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Heat Transfer Final Exam

TYPE CORRECTION

Problem n° 01:

Taking the **general heat conduction equation on Cartesian coordinates**,

Using assumptions: constant thermal conductivity $\boldsymbol{k}$ and transient two-dimensional heat transfer with no heat generation, so

a) Heat conduction equation is: $\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{1}{\alpha}.\frac{\partial T}{\partial t}$ (1)

b) Boundary Conditions: we must put 4 BCs

$\frac{\partial T(0,y,t)}{\partial x} = 0$ (1) $\frac{\partial T(x,0,t)}{\partial y} = 0$ (1)

$-k.\frac{\partial T(a,y,t)}{\partial x} = h.[T(a,y,t) - T_\infty]$ (1) $-k.\frac{\partial T(x,b,t)}{\partial y} = h.[T(x,b,t) - T_\infty]$ (1)

Initial condition: 1 IC: $T(x,y,0) = T_i$ (1)

Problem n° 02:

1) The rate of heat transfer of a single fin (q_f) as function of $\theta_{gas} = (T_b - T_\infty)$:

The heat transfer from a single fin can be calculated from Table 1 for a fin with convection from the tip (case A):

$$q_f = M.\frac{\sinh mL + \left({h}/{mk}\right)\cosh mL}{\cosh mL + \left({h}/{mk}\right)\sinh mL}$$ (1)

Where $$m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{h.(2t+2w)}{k.(w.t)}} = \sqrt{\frac{57.(6+0,01)}{3.(3.0,005)}} = 87,25\ m^{-1}$$

$$mL = 87,25\ \times 0,0125 = 1,091 \quad \text{and} \quad {h}/{mk} = \frac{57}{87,25 \times 3} = 0,2178$$

$$M = \sqrt{h.P.K.A_c}.\theta_b = \sqrt{57 \times 6,01 \times 3 \times 0,015} \times (T_b - T_\infty)$$

$$\boldsymbol{M = 3,926(T_b - T_\infty)}$$

$$q_f = 3,926(T_b - T_\infty).\frac{\sinh(1,091) + (0,2178)\cosh(1,091)}{\cosh(1,091) + (0,2178)\sinh(1,091)} \rightarrow \boldsymbol{q_f = 3,395(T_b - T_\infty)}$$ (1)

2) The total rate of heat transfer on the gas side (q_{Gas}), then predict thermal resistance (R_{Gas}):

The rate of heat transfer on the gas side is the sum of the convection from the fins and the convection from the bare wall between the fins. The bare area is:

$$A_{bare} = A_{wall} - (number\ of\ fins).(area\ of\ one\ fin)$$

$$A_{bare} = 1{,}8 - (96\ fins).(3 \times 0{,}005) = 0{,}36\ m^2$$

The total rate of heat transfer to the gas is:

$$q_{Gas} = q_{bare} + (number\ of\ fins).q_f = h_G \times A_{bare} \times (T_b - T_\infty) + 96 \times 0{,}395(T_b - T_\infty)$$

$$q_{Gas} = 57 \times 0{,}36 \times (T_b - T_\infty) + 96 \times 0{,}395(T_b - T_\infty) = 346{,}4 \times (T_b - T_\infty)$$

$$\boldsymbol{q_{Gas} = 346{,}4 \times (T_b - T_\infty)}$$ 1

❖ The thermal resistance on the gas side is:

$$q_{Gas} = 346{,}4 \times (T_b - T_\infty) = \frac{(T_b - T_\infty)}{R_{Gas}} \rightarrow R_{Gas} = \frac{1}{346{,}4} \rightarrow \boldsymbol{R_{Gas} = 0{,}002887\ K/W}$$ 1

3) The total rate of heat transfer on the liquid side (q_{Liquid}), then predict thermal resistance (R_{Liquid}):

$$q_{Liquid} = h_L \times A_{wall} \times (T_b - T_\infty) = 255 \times (0{,}6 \times 3) \times (T_b - T_\infty) \rightarrow \boldsymbol{q_{Liquid} = 459 \times (T_b - T_\infty)}$$ 1

❖ The thermal resistance on the liquid side is:

$$q_{Liquid} = 459 \times (T_b - T_\infty) = \frac{(T_b - T_\infty)}{R_{Liquid}} \rightarrow R_{Liquid} = \frac{1}{459} \rightarrow \boldsymbol{R_{Liquid} = 0{,}002179\ K/W}$$ 1

4) The total rate of heat transfer (q) if the overall temperature difference is ΔT = 38 °C:

$$q = \frac{\Delta T}{R_{tot}} = \frac{\Delta T}{R_{Gas} + R_{Liquid}} \rightarrow q = \frac{38}{0{,}002887 + 0{,}002179} \rightarrow \boldsymbol{q = 7500\ W}$$ 1

Problem n° 03:

