Chapitre 03

Electrokinetics

Introduction

3- **Electrokinetics** is the set of phenomena and laws relating to moving electric charges. It is a branch of physics that studies the transport of electricity in electrical circuits.

3.1- General definitions

3.1.1- Electric current

The amount of charge flowing through a section of a conductor per unit of time is called electric current. The intensity of electric current is expressed in amperes:

$$i = \frac{dq}{dt}$$

When the current is constant over time, it is called direct current andrepresented by **I** : $I = \frac{Q}{t}$

An electric current appears in a conductor when a potential difference is established between the terminals of the latter.

3.1.2- Dipole

A dipole is an electrical element that has an input terminal and an output terminal. It is considered active when it supplies energy and passive when it consumes it.

Examples Resistors, capacitors, and coils are **passive elements**, while voltage sources and current sources are **active elements**.

3.1.3- Electrical circuit

An electrical circuit is a simple or complex set of conductors and electrical or electronic components carrying an electric current.

- A node is an interconnection point connected to at least three dipoles.
- A branch is a part of the electrical circuit between two consecutive nodes.
- A mesh is a set of branches forming a closed loop without passingthrough the same node twice.

3.1.4- Electric generator

An electrical generator is an active dipole that can cause an electric current to flowthrough a circuit. For an electrical circuit to be complete, it mustnecessarily contain at least one generator.In a circuit, a generator is represented by the following two symbols:

3.1.5- Electrical receiver

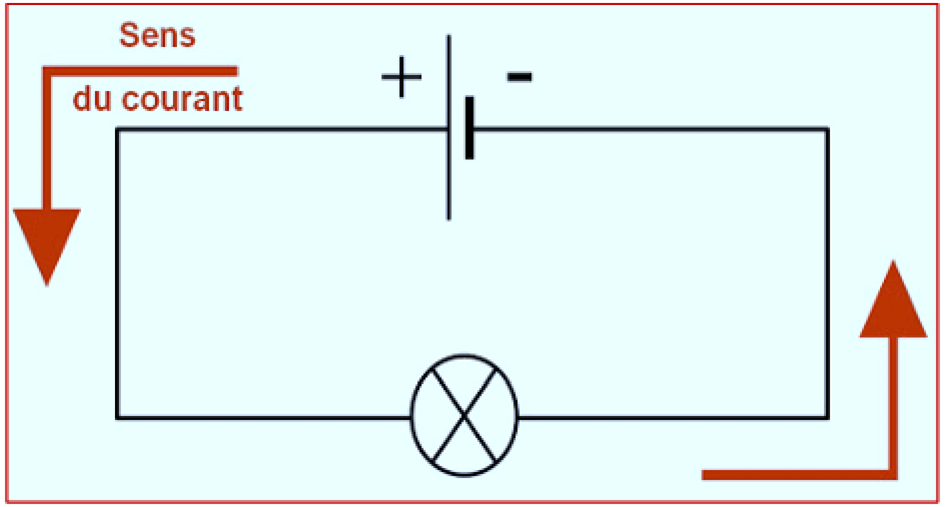
A receiver is a dipole that receives electrical energy and converts it into another form of energy.

Examples

- Electrical wires of all kinds, incandescent lamps, and heating wires in household appliances transform electrical energy into thermal energy.
- Electric motors transform electrical energy into mechanical energy and thermal energy.

3.1.6- Sens du courant continu

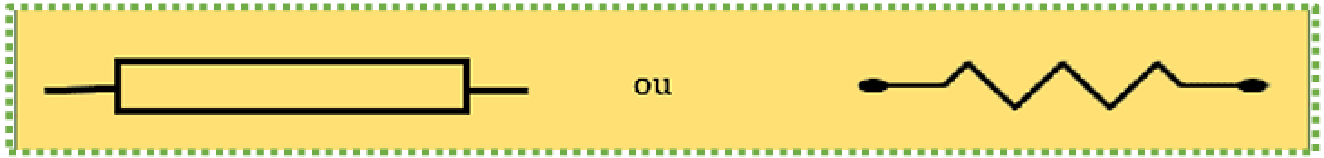
By convention, in a single-loop DC circuit, the electric current exits the generator through the positive terminal, passes through the circuit, and returns to the generator through its negative terminal.



the circulation of electric **charges** is in the opposite direction

3.1.7- Résistance électrique

Electrical resistance refers to a component's ability to resist the flow of an electric current. It is often denoted by the letter **R** and its unit of measurement is ohms **(Ω)**. A resistor can be represented by the following two symbols:

