University Center Mila.
Institute of Mathematics and Computer Science.
1^{st} **Year Mathematics (Algebra 1)**

Final exam

Exercise 1. *(4 pts)*

1. *Let's consider a propositional language where:*
   - P *means "Rima is happy",*
   - Q *means "Rima paints a picture",*
   - R *means "Khaled is happy".*

   Formalize the following sentences:

   (a) *"If Rima is happy and paints a picture, then Khaled isn't happy"*
   (b) *"If Rima is happy, then she paints a picture"*

2. *Determine whether each statement is true or false*

   (a) $7 < 5 \vee 3 > 1$
   (b) $3 \leq 9$
   (c) $(-1)^{50} = 1 \wedge (-1)^{99} = -1$
   (d) $7 \neq 3 \vee 9$ *is a prime number*
   (e) $-5 \geq -11$
   (f) $4.5 \leq 5.4$
   (g) *2 is an odd number or 2 is an even number*

Exercise 2. *(4 pts)*
Let $f : [-1;1] \longrightarrow \mathbb{R}$ *be a function defined by* $f(x) = \sqrt{1-x^2}$.

1. *Determine* $f\{\frac{-1}{2}, \frac{1}{2}, 1\}$ *and* $f^{-1}\{\frac{1}{2}, 2\}$.
2. *Is the function* f *injective? Is it surjective?*
3. *Show that the restriction* $g : [-1;0] \longrightarrow [0;1]$, *where* $g(x) = f(x)$, *is bijective.*
4. *Determine the inverse function* g^{-1}.

Exercise 3. *(4 pts)*
Let $\mathcal{R}$ *be the binary relation defined on* $\mathbb{R}$ *by:*

$$\forall x, y \in \mathbb{R}, \quad x\mathcal{R}y \text{ if and only if } x^4 - y^4 = x^2 - y^2.$$

1. *Show that* $\mathcal{R}$ *is an equivalence relation on* $\mathbb{R}$.
2. *Determine the equivalence class of 0, and deduce the equivalence class of 1.*

Exercise 4. *(8 pts)*
Let $G = \mathbb{R} - \{-2\}$ *and* $*$ *be the internal composition law in* G *defined by:*

$$\forall x, y \in G, \quad x * y = xy + 2(x + y) + 2.$$

1. *Show that* $(G, *)$ *is an abelian group.*
2. *Let* $H = \{x \in \mathbb{R} \mid x > -2\}$*. Show that* $(H, *)$ *is a subgroup of* $(G, *)$*.*
3. *Let the application* f *be defined from the group* $(G, *)$ *to the multiplicative group* $(\mathbb{R} - \{0\}, \times)$ *by:*
$$f(x) = x + 2.$$
Show that f *is an isomorphism from the group* $(G, *)$ *to* $(\mathbb{R} - \{0\}, \times)$*.*

Good luck.