Lecture 7

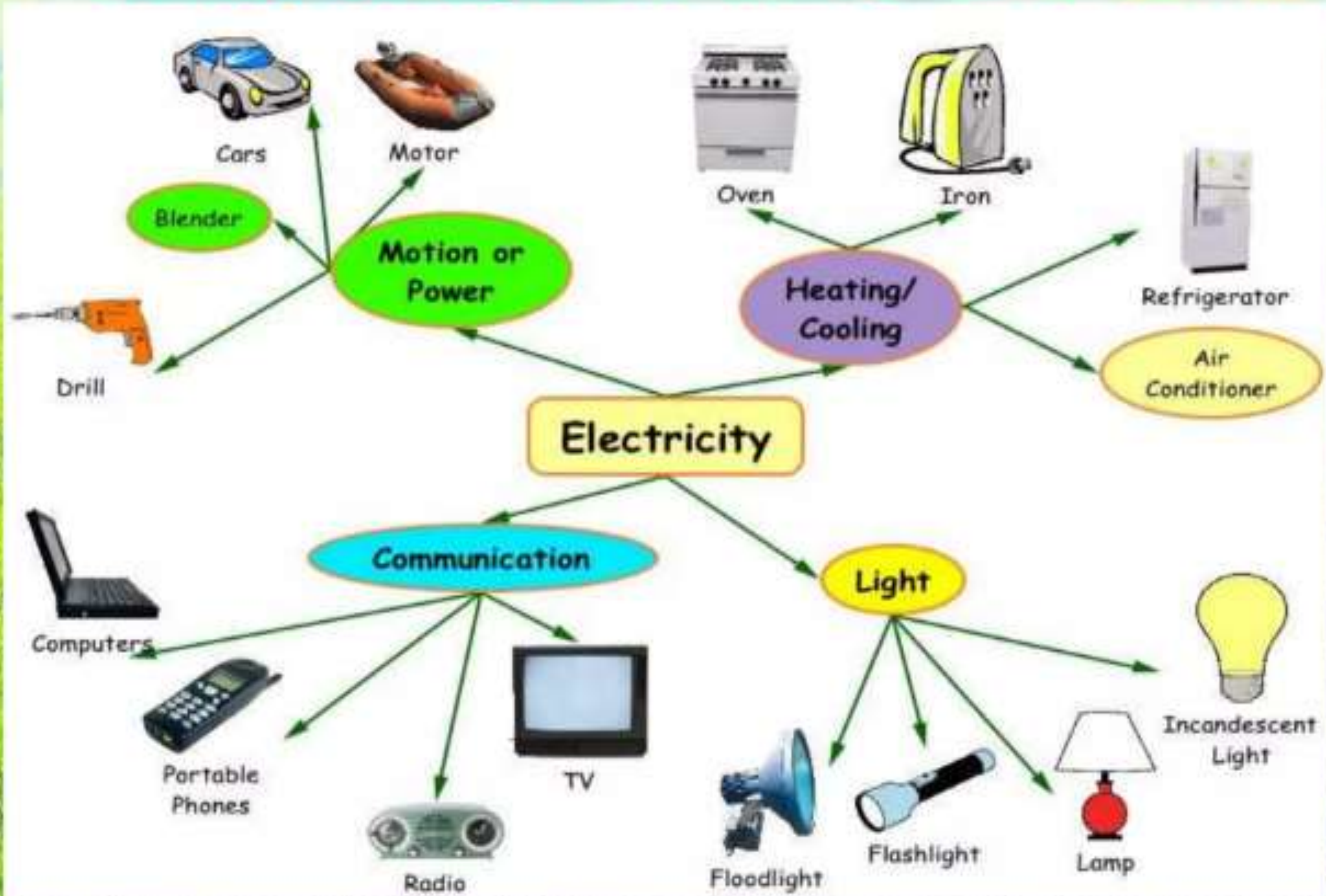
The Electric Current and the resistance

- **Introduction**
- **Electric current**
- **The Electromotive Force and Internal Resistance**
- **Electrical energy and thermal energy**
- **Resistors in series.**
- **Resistors in parallel.**
- **Kirchhoff’s Laws and its applications.**
- **Charging and Discharging Processes in RC**

Introduction

The results of the last two chapters (particularly those involving conductors) apply to the special case that electric charges are not in motion, the electrostatic case. For that case, all the points of a single conductor were at the same potential and the electric field was zero within the material of the conductor. In certain situations, we can maintain the motion of charges through a conductor, as when we connect a battery across the ends of a wire. In that case electric charge (negative charge, as it turns out) moves through the wire and there will be potential differences between the points of the conductor.

Electricity is a form of a energy that can be easily changed to many other forms

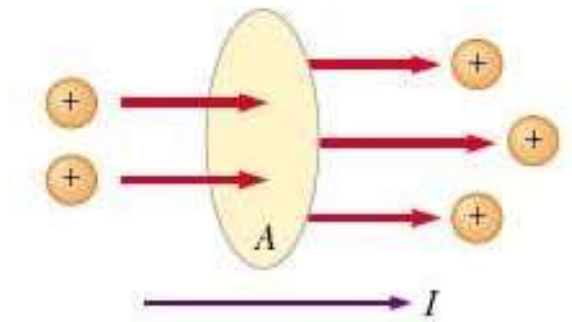


Electric current

The **current is defined as the flow of the charge**.

- The current is the rate at which charge flows through a surface of area *A*,
- If ΔQ is the amount of charge that passes through this area in a time interval Δt, the average current I_{av} is equal to the charge that passes through *A* per unit time:

$$I_{av} = \frac{\Delta Q}{\Delta t}$$



The SI unit of current is the ampere (A):That is, 1 A of current is equivalent to 1 C of charge passing through the surface area in 1 s.

$$1\ \mathrm{A} = \frac{1\ \mathrm{C}}{1\ \mathrm{s}}$$

The current density

Consider a conductor of cross-sectional area ***A*** carrying a current ***I***. The current density ***J*** in the conductor is defined as the current per unit area. Because the current $I = nqv_dA$, the current density is

The current density $$J \equiv \frac{I}{A} = nqv_d$$

where J has SI units of A/m^2.

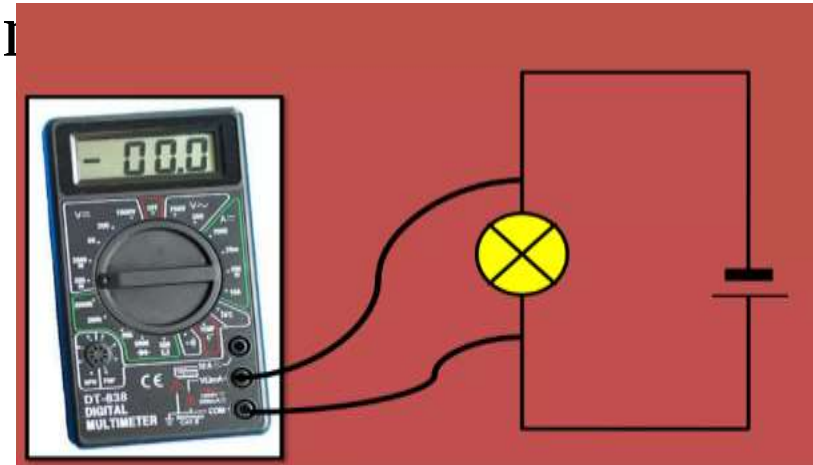
A current density ***J*** and an electric field ***E*** are established in a conductor, whenever a potential difference is maintained across the conductor, the current density is proportional to the electric field:

$$\mathbf{J} = \sigma \mathbf{E}$$

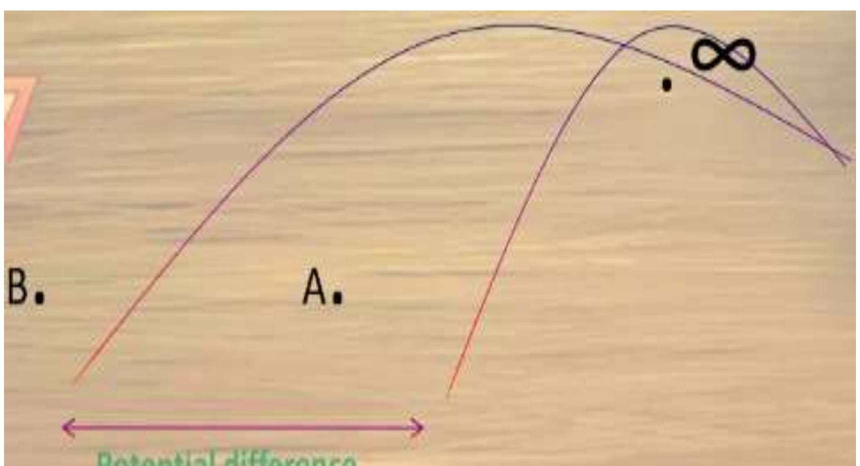
σ is called the conductivity of the conductor.