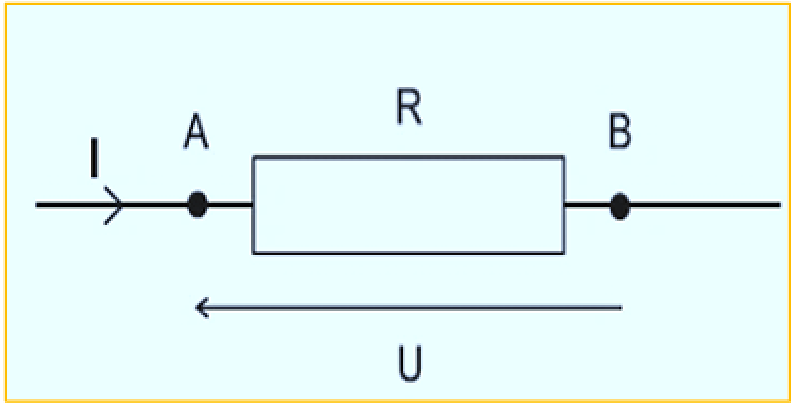


conductors have relatively low resistances while insulators have extremely high resistances

3.2- Ohm's Law

Let **U** be the voltage across a resistor **R** at constant temperature. Experienceshows that **U** is proportional to the intensity **I** of the current flowing through this resistor, such that:

$$U = RI$$



3.3- Kirchhoff's Laws

3.3.1- Kirchhoff's first law (law of knots)

The law of nodes is used to establish a mathematical relationship concerning the electric currents flowing through a node. This is a consequence of the conservation of electric charge.

statement of the law:

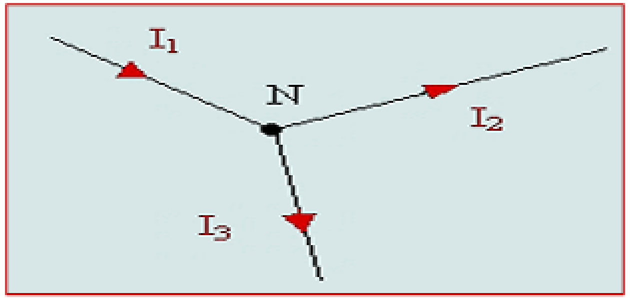
the sum of the intensities of the currents entering through a node is equal to the sum of the intensities of the currents leaving it:

$$\sum \mathbf{I}_{\mathbf{entrant}} = \sum \mathbf{I}_{\mathbf{sortant}} .$$

This law can be translated mathematically by the expression: $\sum_{i=1}^{n} I_i = 0$

where Ii denotes the algebraic value of the ith current flowing through a node. The value of Ii is assigned a (+) sign if the current reaches the node in question and a (-) sign if it moves away from it.

Examples



Kirchhoff's first law applied to node N gives:

$$\boxed{I_1 = I_2 + I_3}$$

3.3.2- Kirchhoff's Second Law (Lattice Law)

The mesh law is used to establish a mathematical relationship concerning the voltages within a mesh of an electrical circuit.

<u>statement of the law:</u>

in a mesh of an electric circuit, the sum of the voltages along this mesh is always zero.

This law constitutes a generalization of Ohm's law: $\sum(E - RI) = 0$

how to apply the law of meshes ?

1- On each branch, *we choose an arbitrary direction of the current.*

2- We choose *an arbitrary direction of travel of the mesh.*

3- Each **RI** term is assigned a **(+)** sign if the direction of the chosen mesh path coincides with the direction of the current. Otherwise, it is assigned a **(-)** sign.

4- The electromotive force **E** is affected by the sign of the pole of the generator through which the direction of the chosen path enters.

