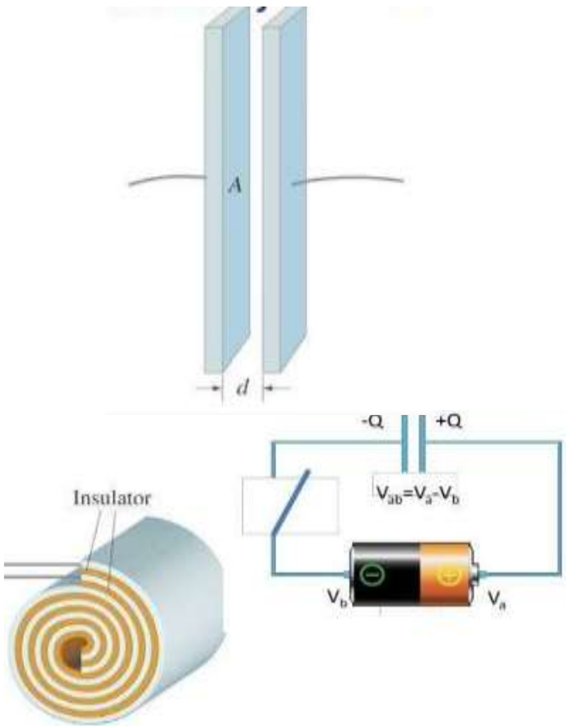
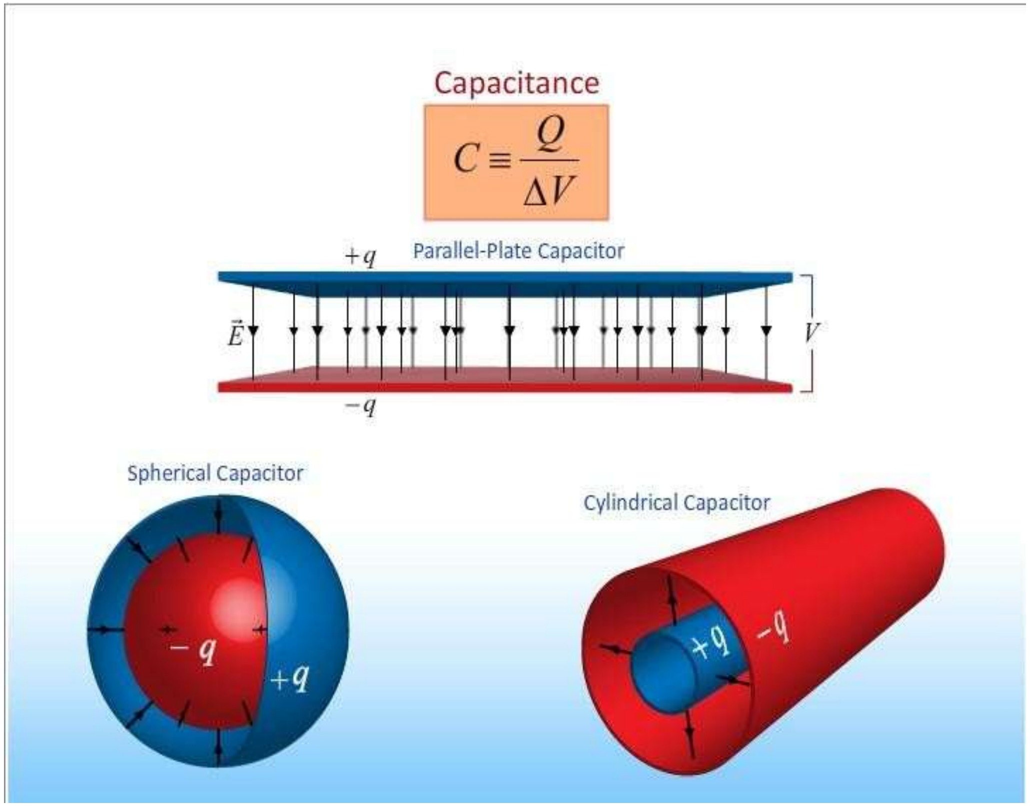
Capacitors

A capacitor is the device that stores charge and electrical energy.

A capacitor consists of two conductors separated by an insulator.



