Series no. 1 solution

Exercise 1

1. Binary system.
2.

Explanation:

- Number of values that can be represented with 1 bit: $2^1 = 2$

  Values:

Binary	Decimal
0	0
1	1

[min, max] = [0, 1]

- Number of values that can be represented with 2 bits: $2^2 = 4$

  Values:

Binary	Decimal
00	0
01	1
10	2
11	3

[min, max] = [0, 3]

- Number of values that can be represented with 3 bits: $2^3 = 8$

  Values:

Binary	Decimal
000	0
….	….
111	7

[min, max] = [0, 7]

- Number of values that can be represented with n bits : 2^n

  [min, max] = $[0, 2^n - 1]$

→ The largest decimal value that can be represented in binary with n bits is : $2^n - 1$

3. Number of unique binary numbers that can be represented with n bits : 2^n
4.

In base 2: ($16 \leq 2^4$ → 4 bits to represent numbers)

In base 8: ($16 \leq 8^2$ → 2 digits to represent numbers)

In base 16: ($16 \leq 16^1$ → 1 digit to represent numbers)

Base 10 (Decimal)	Base 2 (Binary)	Base 8 (Octal)	Base 16 (Hexadecimal)
0	0000	00	0
1	0001	01	1
2	0010	02	2
3	0011	03	3
4	0100	04	4
5	0101	05	5
6	0110	06	6
7	0111	07	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

5.

- $(12)_2$ incorrect
- $(14)_{12}$
- $(BAC2023)_{16}$
- $(318)_8$ incorrect
- $(2A0GF00)_{16}$ incorrect

Exercise 2

1. $(54)_{10} = (?)_2$

   $54 \div 2 = 27$ remainder = 0

   $27 \div 2 = 13$ remainder = 1

   $13 \div 2 = 6$ remainder = 1

   $6 \div 2 = 3$ remainder = 0

   $3 \div 2 = 1$ remainder = 1

   $1 \div 2 = \mathbf{0}$ remainder = 1

   $(54)_{10} = (110110)_2$

2. $(18.6875)_{10} = (?)_2$

   - Integral part : $(18)_{10}$

     $18 \div 2 = 9$ remainder = 0

     $9 \div 2 = 4$ remainder = 1

     $4 \div 2 = 2$ remainder = 0

     $2 \div 2 = 1$ remainder = 0

     $1 \div 2 = \mathbf{0}$ remainder = 1

     $(18)_{10} = (10010)_2$

- Fractional part $(0.6875)_{10}$

  0.6875 x 2 = 1.375 carry = 1 remainder = 0.375

  0.375 x 2 = 0.75 carry = 0 remainder = 0.75

  0.75 x 2 = 1.5 carry = 1 remainder = 0.5

  0.5 x 2 = 1 carry = 1 remainder = **0**

  $(0.6875)_{10} = (0.1011)_2$

  Hence $(18.6875)_{10} = (10010.1011)_2$

3. $(564)_{10} = (?)_8$

   564 ÷ 8 = 70 remainder = 4

   70 ÷ 8 = 8 remainder = 6

   8 ÷ 8 = 1 remainder = 0

   1 ÷ 8 = **0** remainder = 1

   $(564)_{10} = (1064)_8$

4. $(36.75)_{10} = (?)_8$

   - Integral part : $(36)_{10}$

     36 ÷ 8 = 4 remainder = 4

     4 ÷ 8 = **0** remainder = 4

     $(36)_{10} = (44)_8$

   - Fractional part $(0.75)_{10}$

     0.75 x 8 = 6 carry = 6 remainder = **0**

     $(0.75)_{10} = (0.6)_8$

     Hence $(36.75)_{10} = (44.6)_8$

5. $(1564)_{10} = (?)_{16}$

   1564 ÷ 16 = 97 remainder = 12 (C)