Electric potential and potential difference

Electric potential-work done in [partly hidden by an overlapping image] from infinity to a point.



Potential Difference-The difference between potential at two points.



Electromotive Force, emf

- **Electromotive force** is the same as voltage
- When the current in the circuit is constant in magnitude and direction and is called **direct current DC.**
- A battery is called a *source of electromotive force* or, *emf*.
- The emf of a battery is the maximum possible voltage that the battery can provide between its terminals.
- When an electric potential difference exists between two points, the source moves charges from the lower potential to the higher.

Ohm's Law

- For many substances it is found that the current flowing through a wire made of the material is proportional to the potential difference across its ends: I ∝ V . We write this relation in the following way:

$$\frac{V}{I} = R \qquad \text{or} \qquad V = IR$$

- where R is constant which depends on the properties of the wire (its material and its dimensions). R is called the resistance of the wire and this relation is known.

- From the relation R = V/I we see that the units of resistance must $\frac{\mathrm{V}}{\mathrm{A}}$ be. This combination of units is called an ohm:

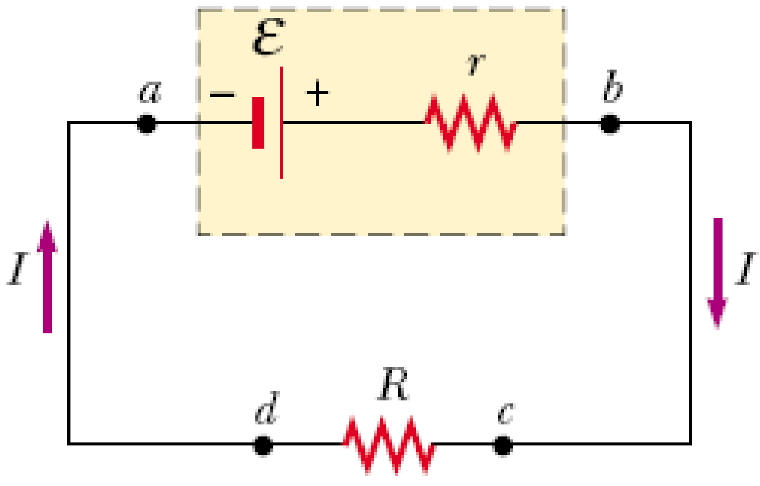
The unit of resistance is Ohms (W): 1 W = 1 V/A

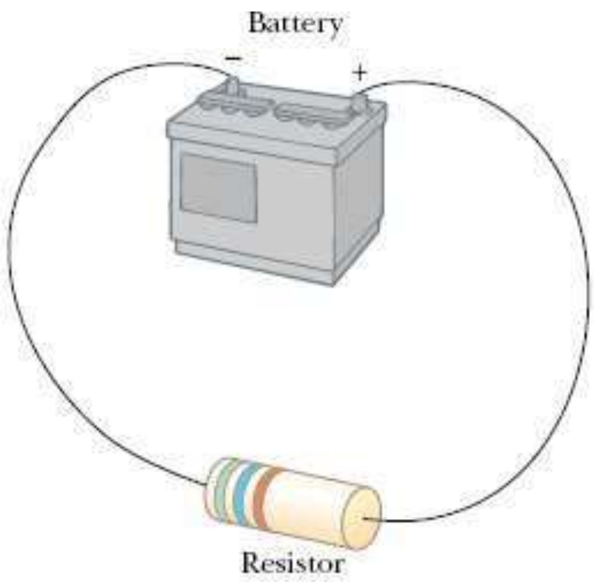
$$1\,\mathrm{ohm} = 1\,\Omega = 1\,\frac{\mathrm{V}}{\mathrm{A}}$$

The internal resistance

- The resistance of the battery is called **internal resistance *r***.
- ***I*** is the current in the circuit, $\boldsymbol{I_r}$ is the current through the resistor, emf is ε
- The terminal voltage of the battery

$\Delta V = V_b - V_a$ is $\qquad \Delta V = \varepsilon - I_r$





Resistance and Resistivity

The resistance of a piece of material depends on the type and shape of the material. If the piece has length *L* and cross-sectional area *A*, the resistance is $R = \rho \frac{L}{A}$

where *ρ* is a constant (for a given material at a given

$$\rho_{\text{Copper}} = 1.72 \times 10^{-8}\,\Omega \cdot \text{m} \qquad \rho_{\text{Aluminum}} = 2.82 \times 10^{-8}\,\Omega \cdot \text{m} \qquad \rho_{\text{Carbon}} = 3.5 \times 10^{-5}\,\Omega \cdot \text{m}$$

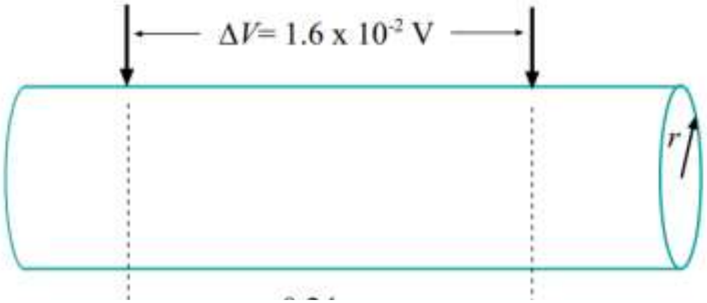
selected values for ρ are:

The resistivity of a material usually increases with temperature. It generally follows an empirical formula given by:

$$\rho = \rho_0[1 + \alpha(T - T_0)]$$

Example 1

A cylindrical copper cable carries a current of 1200 A. There is a potential difference of 1.6×10^{-2} V between two points on the cable that are 0.24 m apart. What is the radius of the cable?



Solution

$$R = \frac{V}{I} = \frac{(1.6 \times 10^{-2}\,\mathrm{V})}{(1200\,\mathrm{A})} = 1.33 \times 10^{-5}\,\Omega$$

Then from Eq. 4.6, knowing R, L and the resistivity of the material (i.e. copper) we can get the cross-sectional area:

$$R = \rho\frac{L}{A} \qquad \Longrightarrow \qquad A = \frac{\rho L}{R}$$

Plug in the numbers:

$$A = \frac{(1.72 \times 10^{-8}\,\Omega \cdot \mathrm{m})(0.24\,\mathrm{m})}{(1.33 \times 10^{-5}\,\Omega)} = 3.10 \times 10^{-4}\,\mathrm{m}^2$$

The cable has a circular cross-section so that $A = \pi r^2$. Solve for r:

$$r^2 = \frac{A}{\pi} = 9.87 \times 10^{-5}\,\mathrm{m}^2 \qquad \Longrightarrow \qquad r = 9.93 \times 10^{-3}\,\mathrm{m} = 9.93\,\mathrm{mm}$$

Table 5.1 Values of Resistivity of Materials

Material	Resistivity ($\Omega \cdot m$)
Metals:	
Silver	1.47×10^{-8}
Copper	1.72×10^{-8}
Gold	2.44×10^{-8}
Aluminum	2.63×10^{-8}
Tungsten	5.51×10^{-8}
Steel	20×10^{-8}
Lead	22×10^{-8}
Mercury	95×10^{-8}
Semiconductors:	
Pure carbon	3.5×10^{-5}
Pure germanium	0.60
Pure silicon	2300
Insulators:	
Amber	5×10^{14}
Mica	10^{11}–10^{15}
Teflon	10^{16}
Quartz	7.5×10^{17}

Electrical Power and Electrical Work

All electrical circuits have three parts in common.

1. A voltage source.
2. An electrical device
3. Conducting wires.

- The work done (W) by a voltage source is equal to the work done by the electrical field in an electrical device,

1. Work = Power x Time.

The electrical potential is measured in joules/coulomb and a quantity of charge is measured in coulombs, so the electrical work is measure in joules.