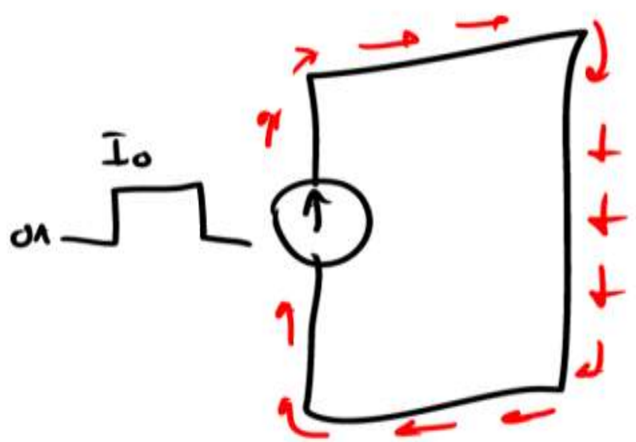


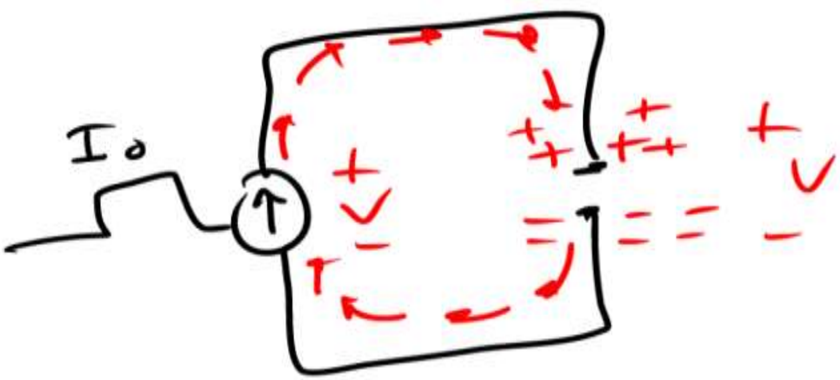
Thought Experiment

• Imagine a current source connected to an ideal short circuit as shown in figure 2. The waveform for the current is shown. It’s a constant current I_0 for a time T, so a total charge of $Q = I_0 \cdot T$ circulates around but no net work is done since $v(t) = 0$ (short circuit).

• Now imagine that we break the conductor in two and leave a gap between the conductors. Let’s repeat the experiment by applying the same current waveform.

• What happens? The current flow is interrupted but the same amount of charge Q leaves the positive terminals of the current source. Where does it go?



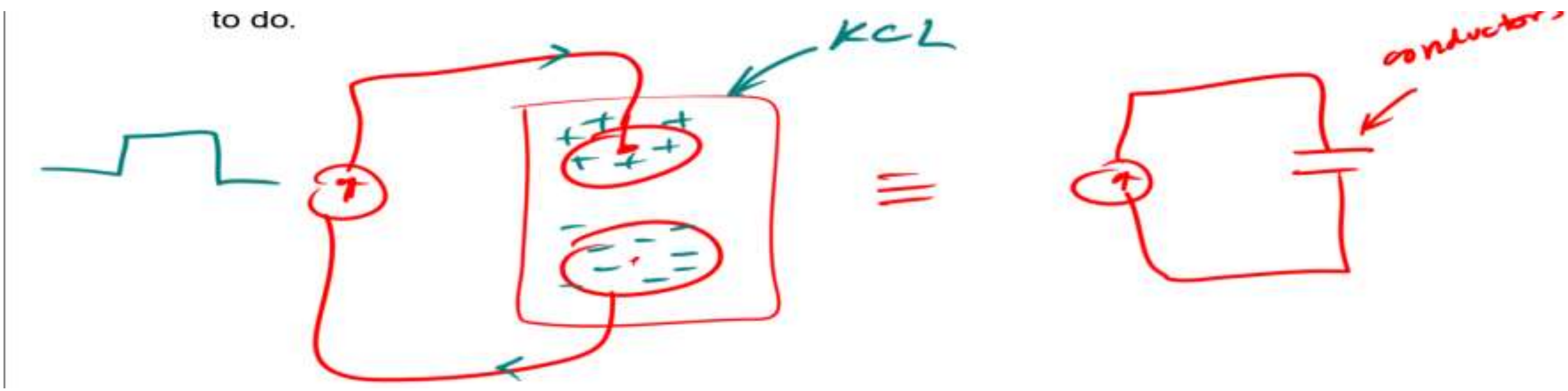


Capacitor Charge

• In addition to the positive charge Q leaving the positive terminal, the same amount of charge enters the negative terminal. Equivalently, a charge of −Q leaves the negative terminal and goes into the conductors.