   97 ÷ 16 = 6 remainder = 1

   6 ÷ 16 = **0** remainder = 6

   $(564)_{10} = (61C)_{16}$

6. $(57.71875)_{10} = (?)_{16}$

   - Integral part : $(57)_{10}$

     57 ÷ 16 = 3 remainder = 9

     3 ÷ 16 = **0** remainder = 3

     $(57)_{10} = (39)_{16}$

   - Fractional part $(0.71875)_{10}$

     0.71875 x 16 = 11.5 carry = 11 (B) remainder = 0.5

     0.5 x 16 = 8 carry = 8 remainder = **0**

     $(0.71875)_{10} = (0.B8)_{16}$

     Hence $(57.71875)_{10} = (39.B8)_{16}$

7. $(101011101)_2 = (?)_{10}$

   $(101011101)_2 = (1 \times 2^0) + (0 \times 2^1) + (1 \times 2^2) + (1 \times 2^3) + (1 \times 2^4) + (0 \times 2^5) + (1 \times 2^6) + (0 \times 2^7) + (1 \times 2^8) = 1 + 0 + 4 + 8 + 16 + 0 + 64 + 256 = (349)_{10}$

8. $(101101.1101)_2 = (?)_{10}$

   $(101101.1101)_2 = (1 \times 2^0) + (0 \times 2^1) + (1 \times 2^2) + (1 \times 2^3) + (0 \times 2^4) + (1 \times 2^5) + (1 \times 2^{-1}) + (1 \times 2^{-2}) + (0 \times 2^{-3}) + (1 \times 2^{-4}) = 1 + 0 + 4 + 8 + 0 + 32 + 0.5 + 0.25 + 0 + 0.0625 = (45.8125)_{10}$

9. $(745)_8 = (?)_{10}$

   $(745)_8 = (5 \times 8^0) + (4 \times 8^1) + (7 \times 8^2) = 5 + 32 + 448 = (485)_{10}$

10. $(2454.46)_8 = (?)_{10}$

    $(2454.46)_8 = (4 \times 8^0) + (5 \times 8^1) + (4 \times 8^2) + (2 \times 8^3) + (4 \times 8^{-1}) + (6 \times 8^{-2}) = 4 + 40 + 256 + 1024 + 0.5 + 0.09375 = (1324.59375)_{10}$

11. $(A9C)_{16} = (?)_{10}$

    $(A9C)_{16} = (12 \times 16^0) + (9 \times 16^1) + (10 \times 16^2) = 12 + 144 + 2560 = (2716)_{10}$

12. $(C0E.1)_{16} = (?)_{10}$

    $(C0E.1)_{16} = (14 \times 16^0) + (0 \times 16^1) + (12 \times 16^2) + (1 \times 16^{-1}) = 14 + 0 + 3072 + 0.0625 = (3086.0625)_{10}$

13. $(23)_5 = (?)_4$

    - $(23)_5 = (?)_{10}$

      $(23)_5 = (3 \times 5^0) + (2 \times 5^1) = 3 + 10 = (13)_{10}$

    - $(13)_{10} = (?)_4$

      $13 \div 4 = 3$ remainder = 1

      $3 \div 4 = 0$ remainder = 3

      $(13)_{10} = (31)_4$

      $(23)_5 = (13)_{10} = (31)_4$

14. $(1323.23)_4 = (?)_8$

    $(1323.23)_4 = (01111011.1011)_2 = (001111011.101100)_2 = (173.54)_8$

Exercise 3

1.
   - $(607)_8 = (110000111)_2$
   - $(501.4)_8 = (101000001.100)_2 = (101000001.1)_2$
   - $(A8D)_{16} = (101010001101)_2$
   - $(A4.F)_{16} = (10100100.1111)_2$

2.
   - $(10111010)_2 = (010111010)_2 = (272)_8$
   - $(1011.01101)_2 = (001011.011010)_2 = (13.32)_8$
   - $(F1E)_{16} = (111100011110)_2 = (7436)_8$
   - $(A.3E2F)_{16} = (1010.0011111000101111)_2 = (001010.001111100010111100)_2 = (12.174274)_8$

3.

