebraic writing of the following numbers :

a. $z_1 + z_2$ b. $z_1 - z_2$ c. $z_1 - 2z_2$
 d. $z_1 \times z_2$ e. $\frac{z_1}{z_2}$ f. $\frac{z_2}{z_1 - z_2}$

Exercise 2

Write in algebraic form : $z_1 = \frac{7+i}{3-2i}$ $z_2 = \frac{-3}{(1+i)(2-i)}$

Exercise 3

Determine the conjugate of the following complex number and write it in algebraic form

$z_1 = \frac{2+i}{1-2i}$

Exercise 4

Calculate the modulus of each of the given complex numbers :

1. $z_1 = 1+3i$ 2. $z_2 = 3-4i$
 3. $z_3 = -1+7i$ 4. $z_4 = -5-3i$

Exercise 5

Solve in \mathbb{C} the following equations :

a. $3z + iz = 0$ b. $z + 2iz = i$ c. $z + 2 - i(z+1) = 0$
 d. $\frac{z-5}{z-i} = i$ e. $2iz - 3 = z + 1$ f. $3z - 5 + 2iz = 2i - 3z + 4iz$

Exercise 6

Solve the following quadratic equations :

1. $2z^2 - 6z + 5 = 0$ 2. $z^2 + z + 1 = 0$ 3. $z^2 - 5z + 9 = 0$
 4. $z^2 - 3z + 4 = 0$ 5. $z^2 - z + 10 = 0$ 6. $z^2 - 4z - 1 = 0$

Exercise 7

Determine an argument of each of the given complex numbers :

1. $z_1 = -1+i$ 3. $z_3 = \sqrt{6} + i\sqrt{2}$
 2. $z_2 = i$ 4. $z_4 = (2+2i)(1-i)$

Exercise 8

The complex plane is related to a direct orthonormal reference frame O, \vec{u}, \vec{v} .

We designate by A, B, C and G the points of the plane of respective affixes $z_A = -1$,

$z_B = 2+i\sqrt{3}$, $z_C = 2-i\sqrt{3}$ et $z_G = 3$.

- a. Create a figure and place points A, B, C and G.
- b. Calculate the distances AB, BC and AC. Deduce the nature of triangle ABC.
- c. Calculate an argument of the complex number

$$\frac{z_A - z_C}{z_G - z_C}$$

Deduce the nature of the triangle GAC.

Exercise 9

Write the following numbers in exponential form:

$z_1 = (3+i\sqrt{3})^4$ $z_2 = \frac{1+i\sqrt{3}}{\sqrt{3}-i}$ $z_3 = \frac{\sin \frac{\pi}{12} - i \cos \frac{\pi}{12}}{\cos \frac{\pi}{12} + i \sin \frac{\pi}{12}}$

Exercise 10

We consider the complex number $a = (-\sqrt{3} + i)^{2013}$.

- 1) Determine the exponential form of: $-\sqrt{3} + i$
- 2) Show that a is pure imaginary.