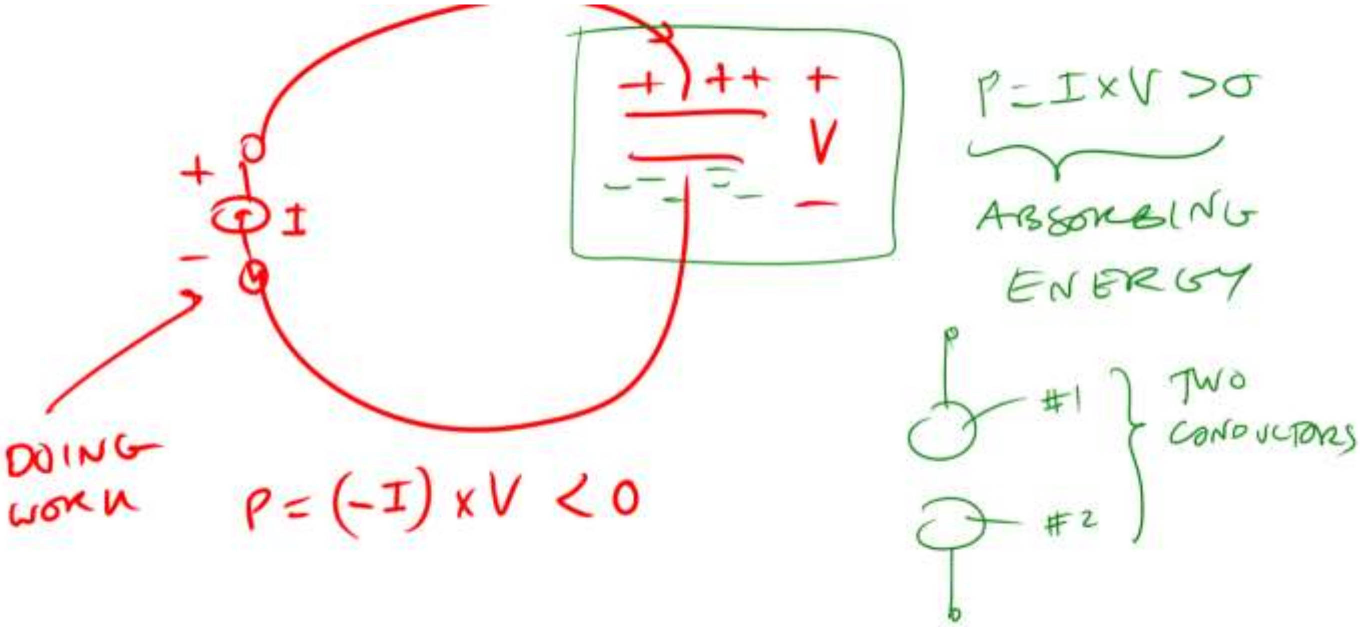
• We see that the charge cannot go anywhere but into the conductors, and therefore the charge is stored there.

• Since like charges repel, we have to do work to force the charges to accumulate on the conductors. In fact, the smaller the conductor, the more work that we have to do.



Capacitor Voltage

- By definition, the potential v across the capacitor represents the amount of work required to move a unit of charge onto the capacitor plates. This is the work done by the current source.
- For linear media, we observe that as we push more charge onto the capacitor with a fixed current, it's voltage increases linearity because it's more and more difficult to do it (like charges repel). $V \propto Q$



1. Definition of Capacitance

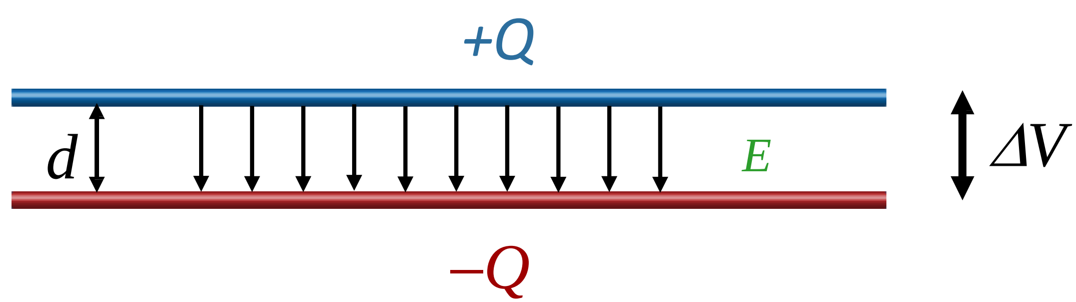
$$C = \frac{Q}{\Delta V}$$

Capacitance is defined for any pair of spatially separated conductors.