A joule/second is a unit of power called the watt.

2. Power = current x potential

Or, $P = I V = I^2 R$

- Energy = Power / Time

Electric Power

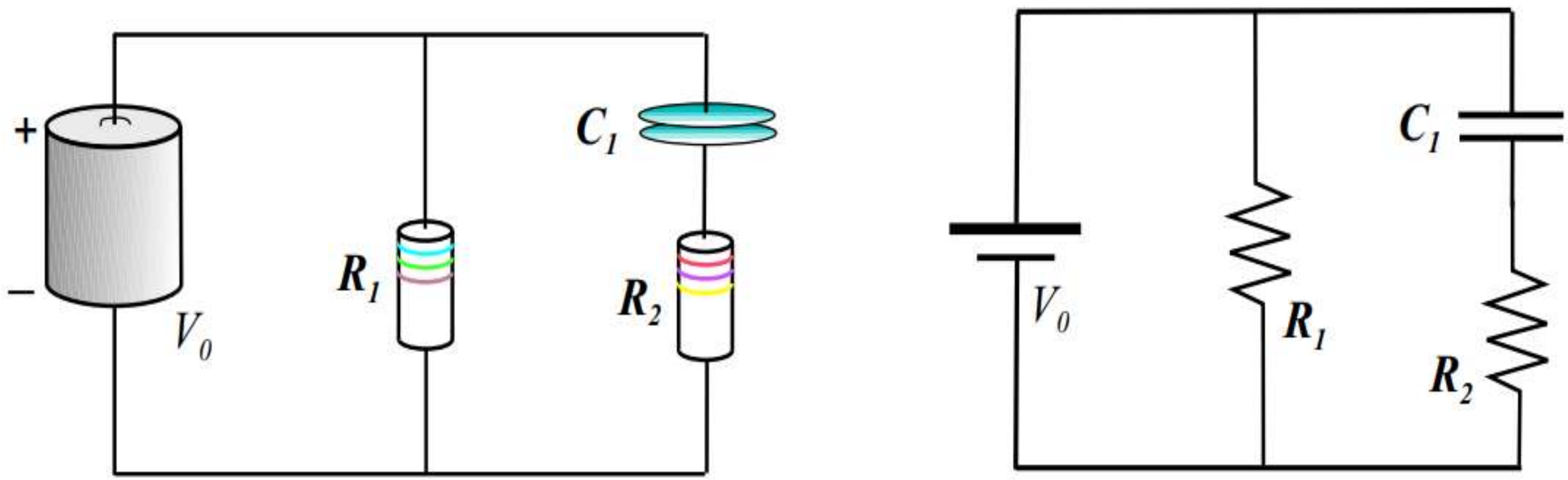
As charge moves through the wires of an electric circuit, they lose electric potential energy. (When charge Δq moves through a potential difference V , it loses ΔqV of potential energy.) The rate of energy loss is the power P delivered to the circuit elements,

$$P = \frac{\Delta q V}{\Delta t} = \frac{\Delta q}{\Delta t} V = IV$$

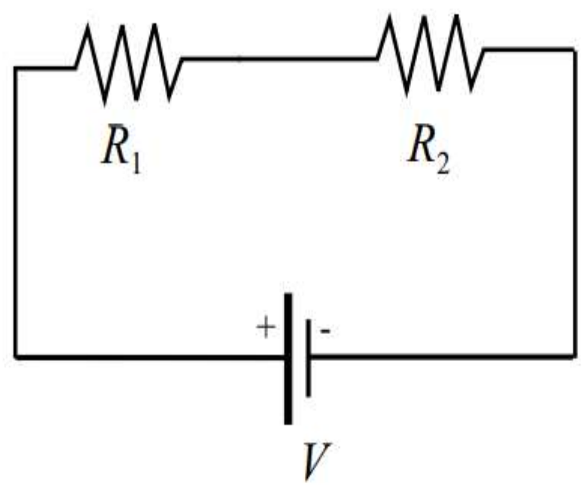
$$P = IV$$

Electric power is measured in joules per second, or watts: 1 J s = 1W. (We have already met this unit when we considered mechanical work done per unit time in first-semester physics.) The energy goes into heating the resistor. Using Ohm's law, (V = IR, or I = V/R) we can show that the power deliver to a circuit element of resistance R can also be written as

$$P = I^2 R \qquad \text{or} \qquad P = \frac{V^2}{R}$$



(a) Battery connected to two resistors and a capacitor. (b) Schematic diagram for this circuit.



Circuit with two resistors in series.

Voltage sources

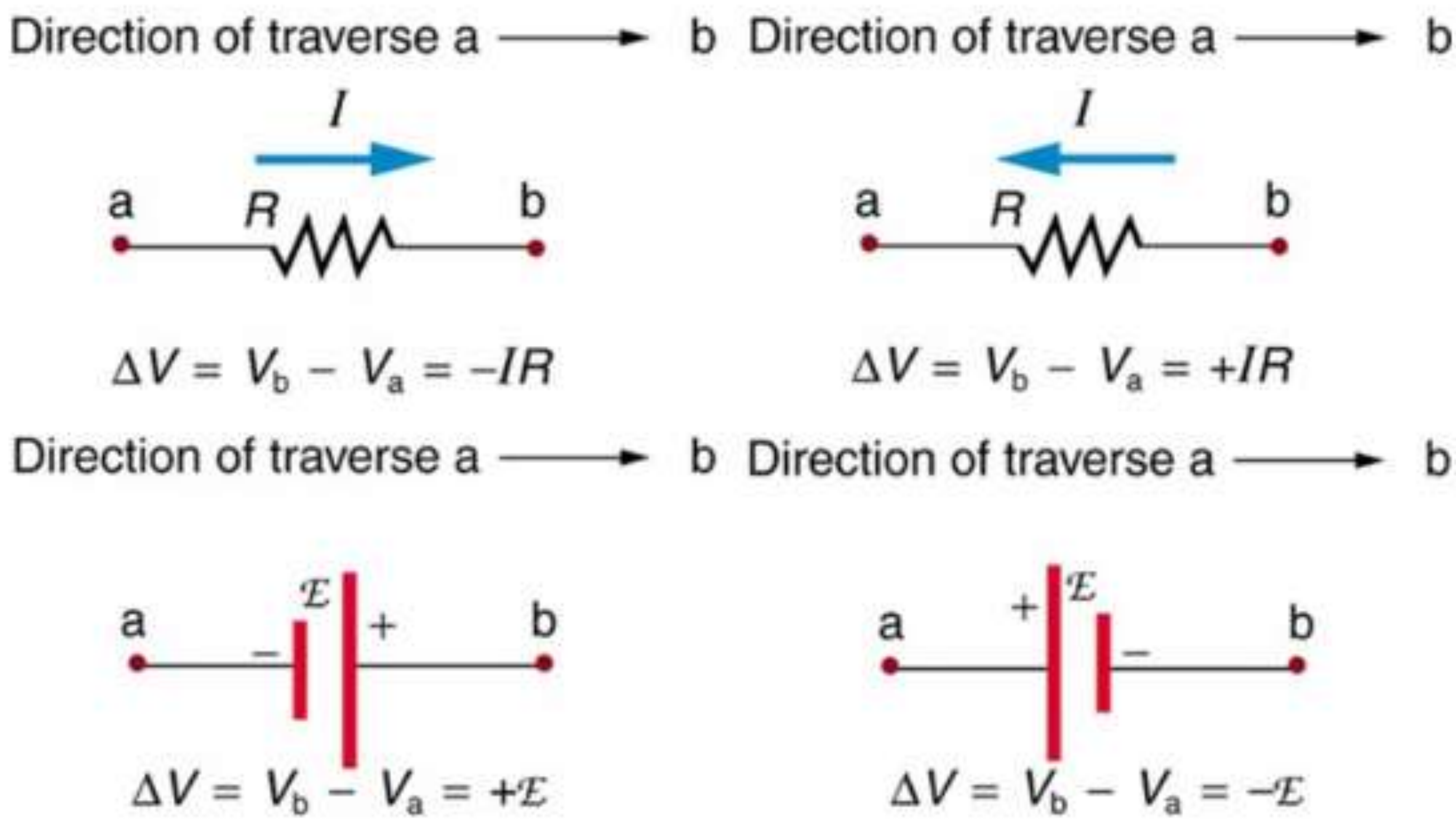


Figure . Each of these resistors and voltage sources is traversed from *a* to *b*. The potential changes are shown beneath each element and are explained in the text. (Note that the script E stands for emf.)