From Appendix A, Table A.4, for hydrogen at 43 °C:

$\nu = 119{,}9 \times 10^{-6}\ m^2/s$
$\Pr = 0{,}703$
$\rho = 0{,}07811\ kg/m^3$
$k = 0{,}190\ W/mK$

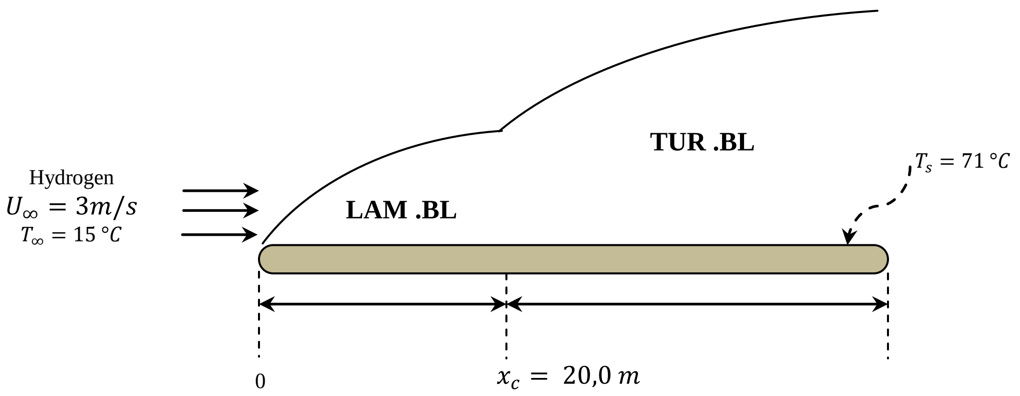
0,5

1) the regime flow of hydrogen over the flat plate:

Transition to turbulence occurs around: $Re_x = \frac{U_\infty . x_c}{\nu} = 5 \times 10^5$

$$x_c = \frac{5 \times 10^5 \times 119{,}9 \times 10^{-6}}{3} \rightarrow \boldsymbol{x_c = 20{,}0\ m}$$ 1

2) a representative scheme:



0,5

3) <u>hydrodynamic boundary layer thickness (δ), in *cm*,</u>

$$\delta = \frac{5x}{\sqrt{Re_x}} \rightarrow \delta = \frac{5\times0{,}3}{\sqrt{7506}} \rightarrow \boldsymbol{\delta = 0,017\ m} \qquad (Re_{0,3} = 7506)$$ 0,5

4) <u>thickness of thermal boundary layer (δ_T), in *cm*,</u>

$$\delta_t = \frac{\delta}{Pr^{\frac{1}{3}}} \rightarrow \delta_t = \frac{0{,}017}{(0{,}703)^{\frac{1}{3}}} \rightarrow \boldsymbol{\delta_t = 1,91}$$ 1

5) <u>local friction coefficient ($C_{f,x}$),</u>

$$C_{f,x} = \frac{0{,}664}{\sqrt{Re_{0,3}}} \rightarrow C_{f,x} = \frac{0{,}664}{\sqrt{7506}} \rightarrow \boldsymbol{C_{f,x} = 0,0077}$$ 0,5

6) <u>average friction coefficient ($\overline{C_f}$),</u>

$$\overline{C_f} = \frac{1{,}33}{\sqrt{Re_{0,3}}} \rightarrow \boldsymbol{\overline{C_f} = 0,0154}$$ 0,5

7) <u>drag force (F_D), in *N*,</u>

$$F_D = A.\tau_s = \frac{\rho.A.U_\infty^2.C_f}{2} \rightarrow F_D = \frac{0{,}07811\times(0{,}3\times0{,}3)\times3^2\times0{,}0154}{2} \rightarrow \boldsymbol{F_D = 0,00049\ N}$$ 1

8) <u>local convection heat transfer coefficient (h_x), in W/m^2•K,</u>

$$Nu_x = \frac{h_x.x}{k} = 0.332Re_x^{0.5}Pr^{\frac{1}{3}} \rightarrow h_x = \frac{k}{x}\times 0.332.Re_x^{0.5}.Pr^{\frac{1}{3}} \rightarrow \boldsymbol{h_x = 16,2\left(\frac{W}{m^2.°C}\right)}$$ 0,5

9) <u>average convection heat transfer coefficient ($\overline{h}$), in W/m^2•K,</u>

$$\overline{h} = 2.h_x = 2\times16{,}2 \rightarrow \boldsymbol{\overline{h} = 32,4\ \left(\frac{W}{m^2.°C}\right)}$$ 0,5

10) <u>rate of heat transfer (q), in W.</u>

$$q = \overline{h}.A.(T_s - T_\infty) \rightarrow q = 32{,}4\ \times(0{,}3\times0{,}3)\times(71-15) \rightarrow \boldsymbol{q = 163\ W}$$ 0,5