The quantity ***C*** is called the *capacitance*. It is always a positive quantity. Furthermore, the charge Q and the potential difference ΔV are positive quantities. Because the potential difference increases linearly with the stored charge, the ratio $Q / \Delta V$ is constant for a given capacitor.

How do we understand this definition ?

- Consider two conductors, one with excess charge = $+Q$ and the other with excess charge = $-Q$



- These charges create an electric field in the space between them
- We can integrate the electric field between them to find the potential difference between the conductor
- This potential difference should be proportional to Q !
  - The ratio of Q to the potential difference is the capacitance and only depends on the geometry of the conductors

We expect that a physically larger conductor should heave a larger capacitance $C_1 > C_2$, because it has more surface area for the charges to reside. The average distance between the like charges determines how much energy you have to provide to push additional charges onto the capacitor.

The SI unit of capacitance is the farad (F), $1\,\mathrm{F} = 1\,\mathrm{C/V}$

- Charge Q is measured in coulombs, C.
- Potential difference, V, is measured in volts, V.
- Capacitance, C, is measured in farads, F.
- 1 farad is 1 coulomb per volt: $1\ \mathrm{F} = 1\ \mathrm{C\ V^{-1}}$

- 1 farad is a very large unit. It is much more common to use the following:

  mF = 10^{-3} F

  μF = 10^{-6} F

  nF = 10^{-9} F

  pF = 10^{-12}F

Capacitor Energy

- Where v is the potential energy of the capacitor in a given state.
- Since $q = Cv$, we have $dq = Cdv$, or

$$dE = Cvdv$$

- If we now integrate from zero potential (no charge) to some final voltage

- The incremental of amount of work done to move a charge dq onto the plates of the capacitor is given by

$$dE = vdq$$

‘

- This is the energy stored in the capacitor. Just like the water tank, it's stored as potential energy that we can later recover.

$$E = \int_0^{V0} C\,V\,dV = \frac{1}{2}\,CV^2{}_0$$

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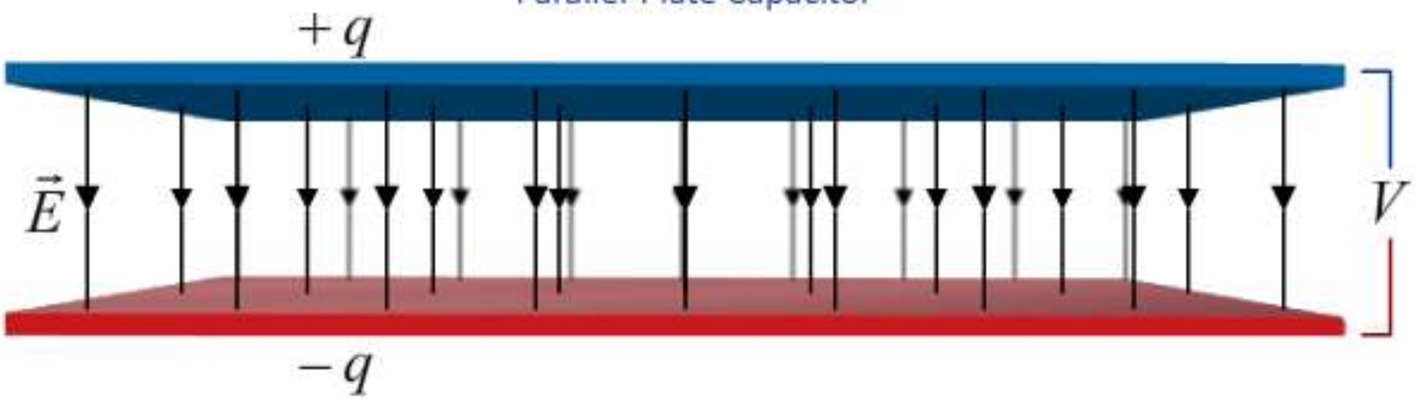
Capacitors Store Energy in E