Electrical energy and thermal energy.

- The resistor represents a *load* on the battery because the battery must supply energy to operate the device.
- The potential difference across the load resistance is
- $\Delta \boldsymbol{V} = \mathbf{IR}$ and

$$I = \frac{\mathcal{E}}{R + r}$$

- The total power output $I\varepsilon$ of the battery is delivered to the external load resistance in the amount I^2R and to the internal resistance in the amount I^2r.

$$I\mathcal{E} = I^2R + I^2r$$

Example 2

A battery has an emf of 12 V and an internal resistance of 0.05 Ohm. Its terminals are connected to a load resistance of 3/00 Ohm.

a- Find the current in the circuit and the terminal voltage of the battery.

Solution

$$I = \frac{\mathcal{E}}{R + r} = \frac{12.0\ \text{V}}{3.05\ \Omega} = 3.93\ \text{A}$$

We find the terminal voltage

$$\Delta V = \mathcal{E} - Ir = 12.0\ \text{V} - (3.93\ \text{A})(0.05\ \Omega) = 11.8\ \text{V}$$

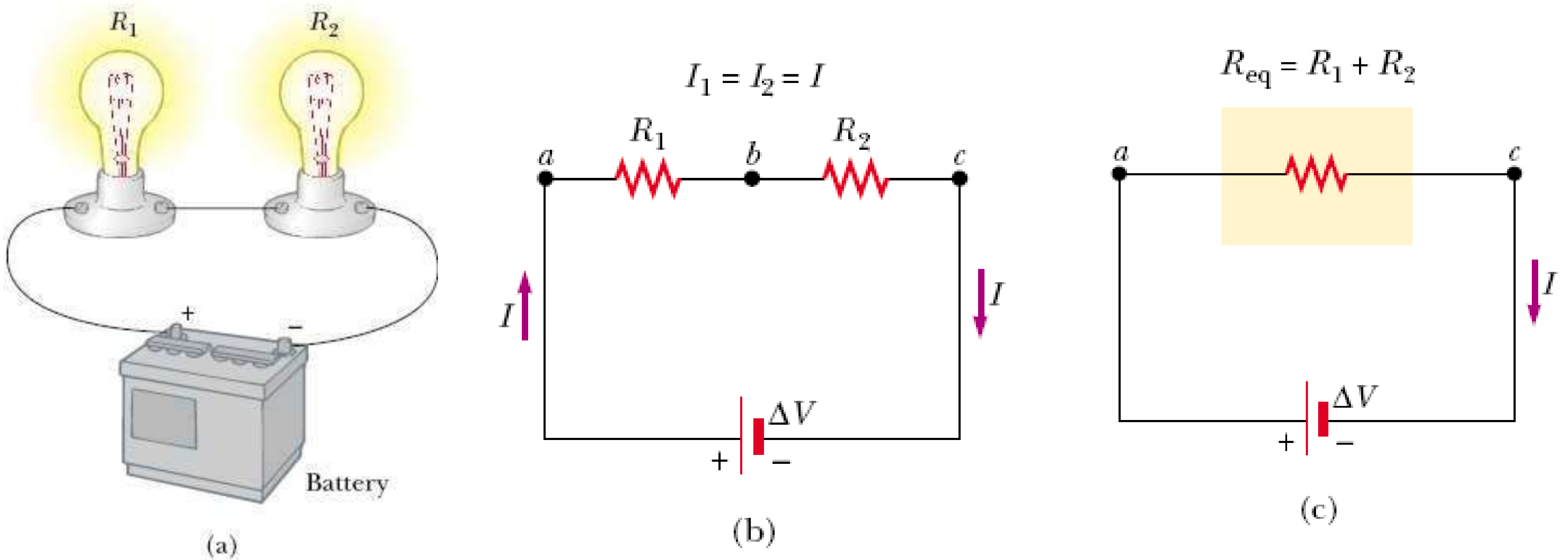
- Calculate the power delivered to the load resistor of 3 ohm when the current in the circuit is 3.93 A.

Solution The power delivered to the load resistor is

$$\mathcal{P}_R = I^2 R = (3.93\ \text{A})^2\ (3.00\ \Omega) = 46.3\ \text{W}$$

Resistors in Series

- for a series combination of two resistors, the currents are the same in both resistors because the amount of charge that passes through R_1 must also pass through R_2 in the same time interval.



The potential difference across the battery is applied to the **equivalent resistance** $\boldsymbol{R_{eq}}$:

$$\Delta V = IR_{eq}$$

Resistors in Series

$\Delta V = IR_{eq}$

$$\Delta V = IR_{eq} = I(R_1 + R_2) \longrightarrow R_{eq} = R_1 + R_2$$

The equivalent resistance of three or more resistors connected in series is

$$R_{eq} = R_1 + R_2 + R_3 + \cdots$$

Problem 5.8. Consider the series portion of a circuit shown in Fig. 5-4. The current in the circuit flows from *a* to *b* and is 2.3 A.

(*a*) What is the equivalent resistance?

(*b*) What is the voltage across the entire circuit? Which point, *a* or *b*, is at the higher potential?

(*c*) What is the voltage across each resistor?

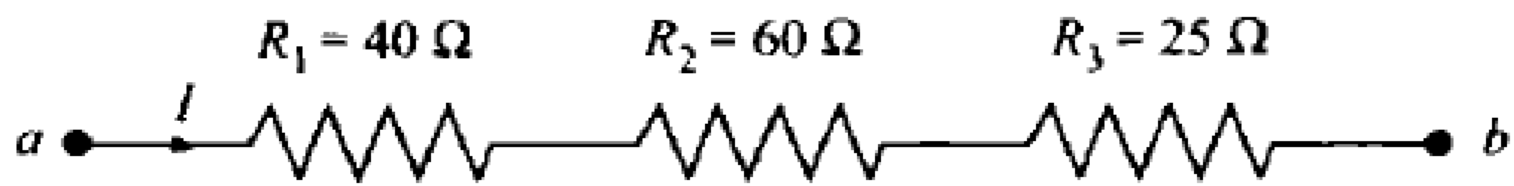


Fig. 5-4

Problem 5.8. Consider the series portion of a circuit shown in Fig. 5-4. The current in the circuit flows from a to b and is 2.3 A.

(*a*) What is the equivalent resistance?

(*b*) What is the voltage across the entire circuit? Which point, a or b, is at the higher potential?

(*c*) What is the voltage across each resistor?

Solution

(*a*) The equivalent resistance is the sum of all the resistances, or $R_{eq} = 40 + 60 + 25 = 125\ \Omega$.

(*b*) The voltage across the entire circuit is $V_{total} = IR_{eq} = (2.3\ \text{A})(125\ \Omega) = 288\ \text{V}$. Since the current flows from a to b, and the electric field does positive work in pushing charges through the resistors, energy is lost as the charges move through. Thus the potential at a is higher (by 288 V) than the potential at b.

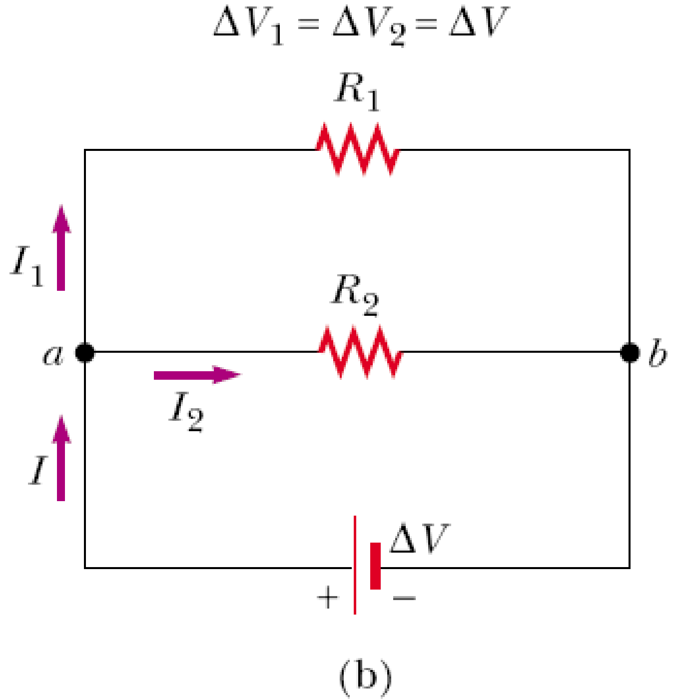
(*c*) The voltage across each resistor is IR_i. Thus $V_1 = (2.3\ \text{A})(40\ \Omega) = 92\ \text{V}$, $V_2 = (2.3\ \text{A})(60\ \Omega) = 138\ \text{V}$ and $V_3 = (2.3\ \text{A})(25\ \Omega) = 58\ \text{V}$.