- $(10110110011101)_2 = (0010110110011101)_2 = (2D9D)_{16}$
- $(7106)_8 = (111001000110)_2 = (E46)_{16}$
- $(10110.11001)_2 = (00010110.11001000)_2 = (16.C8)_{16}$
- $(123.55)_8 = (001010011.101101)_2 = (01010011.10110100)_2 = (53.B4)_{16}$

Exercise 4

1. $(1100011)_2 + (10111)_2$

	1	1	0	$^{1}0$	$^{1}0$	$^{1}1$	1
+			1	0	1	1	1
	1	1	1	1	0	1	0

$(1100011)_2 + (10111)_2 = (1111010)_2$

2. $(101010)_2 + (11110)_2 + (101010)_2$

	^{1}	$^{1}1$	$^{1}0$	$^{1}1$	$^{1}0$	1	0
+			1	1	1	1	0
	1	0	$^{1}0$	1	0	0	0
+		1	0	1	0	1	0
	1	1	1	0	0	1	0

$(101010)_2 + (11110)_2 + (101010)_2 = (1110010)_2$

3. $(1011.0011)_2 + (1100.11)_2 + (10010.101)_2$

		^{1}	1	0	1	1	.	0	0	1	1
	+		1	1	0	0	.	1	1		
	^{1}	1	$^{1}0$	$^{1}1$	$^{1}1$	$^{1}1$	.	$^{1}1$	$^{1}1$	1	1
+		1	0	0	1	0	.	1	0	1	
	1	0	1	0	1	0	.	1	0	0	1

$(1011.0011)_2 + (1100.11)_2 + (10010.101)_2 = (101010.1001)_2$

4. $(274)_8 + (136)_8$

	$^{1}2$	$^{1}7$	4
+	1	3	6
	4	3	2

$(274)_8 + (136)_8 = (432)_8$

5. $(6AB)_{16} + (A97)_{16}$

		${}^{1}6$	^{1}A (=10)	B (=11)
+		A (=10)	9	7
	1	1	4	2

$(6AB)_{16} + (A97)_{16} = (1142)_{16}$

6. $(2054)_7 + (156)_7$

	2	${}^{1}0$	${}^{1}5$	4
+		1	5	6
	2	2	4	3

$(2054)_7 + (156)_7 = (2243)_7$

7. $(111011)_2 – (11101)_2$

	1	${}_{1}1$	${}_{1}1$	${}_{1}0$	1	1
-	${}_{1}$	1_{1}	1_{1}	1	0	1
	0	1	1	1	1	0

$(111011)_2 – (11101)_2 = (11110)_2$

8. $(101100.01)_2 – (10011.011)_2$

	1	${}^{1}0$	1	1	${}^{1}0$	${}^{1}0$	.	${}^{1}0$	${}^{1}1$	${}^{1}0$
+	${}_{1}$	1	0	0_{1}	1_{1}	1_{1}	.	0_{1}	1_{1}	1
	0	1	1	0	0	0	.	1	1	1

$(101100.01)_2 – (10011.011)_2 = (11000.111)_2$

9. $(574)_8 – (436)_8$

	5	7	${}^{8}4$
-	4	3_{1}	6
	1	3	6

$(574)_8 – (436)_8 = (136)_8$

10. $(A6E)_{16} – (9D)_{16}$

	A	^{16}6	E
-	$_{1}$	9	D
	9	D	1

$(A6E)_{16}$ – $(9D)_{16}$ = $(9D1)_{16}$

11. $(110110)_2$ * $(1101)_2$

					1	1	0	1	1	0
*							1	1	0	1
	^{1}	^{1}	^{1}	^{1}	^{1}1	1	0	1	1	0
				0	0	0	0	0	0	
			1	1	0	1	1	0		
		1	1	0	1	1	0			
	1	0	1	0	1	1	1	1	1	0