Parallel-Plate Capacitor



$$u = \frac{1}{2}\,\varepsilon_o\,E^2$$ Energy Density

Energy Stored in Capacitors

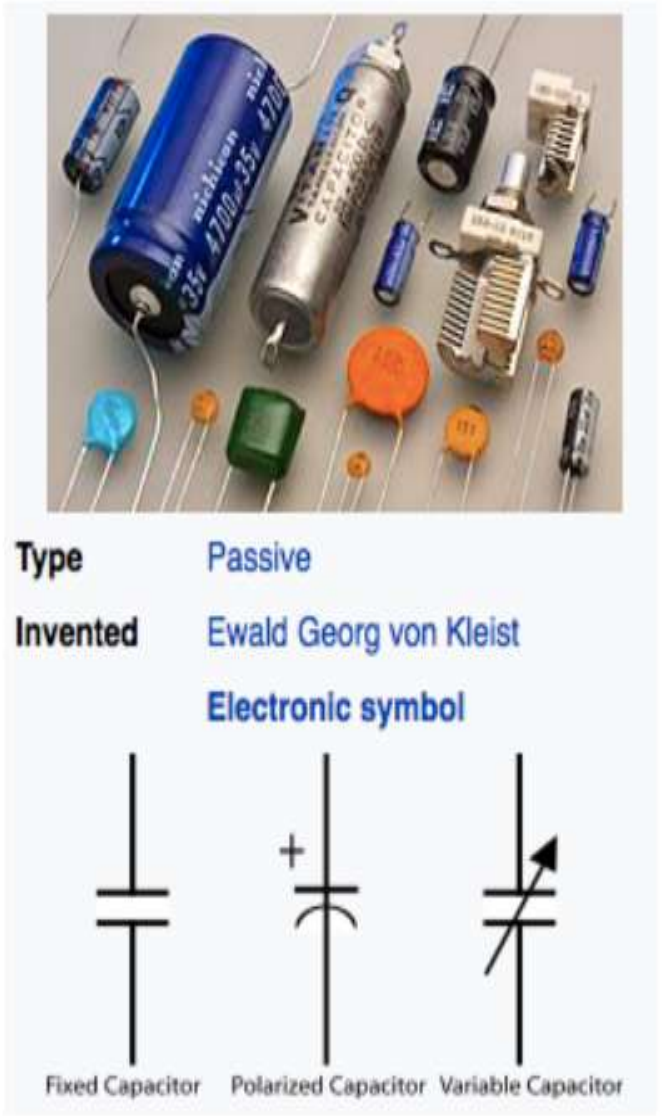
$$U = \frac{1}{2}QV$$ or $$U = \frac{1}{2}\frac{Q^2}{C}$$ or $$U = \frac{1}{2}CV^2$$

Capacitor Types

- There are many different types of capacitors

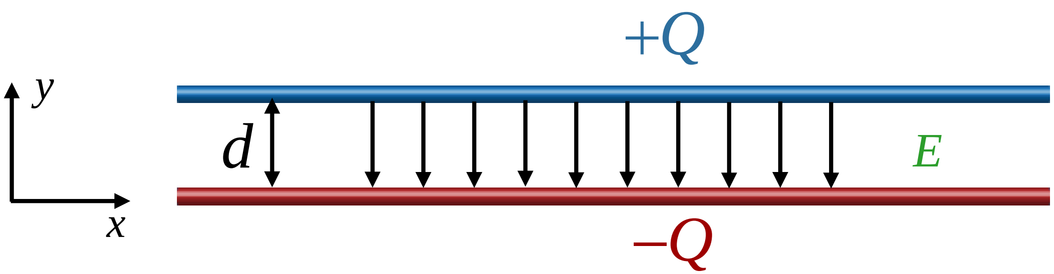
– Electrolytic, tantalum, ceramic, mica,

- They come in different sizes
  - Larger capacitance
    - Generally larger size
  - Higher voltage
    - Larger size



- Electrolytic have largest cap/volume
  - But they have limited voltage
  - They are polarized
    - One terminal must be + vs. other

Example



What is σ ?

$$E = \frac{\sigma}{\varepsilon_o} \qquad \sigma = \frac{Q}{A}$$

A = area of plate

Second, integrate E to find the potential difference V

$$V = -\int_0^d \vec{E} \cdot \vec{dy} \longrightarrow V = -\int_0^d (-E dy) = E \int_0^d dy = \frac{Q}{\varepsilon_o A} d$$