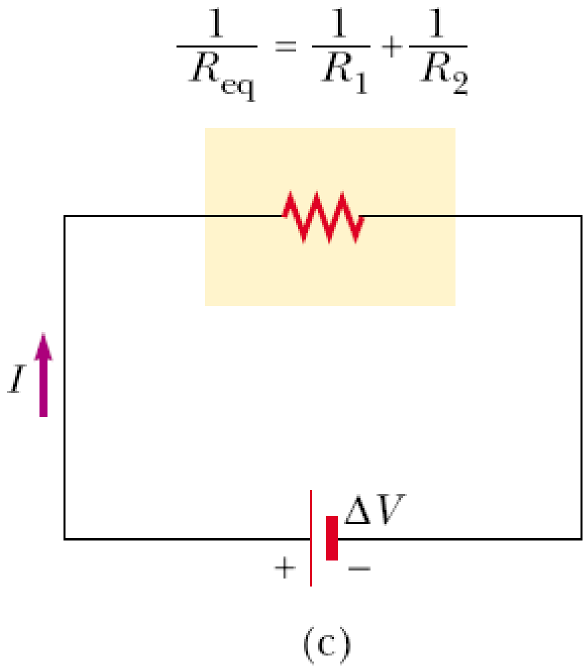
> ***Note.*** Adding the voltages gives $92 + 138 + 58 = 288\ \text{V}$, which is the voltage we calculated in part(*b*).

Resistors in Parallel

- The current I that enters point a must equal the total current leaving that point:

$$I = I_1 + I_2$$





When resistors are connected in parallel, the potential differences across the resistors is the same.

Resistors in Parallel

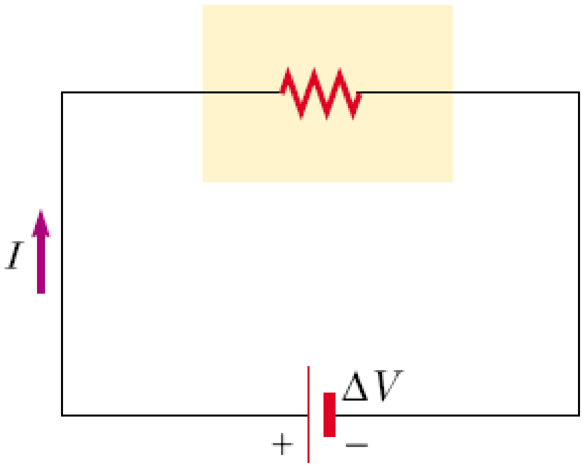
- Because the potential differences across the resistors are the same, the expression
- $\Delta V = IR$ gives

$$I = I_1 + I_2 = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \Delta V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{\Delta V}{R_{\text{eq}}}$$

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

the equivalent resistance

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$



(c)

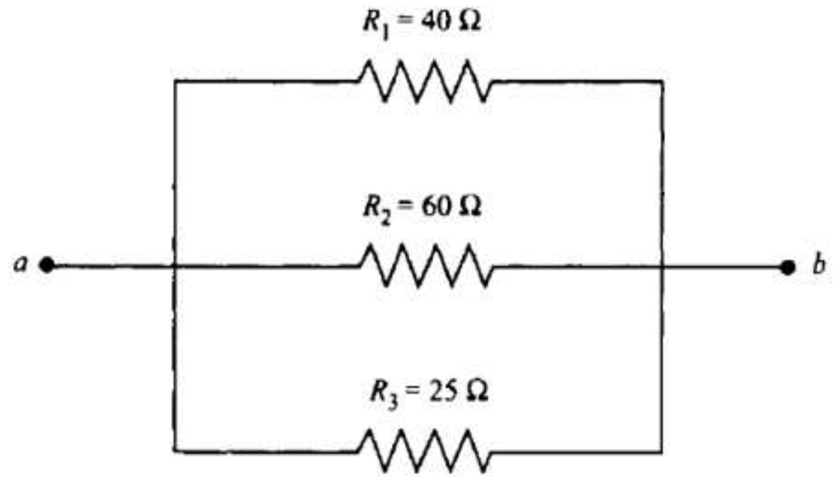
Example

Three resistors are connected in parallel, The potential difference between a and b is 75 V.

a) What is the equivalent resistance of this circuit?

a) What is the current flowing from point a

b) What is the the current in each resistor



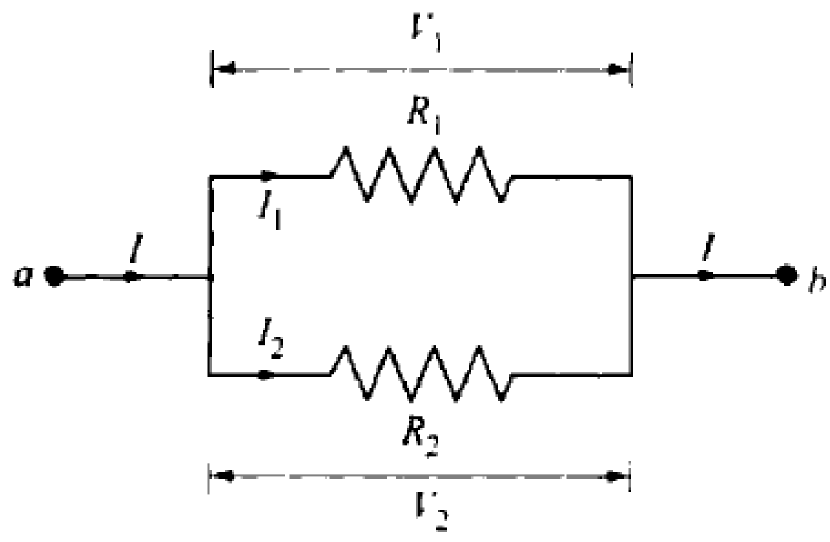
Solution

(*a*) The equivalent resistance is given by $1/R_{eq} = \Sigma(1/R_i) = 1/40 + 1/60 + 1/25 = 0.817$, or $R_{eq} = 12.2\ \Omega$.

(*b*) The total current is $I_{tot} = V/R_{eq}$. Thus $I_{tot} = 6.13$ A.

(*c*) The current in each resistor is $I_i = V/R_i$. Thus $I_1 = (75\ \text{V})/(40\ \Omega) = 1.88$ A, $I_2 = (75\ \text{V})/(60\ \Omega) = 1.25$ A, $I_3 = (75\ \text{V})/(25\ \Omega) = 3.0$ A. [The total current is $1.88 + 1.25 + 3.0 = 6.13$ A, as in part(b).]

Problem 5.20. In the circuit segment of Fig. 5-2(*b*), the voltage across the circuit is 55V. If $R_1 = 25\ \Omega$ and $R_2 = 35\ \Omega$, calculate (*a*) the current in each resistor; (*b*) the power dissipated in each resistor and (*c*) the power dissipated in the equivalent resistance. Compare the answer to this with the sum of the answers to (*b*).



(*b*)

Solution

(*a*) The voltage across each resistor is 55 V. Thus the current for each resistor is $I = V/R$. Then $I_1 = (55\ \text{V})/(25\ \Omega) = 2.2$ A, and $I_2 = (55\ \text{V})/(35\ \Omega) = 1.57$ A.

Problem 5.20. In the circuit segment of Fig. 5-2(*b*), the voltage across the circuit is 55V. If $R_1 = 25\ \Omega$ and $R_2 = 35\ \Omega$, calculate (*a*) the current in each resistor; (*b*) the power dissipated in each resistor and (*c*) the power dissipated in the equivalent resistance. Compare the answer to this with the sum of the answers to (*b*).

Solution

(*a*) The voltage across each resistor is 55 V. Thus the current for each resistor is $I = V/R$. Then $I_1 = (55\ \text{V})/(25\ \Omega) = 2.2$ A, and $I_2 = (55\ \text{V})/(35\ \Omega) = 1.57$ A.

