Abdelhafid Boussouf University Center – Mila
Institute of Science & Technology
Process Engineering – E2
Heat Transfer

Academic year: **2024-2025** Instructor: **Dr. Mohamed BOUTI**

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Homework n°03

Exercise

A **3-m-high** and **5-m-wide** wall consists of long **16-cm x 22-cm** cross section horizontal bricks (**k = 0.72 W/m·°C**) separated by **3-cm-thick** plaster layers (**k = 0.22 W/m·°C**). There are also **2-cm-thick** plaster layers on each side of the brick and a **3-cm-thick** rigid foam (**k = 0.026 W/m·°C**) on the inner side of the wall, as shown in **Fig. 2–6**. The indoor and the outdoor temperatures are **20°C** and **-10°C**, and the convection heat transfer coefficients on the inner and the outer sides are $\mathbf{h_1 = 10}$ $\mathbf{W/m^2{\cdot}°C}$ and $\mathbf{h_2 = 25}$ $\mathbf{W/m^2{\cdot}°C}$, respectively.

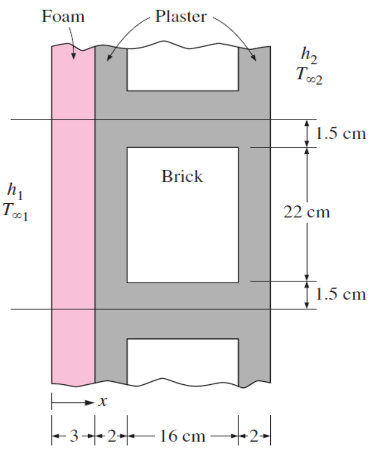
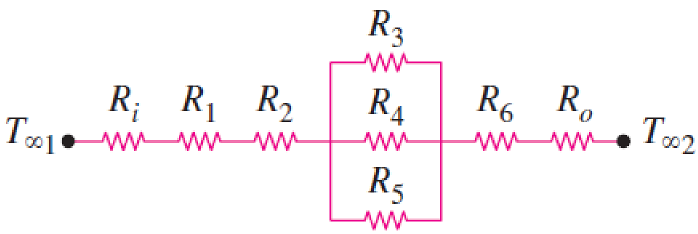


Figure 1

Obtain a general relation for the temperature distribution inside the pipe under steady conditions, and determine the rate of heat loss from the steam through the pipe.



There is a pattern in the construction of this wall that repeats itself every 25-cm distance in the vertical direction. There is no variation in the horizontal direction. Therefore, we consider a 1-m-deep and 0.25-m-high portion of the wall, since it is representative of the entire wall.

Assuming any cross section of the wall normal to the *x*-direction to be *isothermal*, the thermal resistance network for the representative section of the wall becomes as shown in Fig. The individual resistances are evaluated as:

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A} = \frac{1}{(10\text{ W/m}^2 \cdot {}^\circ\text{C})(0.25 \times 1\text{ m}^2)} = 0.4^\circ\text{C/W}$$

$$R_1 = R_{\text{foam}} = \frac{L}{kA} = \frac{0.03\text{ m}}{(0.026\text{ W/m} \cdot {}^\circ\text{C})(0.25 \times 1\text{ m}^2)} = 4.6^\circ\text{C/W}$$

$$R_2 = R_6 = R_{\text{plaster, side}} = \frac{L}{kA} = \frac{0.02\text{ m}}{(0.22\text{ W/m} \cdot {}^\circ\text{C})(0.25 \times 1\text{ m}^2)} = 0.36^\circ\text{C/W}$$

$$R_3 = R_5 = R_{\text{plaster, center}} = \frac{L}{kA} = \frac{0.16\text{ m}}{(0.22\text{ W/m} \cdot {}^\circ\text{C})(0.015 \times 1\text{ m}^2)} = 48.48^\circ\text{C/W}$$

$$R_4 = R_{\text{brick}} = \frac{L}{kA} = \frac{0.16\text{ m}}{(0.72\text{ W/m} \cdot {}^\circ\text{C})(0.22 \times 1\text{ m}^2)} = 1.01^\circ\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A} = \frac{1}{(25\text{ W/m}^2 \cdot {}^\circ\text{C})(0.25 \times 1\text{ m}^2)} = 0.16^\circ\text{C/W}$$

The three resistances R_3, R_4, and R_5 in the middle are parallel, and their equivalent resistance is determined from

$$\frac{1}{R_{\text{mid}}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{48.48} + \frac{1}{1.01} + \frac{1}{48.48} = 1.03\text{ W/}^\circ\text{C}$$

which gives

$$R_{\text{mid}} = 0.97^\circ\text{C/W}$$

Now all the resistances are in series, and the total resistance is

$$\begin{aligned} R_{\text{total}} &= R_i + R_1 + R_2 + R_{\text{mid}} + R_6 + R_o \\ &= 0.4 + 4.6 + 0.36 + 0.97 + 0.36 + 0.16 \\ &= 6.85^\circ\text{C/W} \end{aligned}$$

Then the steady rate of heat transfer through the wall becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]^\circ\text{C}}{6.85^\circ\text{C/W}} = 4.38\text{ W} \qquad \text{(per 0.25 m}^2\text{ surface area)}$$

or 4.38/0.25 = 17.5 W per m^2 area. The total area of the wall is $A = 3\text{ m} \times 5\text{ m} = 15\text{ m}^2$. Then the rate of heat transfer through the entire wall becomes

$$\dot{Q}_{\text{total}} = (17.5\text{ W/m}^2)(15\text{ m}^2) = 263\text{ W}$$

Of course, this result is approximate, since we assumed the temperature within the wall to vary in one direction only and ignored any temperature change (and thus heat transfer) in the other two directions.