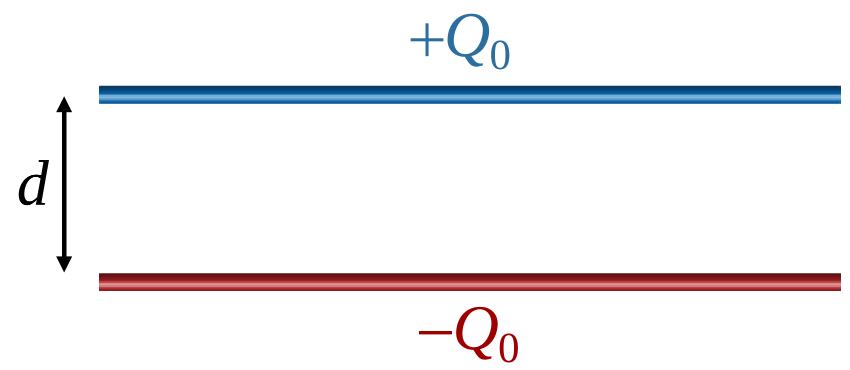
As promised, V is proportional to Q !

$$C \equiv \frac{Q}{V} = \frac{\cancel{Q}}{\cancel{Q} d / \varepsilon_o A} \longrightarrow C = \frac{\varepsilon_0 A}{d}$$

C determined by geometry !

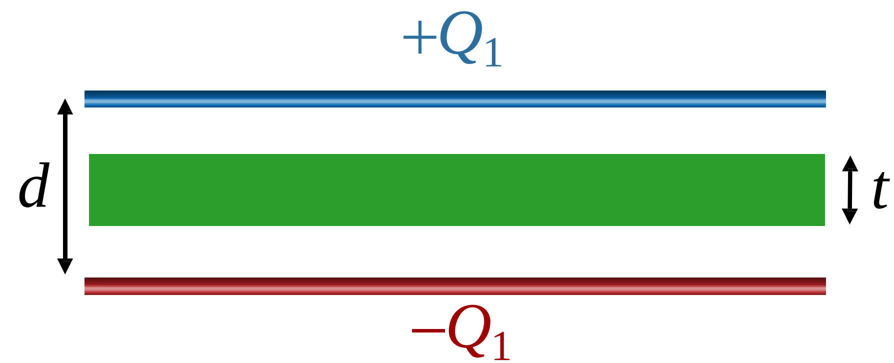
Question

Initial charge on capacitor $= Q_0$



Insert uncharged conductor

Charge on capacitor now $= Q_1$



How is Q_1 related to Q_0 ?

A) $Q_1 < Q_0$
B) $Q_1 = Q_0$
C) $Q_1 > Q_0$

Plates not connected to anything

CHARGE CANNOT CHANGE !

2. Calculating Capacitance

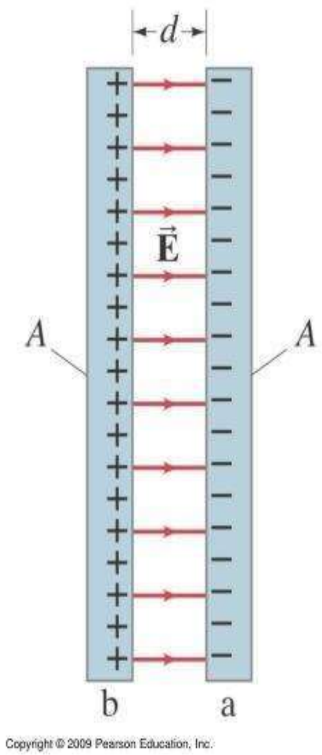
The capacitance of an isolated charged sphere

$$C = \frac{Q}{\Delta V} = \frac{Q}{k_e Q / R} = \frac{R}{k_e} = 4\pi\epsilon_0 R$$

This expression shows that the capacitance of an isolated charged sphere is proportional to its radius and is independent of both the charge on the sphere and the potential difference.

Parallel-Plate Capacitors

Two parallel metallic plates of equal area *A are separated by a distance d*, One
plate carries a charge +*Q* , *and the other carries a charge* -*Q* .