(*b*) Since the voltage across each resistor is the same, the power in each resistor can be calculated using $P = V^2/R$. Thus $P_1 = (55\ \text{V})^2/(25\ \Omega) = 121$ W, and $P_2 = (55\ \text{V})^2/(35\ \Omega) = 86.4$ W. Alternatively, we could have used $P = I^2R$, using the current appropriate to each resistor. Then $P_1 = (2.2\ \text{A})^2(25\ \Omega) = 121$ W, and $P_2 = (1.57\ \text{A})^2(35\ \Omega) = 86.4$ W.

(c) The equivalent resistance is $R_{eq} = (25)(35)/(25 + 35) = 14.6\ \Omega$. The total power is therefore $P_{tot} = (55)^2/14.6 = 207.4$ W. This equals the sum of $P_1 + P_2 = 121 + 86.4$.

Kirchhoff's Rules

- The procedure for analyzing more complex circuits is greatly simplified if we use two principles called Kirchhoff 's rules:

- **1. Junction rule.** The sum of the currents entering any junction in a circuit must equal the sum of the currents leaving that junction:

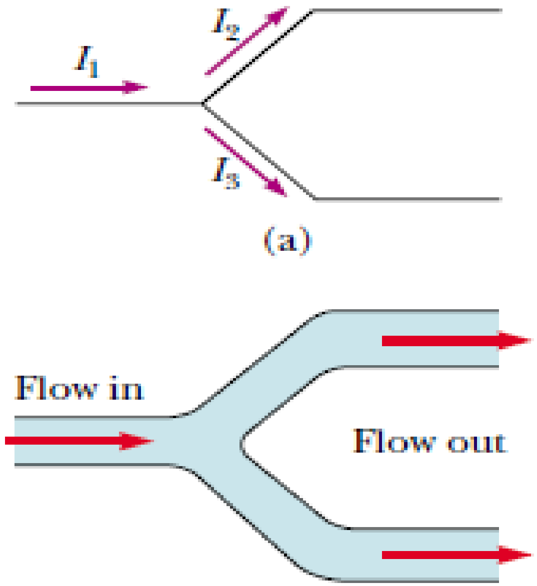
$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

- **2. Loop rule.** The sum of the potential differences across all elements around any closed circuit loop must be zero:

$$\sum_{\substack{\text{closed} \\ \text{loop}}} \Delta V = 0$$

Kirchhoff's Rules

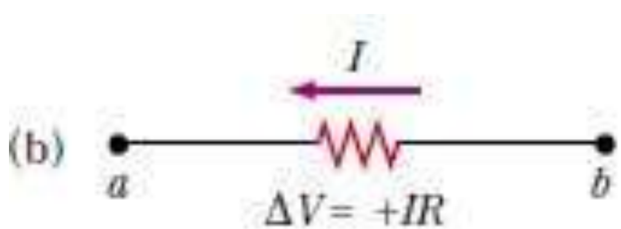
- **Kirchhoff's first rule is a statement of conservation of electric charge.**
- All charges that enter a given point in a circuit must leave that point because charge cannot build up at a point.
- If we apply this rule to the junction shown in Figure, we obtain

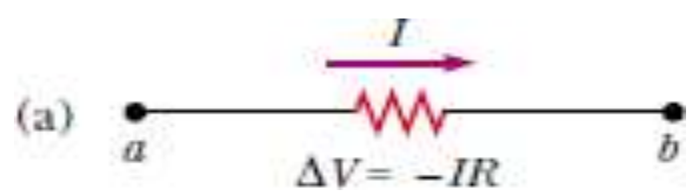


$$I_1 = I_2 + I_3$$

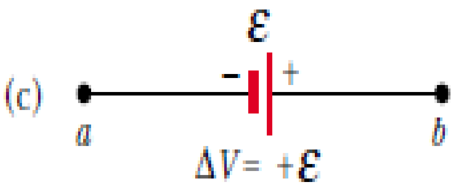
When applying Kirchhoff's second rule in practice, we imagine *traveling* around the loop and consider changes in *electric potential*, rather than the changes in *potential energy* described in the preceding paragraph. You should note the following sign conventions when using the second rule:Because charges move from the high-potential end of a resistor toward the low potential end, if a resistor is traversed in the direction of the current, the potential difference ΔV across the resistor is $-IR$.

- If a resistor is traversed in the direction *opposite* the current, the potential difference ΔV across the resistor is $+IR$.

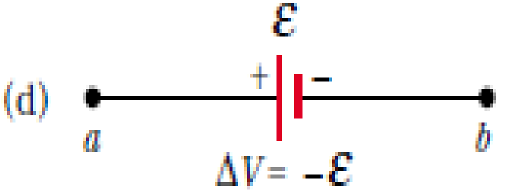




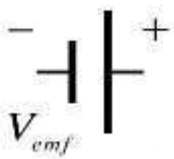
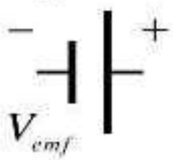
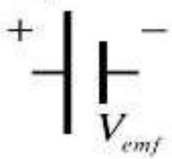
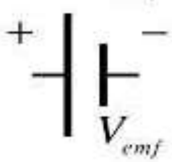
•If a source of emf (assumed to have zero internal resistance) is traversed in the DIRECTION of the emf (from - to +), the potential difference ΔV is +E. The emf of the battery increases the electric potential as we move through it in this direction.



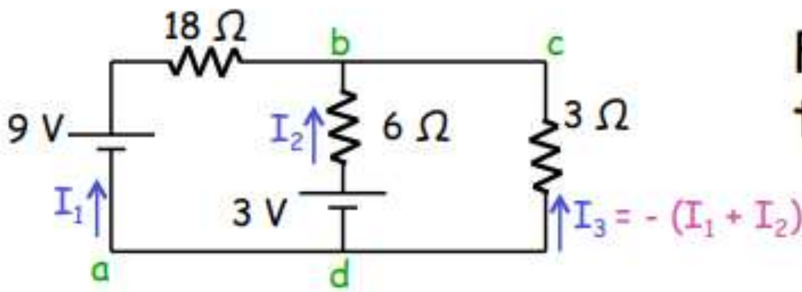
•If a source of emf (assumed to have zero internal resistance) is traversed in the direction OPPOSITE the emf (from + to -), the potential difference ΔV is -E. In the case of the emf of the battery reduces the electric potential as we move through it.



Circuit Analysis Conventions

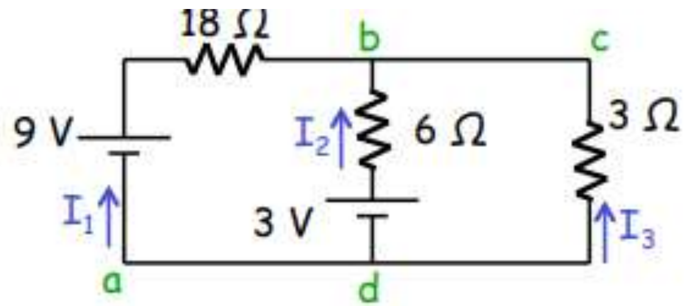
Element	Analysis Direction	Current Direction	Voltage Drop
	$\rightarrow$	$\rightarrow$	$-iR$
	$\leftarrow$	$\rightarrow$	$+iR$
	$\rightarrow$	$\leftarrow$	$+iR$
	$\leftarrow$	$\leftarrow$	$-iR$
	$\rightarrow$		$+V_{emf}$
	$\leftarrow$		$-V_{emf}$
	$\rightarrow$		$-V_{emf}$
	$\leftarrow$		$+V_{emf}$

Example



Find the current through each battery.