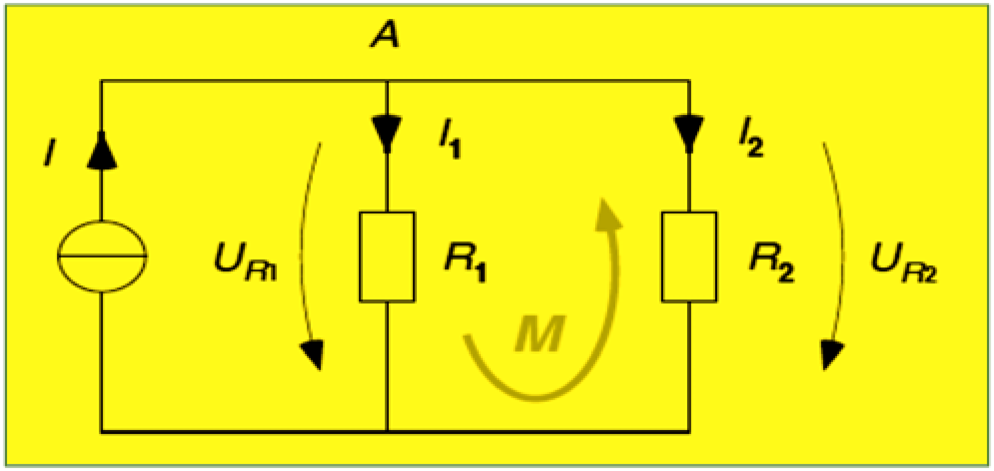
Application

We define the voltage U_{R_1} across resistor R_1 and the voltage U_{R_2} across resistor R_2. We also define the current I_1 flowing through resistor R_1 and the current I_2 flowing through resistor R_2.

1- Calculate the intensities R_2 and I_2

2- Calculate the voltages U_{R_1} and U_{R_2}

Data: $R_1 = 5\Omega$ $R_2 = 10\Omega$ I =2A



Solution

1- We therefore need two linearly independent equations to determine the two unknowns I_1 and I_2.

Applying the mesh law **(mesh M)** gives: $\Sigma U = 0 \Rightarrow R_1 I_1 - R_2 I_2 = 0$

Applying the law of nodes **(node A)** gives: $\Sigma I = 0 \Rightarrow I = I_1 + I_2$

Combining the two equations, we obtain:

$$R_1 (I - I_2) - R_2 I_2 = 0 \Rightarrow \boldsymbol{I_2} = \frac{\boldsymbol{R_1 I}}{\boldsymbol{R_1 + R_2}} \qquad \boldsymbol{I_1 = I - I_2}$$

$A.N : I_2 = 0{,}67\ A$ et $I_1 = 1{,}33\ A$

2- Calculation of tensions:

$$\boldsymbol{U_{R_1} = R_1 I_1} \qquad \boldsymbol{U_{R_2} = R_2 I_2}$$

$A.N : U_{R_1} = 6{,}7\ V$ et $U_{R_2} = 6{,}7\ V$

3.4- Association des résistances

Equivalent resistance is a modeling tool used in the field of electricity. It involves replacing a set of resistors in one part of a circuit with a single resistor that must be equivalent for the rest of the circuit.

3.4.1- Resistors in series

Let $\boldsymbol{R1}$, $\boldsymbol{R2}$ and $\boldsymbol{R3}$ be three resistors connected in series as shown in Figure 1.

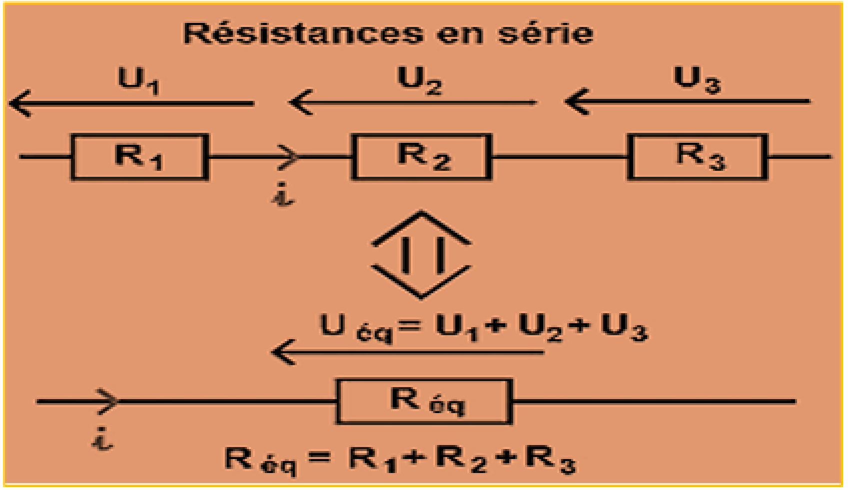


Figure 1.