Capacitance

For a parallel-plate capacitor as shown, the field between the plates is

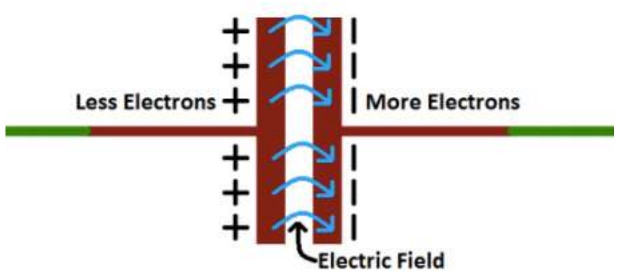
$$E = Q/\varepsilon_0 A.$$

In a uniform field, $V=Ed$:

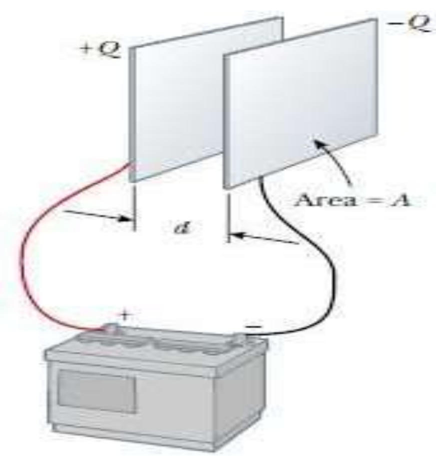
$$V_{ba} = Qd/\varepsilon_0 A.$$

This gives the capacitance:

$$C=Q/V= \varepsilon_0 A/d$$



That is, the capacitance of a parallel- plate capacitor is proportional to the area of its plates and inversely proportional to the plate separation



Example .1 **Parallel-Plate Capacitor**

A parallel-plate capacitor with air between the plates has an area $A = 2.00 \times 10^{-4}\ \text{m}^2$ and a plate separation $d = 1.00$ mm. Find its capacitance.

$$C = \frac{\epsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.00 \times 10^{-4}\ \text{m}^2)}{1.00 \times 10^{-3}\ \text{m}}$$

$$= 1.77 \times 10^{-12}\ \text{F} = \boxed{1.77\ \text{pF}}$$

Combinations of Capacitors

❖ Parallel Combination

• Capacitors in parallel are the same and are equal to the potential difference applied across each one.

• The total charge on capacitors connected in parallel is the sum of the charges on the individual capacitors.

$$Q = Q_1 + Q_2$$

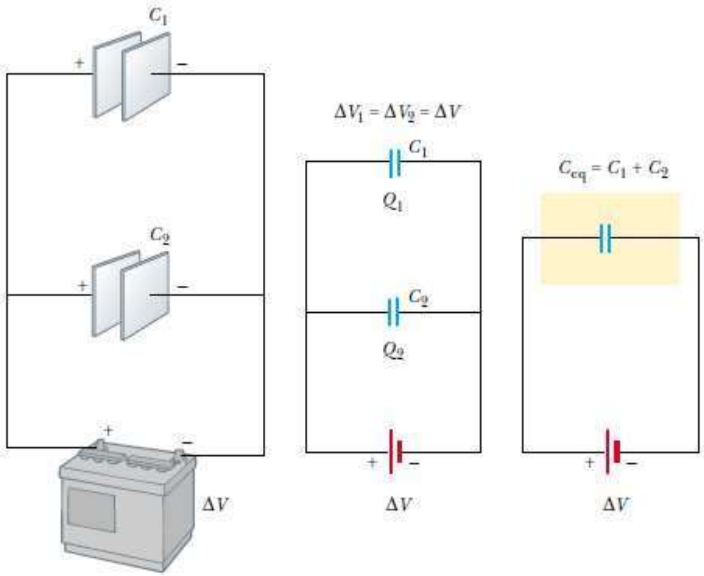
$$Q_1 = C_1\,\Delta V \qquad Q_2 = C_2\,\Delta V$$

$$C_{eq}\,\Delta V = C_1\,\Delta V + C_2\,\Delta V$$

$$C_{eq} = C_1 + C_2 \qquad \text{(parallel combination)}$$

for the equivalent capacitor

$$Q = C_{eq}\,\Delta V$$



$$C_{eq} = C_1 + C_2 + C_3 + \cdots \qquad \text{(parallel combination)}$$

❖ Series Combination

• The charges on capacitors connected in series are the same.