The loop rule applied to loop *abda* will give:

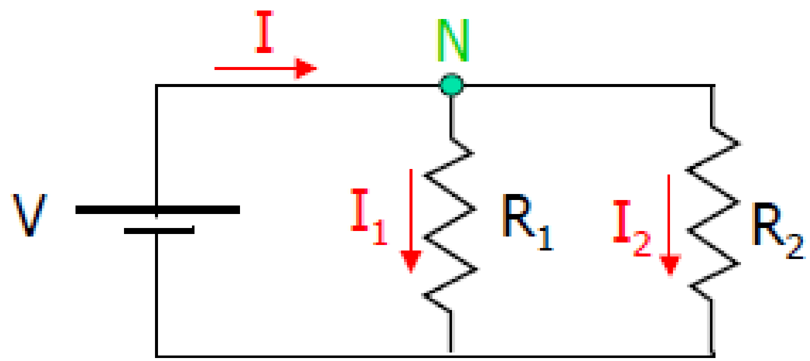


A) $12A - 18I_1 + 6I_2 = 0$
B) $12A - 18I_1 - 6I_2 = 0$
C) $6A - 18I_1 - 6I_2 = 0$
D) $6A + 18I_1 + 6I_2 = 0$
E) $6A - 18I_1 + 6I_2 = 0$

- Solve the circuit:

$$V = I_1 R_1 \quad \Rightarrow \quad I_1 = \frac{V}{R_1}$$

$$V = I_2 R_2 \quad \Rightarrow \quad I_2 = \frac{V}{R_2}$$



- Apply Kirchhoff's first law: $I = I_1 + I_2$

$$I = I_1 + I_2 = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

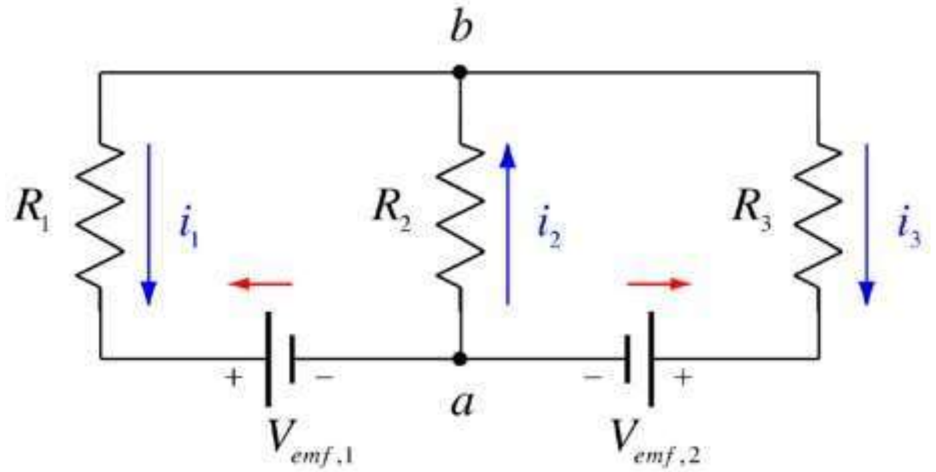
Example - Kirchhoff's Rules

- At junction ***b*** the incoming current must equal the outgoing current

$$i_2 = i_1 + i_3$$

- At junction ***a*** we again equate the incoming current and the outgoing current

$$i_1 + i_3 = i_2$$

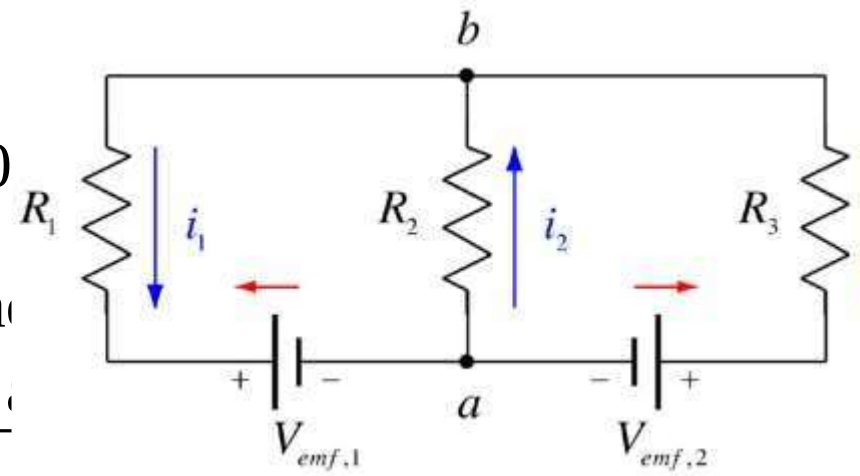


- But this equation gives us the same information as the previous equation!

- We need more information to determine the three currents – 2 more independent equations

- We now have three equations

$$i_1 + i_3 = i_2 \qquad i_1R_1 + V_{emf,1} + i_2R_2 = 0$$



- And we have three unknowns i_1, i_2, an
- We can solve these three equations in

$$i_1 = -\frac{(R_2 + R_3)V_{emf,1} - R_2V_{emf,2}}{R_1R_2 + R_1R_3 + R_2R_3}$$

$$i_2 = -\frac{R_3V_{emf,1} + R_1V_{emf,2}}{}$$

$$i_3 = -\begin{matrix} -R_2V_{emf,1} + (R_1 + R_2)V_{emf,2} \\ R_1R_2 + R_1R_3 + R_2R_3 \end{matrix}$$

$$R_1R_2 + R_1R_3 + R_2R_3$$

- Going around the left loop counterclockwise starting at point *b* we get

- Going around the right loop clockwise starting at point *b* we get

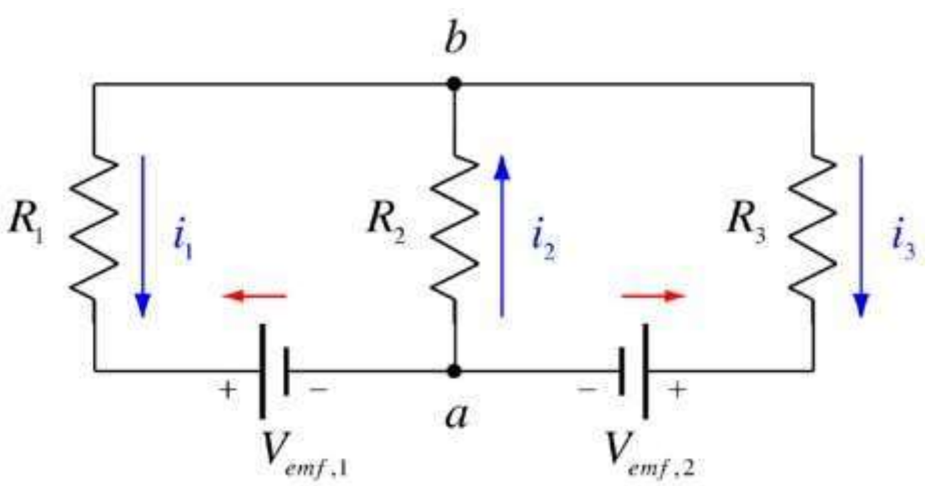
$$-i_1R_1 - V_{emf,1} - i_2R_2 = 0 \quad \Rightarrow \quad i_1R_1 + V_{emf,1} + i_2R_2 = 0$$

- Going around the outer loop clockwise starting at point *b* we get

$$-i_3R_3 - V_{emf,2} - i_2R_2 = 0 \quad \Rightarrow \quad i_3R_3 + V_{emf,2} + i_2R_2 = 0$$

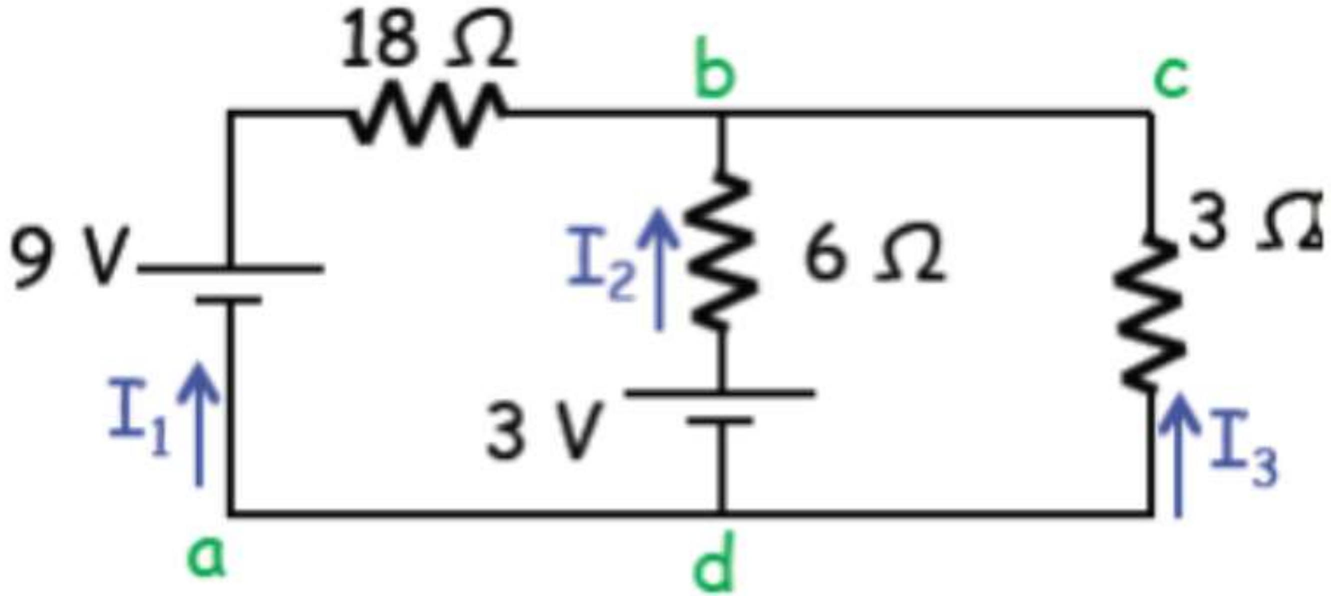
- But this equation gives us no new information!