According to Ohm's law, we have:

$$U_1 = R_1 i$$
$$U_2 = R_2 i$$
$$U_3 = R_3 i$$

- The total voltage across all three of these resistors is:

$$U_{éq} = U_1 + U_2 + U_3$$

$$U_{éq} = R_1 i + R_2 i + R_3\, i$$

$$U_{éq} = (R_1 + R_2 + R_3\,)\mathrm{i}$$

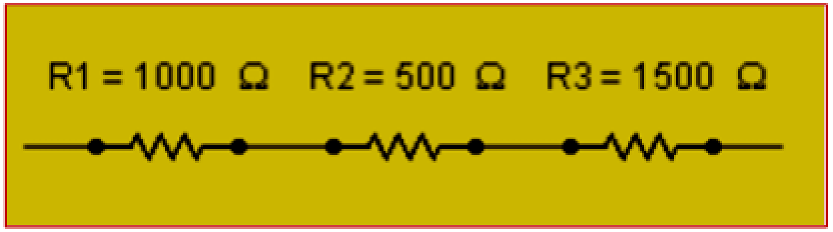
$$U_{éq} = R_{eq} i \Rightarrow$$

$$R_{eq} = R_1 + R_2 + R_3$$

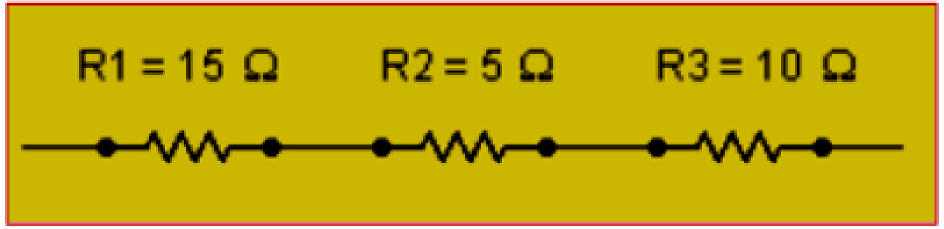
In general, the equivalent resistance of n resistors in series in a circuit is equal to: $\boldsymbol{R_{eq} = \sum_{i=1}^{n} R_i}$

Application

Calculate the equivalent resistance of the following circuits **(A)** and **(B)**:



(A)



(B)

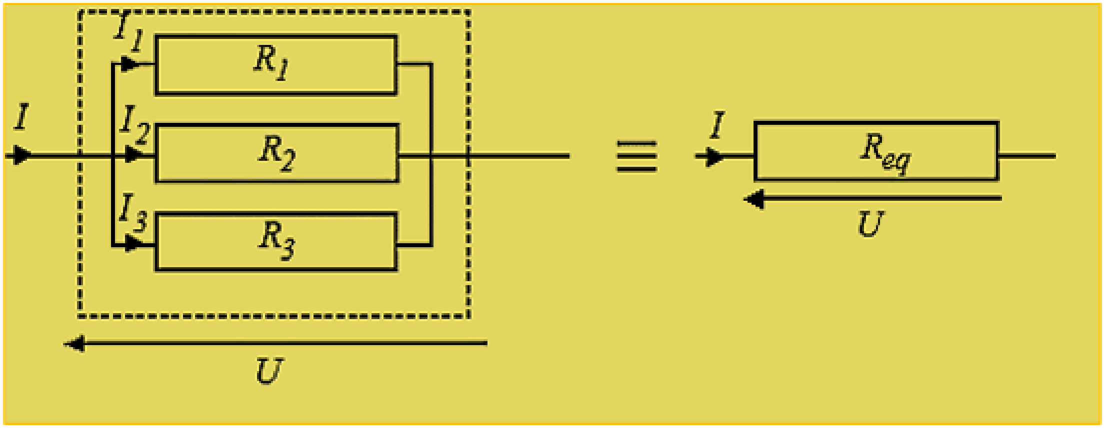
Solution

In circuit **(A)**: $R_{eq} = 3000\Omega$

In circuit **(B)**: $R_{eq} = 30\Omega$

3.4.2- Resistors in parallel

Let $\boldsymbol{R1}$, $\boldsymbol{R2}$ and $\boldsymbol{R3}$ be three resistors associated in parallel as shown in Figure 2.