$(110110)_2$ * $(1101)_2$ = $(1010111110)_2$

12. $(274)_8$ * $(36)_8$

			2	7	4
*				3	6
		2	1	5	0
	1	0	6	4	
	1	3	0	1	0

$(274)_8$ * $(36)_8$ = $(13010)_8$

13. $(E4C)_{16}$ * $(A3)_{16}$

			E	4	C
*				A	3
		2	A	E	4
	8	E	F	8	
	9	1	A	6	4

$(E4C)_{16}$ * $(A3)_{16}$ = $(91A64)_{16}$

14. $(11011010001)_2 / (1011)_2$

```
              00010011110.101…
1011     ┌  11011010001
         -  1011
            0010101
         -       1011
            010100
         -         1011
            010010
         -          1011
            0001110
         -           1011
                001110
         -             1011
                 001100
         -               1011
                      0001
```

$(11011010001)_2 / (1011)_2 = (10011110.101)_2$

15. $(111110001.0111)_2 / (10101)_2$

```
            000010111.1011
10101   ┌  111110001.0111
        -  10101
            0101000
        -       10101
              0100110
        -         10101
                0100011
        -           10101
                  0011100
        -              10101
                     0011111
        -                10101
                       010101
        -                 10101
                         00000
```

$(111110001.0111)_2 / (10101)_2 = (10111.1011)_2$

16. $(3023)_4$ / $(32)_4$

We have:

$(32)_4 \times (0)_4 = (0)_4$

$(32)_4 \times (1)_4 = (32)_4$

$(32)_4 \times (2)_4 = (130)_4$

$(32)_4 \times (3)_4 = (222)_4$

Using that, we have:

```
          0032.2
32 |      3023
   -       222
          0203
   -       130
           0130
   -        130
            000
```

Additional exercises:

Exercise 5

$(11111001)_2 = (249)_{10}$

$(1101)_{16} = (4353)_{10}$

$(1000)_{16} = (4096)_{10}$

$(1000)_2 = (8)_{10}$

Ascending order ranking:

$(1000)_2 < (11111001)_2 < (1101)_{10} < (1000)_{16} < (1101)_{16} < (10000)_{10}$

Exercise 6

1. $\frac{18}{128} = \frac{2^4 + 2^1}{2^7} = 2^{-3} + 2^{-6} = (0.001001)_2$
2. $(75)_{\mathbf{B}} + (46)_{\mathbf{B}} = (132)_{\mathbf{B}} \Rightarrow 7B^1 + 5B^0 + 4B^1 + 6B^0 = 2B^0 + 3B^1 + 1B^2$

   $\Rightarrow 11B + 11 = 2 + 3B + B^2$

   $\Rightarrow B^2 - 8B - 9 = 0$

   $\Delta = (-8)^2 - (4 * 1 * (-9)) = 64 + 36 = 100$

   $B_1 = (-(-8) + 10) / 2 = 9$

   $B_2 = (-(-8) - 10) / 2 = -1$

   Hence B = 9

3.

a.

N1 = $(11)_x$ => N1 = x + 1

N2 = $(121)_x$ => N2 = $x^2 + 2x + 1 = (x + 1)^2$

We notice that N2 = N1 * N1 therefore N1 is a divisor of N2.

b.

According to the previous question: N2 = N1 * N1, so N2 / N1 = (x + 1)

the quotient: Q = x + 1

- In base x = 3: Q = x + 1 = 4,
  we verify that N2 = $(121)_3 = (16)_{10}$ and N1 = $(11)_3 = (4)_{10}$; 16 / 4 = 4 => Q = 4
- In base x = 8: Q = x + 1 = 9,
  we verify that N2 = $(121)_8 = (81)_{10}$ and N1 = $(11)_8 = (9)_{10}$; 81 / 9 = 9 => Q = 9