$$-i_3R_3 - V_{emf,2} + V_{emf,1} + i_1R_1 = 0$$



Example

Find the current through each branch.



According to Kirchhoff's Current Law:

$\boldsymbol{I_1 + I_2 + I_3 = 0}$……………………eq.1

According to Kirchhoff's Voltage Law:

From loop 1:

$\mathbf{9\ V = 18\, I_1 - 6\, I_2 + 3\, V}$ ……………………………eq.2

From loop 2:

$\mathbf{3\ V = 6\, I_2 - 3 I_3}$ ……………………………………eq.3

eq.2 $\Rightarrow 6 = 18 I_1 - 6\, I_2 \Rightarrow I_1 = \frac{6+6I_2}{18} \Rightarrow I_1 = \frac{1+I_2}{3}$

eq.3 $\Rightarrow 3 = 6\, I_2 - 3\, I_3 \Rightarrow I_3 = \frac{-3+6I_2}{3}$

$$\boldsymbol{I_1 + I_2 + I_3 = 0}$$

$$\Rightarrow \frac{1 + I_2}{3} + \boldsymbol{I_2} + \frac{-3 + 6I_2}{3} = \mathbf{0}$$

$$\Rightarrow \frac{1+I_2+3I_2-3+6I_2}{18} = 0 \Rightarrow \boldsymbol{I_2 = \frac{2}{10} \Rightarrow I_2 = 0.2\ A = 200\ mA}$$

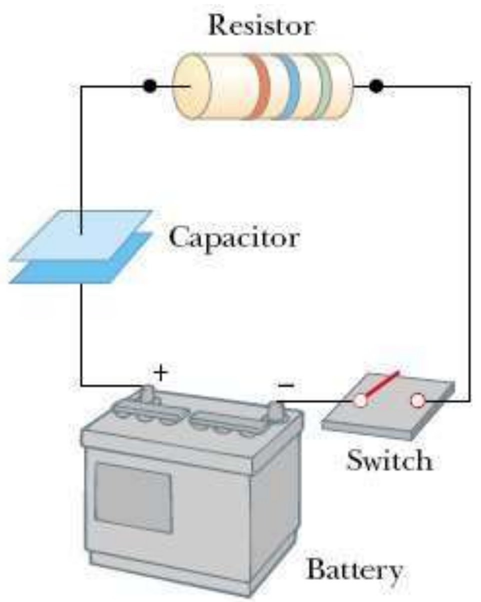
$I_1 = \frac{1+I_2}{3} \Rightarrow I_1 = \frac{1+0.2}{3} \Rightarrow \boldsymbol{I_1 = 0.4\ A = 400\ mA}$

$I_3 = \frac{-3+6I_2}{3} \Rightarrow I_3 = \frac{-3+6\times 0.2}{3} \Rightarrow \boldsymbol{I_3 = -0.6\ A = -600\ mA}$

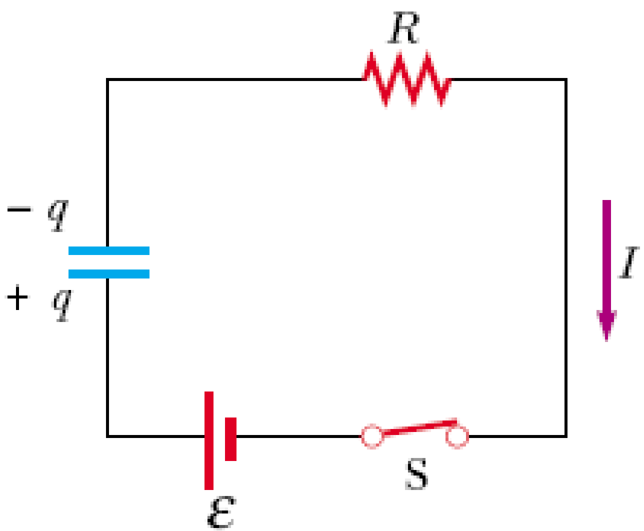
$\boldsymbol{I_3}$: is in the opposite direction: $\boldsymbol{I_1 + I_2 = I_3}$

Charging and Discharging Processes in RC Circuits

- A circuit containing a series combination of a resistor and a capacitor is called An **RC Circuits**



(a)



(c) $t > 0$

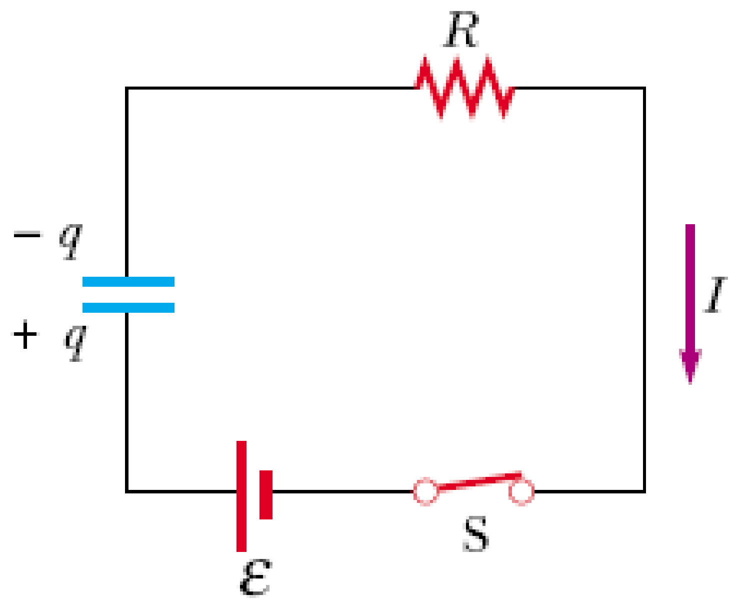
Circuit diagram

A capacitor in series with a resistor, switch, and battery.

RC Circuits

- To analyze this circuit quantitatively, let us apply Kirchhoff's loop rule to the circuit after the switch is closed. Traversing the loop in Fig. c clockwise gives

$$\mathcal{E} - \frac{q}{C} - IR = 0$$



(c) $t > 0$

where q/C is the potential difference across the capacitor and IR is the potential difference across the resistor.

RC Circuits

- **Charging a Capacitor**

Charge as a function of time
for a capacitor being charged

$$q(t) = C\mathcal{E}(1 - e^{-t/RC}) = Q(1 - e^{-t/RC})$$

The charging current is

$$I(t) = \frac{\mathcal{E}}{R} e^{-t/RC}$$

The quantity *RC*, which appears in the exponents of Equations, is called the time constant - of the circuit.
It represents the time interval during which the current decreases to 1/*e* of its initial value.

RC Circuits

- Discharging a Capacitor

Charge as a function of time for a discharging capacitor

$$q(t) = Qe^{-t/RC}$$

Current as a function of time for a discharging capacitor:

$$I(t) = \frac{dq}{dt} = \frac{d}{dt}\left(Qe^{-t/RC}\right) = -\frac{Q}{RC}e^{-t/RC}$$

where $Q/RC = I_0$ is the initial current.
The negative sign indicates that as the capacitor discharges, the current direction is opposite its direction when the capacitor was being charged.