According to Ohm's law, we have:

$$U = R_1 I_1 \Rightarrow I_1 = \frac{U}{R_1}$$

$$U = R_2 I_2 \Rightarrow I_2 = \frac{U}{R_2}$$

$$U = R_3 I_3 \Rightarrow I_3 = \frac{U}{R_3}$$

D’après la loi des nœuds, nous avons :

$$I = I_1 + I_2 + I_3 \Rightarrow I = \frac{U}{R_1} + \frac{U}{R_2} + \frac{U}{R_3}$$

$$I = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) U$$

Ohm's law applied to the equivalent circuit is written: $I = \frac{U}{R_{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) U$

We deduce that the equivalent resistance is expressed as a function of the three resistances by the relation:

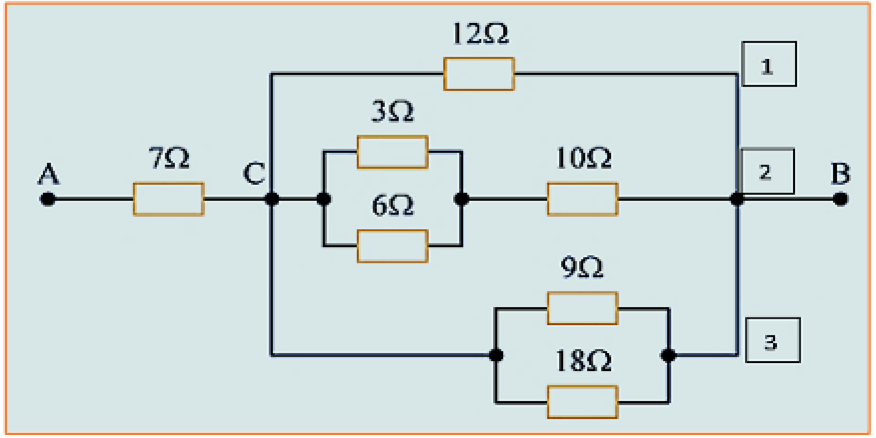
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

In general, the equivalent resistance resulting from the association of n resistors in parallel is equal to:

$$\frac{1}{R_{eq}} = \sum_{i=1}^{n} \frac{1}{R_i}$$

Application

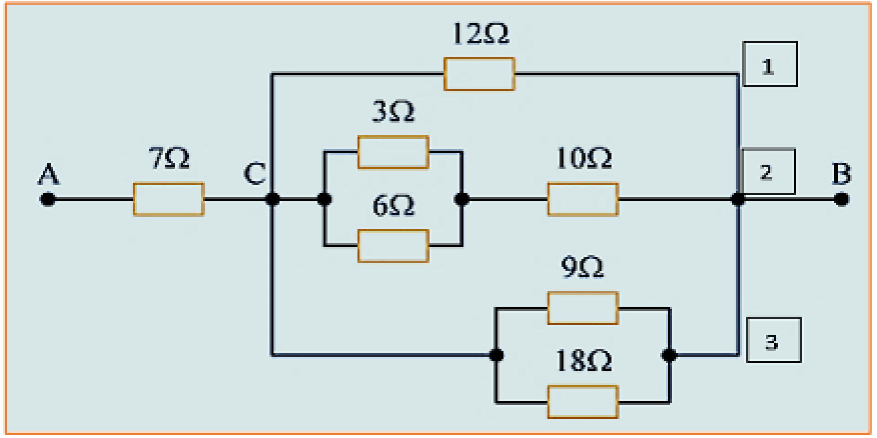
Let the following grouping of resistors be:



1- Calculate the equivalent resistance of each of the branches connecting C and B

2- Deduce the total resistance of the circuit between A and B.

Solution



1- The three branches connecting C and B have the following resistances respectively:

Branch 1: $\boldsymbol{R_1 = 12\ \Omega}$

Branch 2:

The **3 Ω** and **6 Ω** resistors are in parallel, their equivalent resistance is:

$$R_{eq} = \frac{3 \times 6}{3 + 6} \Longrightarrow R_{eq} = 2\ \Omega$$

So, the total resistance of this branch is: $\boldsymbol{R_2 = (10 + 2)\Omega = 12\Omega}$

Branch 3:

The 18 Ω and 9 Ω resistors are in parallel, their equivalent resistance is:

$$R_{eq} = \frac{18 \times 9}{18 + 9} \Longrightarrow \boldsymbol{R_3 = 6\ \Omega}$$

The total resistance between C and B is formed by the parallel connection of the calculated resistances R1, R2 and R3, then:

$$R_{CB} = \frac{1}{\frac{1}{12} + \frac{1}{12} + \frac{1}{6}} \Longrightarrow \boldsymbol{R_{CB} = 3\Omega}$$

The equivalent resistance of circuit AB is formed by the series connection of the two resistors RAC and RCB, then:

$$R_{AB} = 3\Omega + 7\Omega \qquad \boldsymbol{R_{AB} = 10\ \Omega}$$