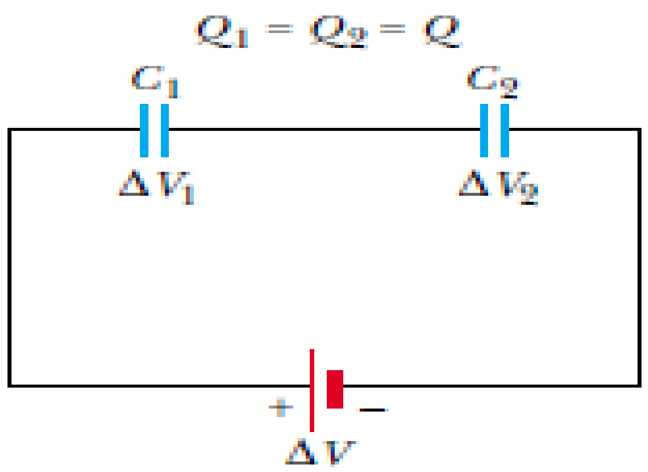
• The total potential difference across any number of capacitors connected in series is the sum of the potential differences across the individual capacitors.

$$\Delta V = \Delta V_1 + \Delta V_2$$

$$\Delta V = \frac{Q}{C_{eq}}$$

$$\Delta V_1 = \frac{Q}{C_1} \qquad \Delta V_2 = \frac{Q}{C_2}$$

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

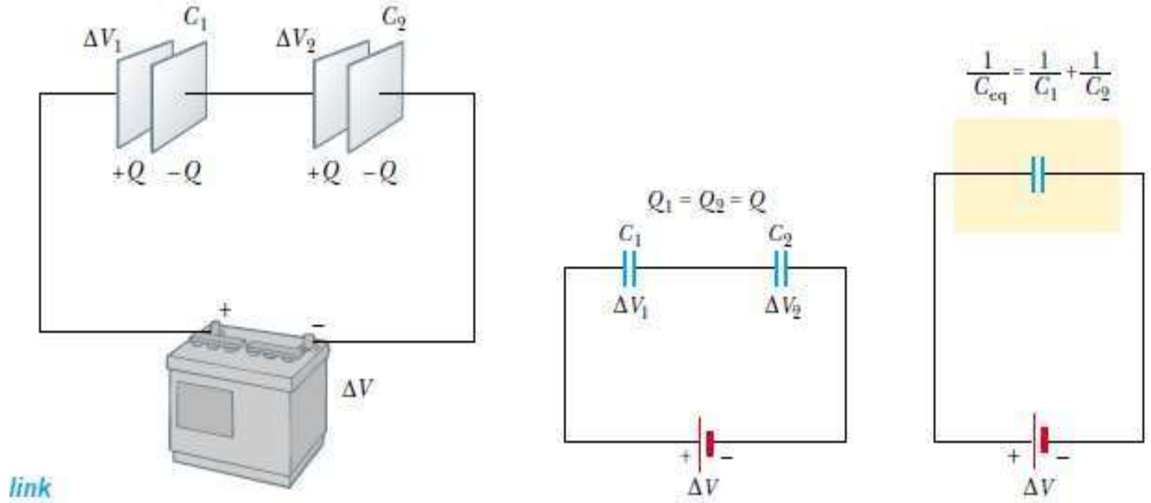


$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \qquad \text{(series combination)}$$

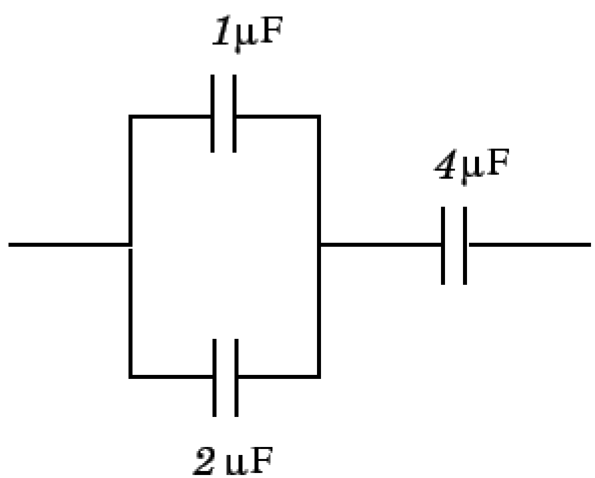
When this analysis is applied to three or more capacitors connected in series, the relationship for the equivalent capacitance is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \qquad \text{(series combination)}$$

The equivalent capacitance of a series combination is always less than any individual capacitance in the combination.



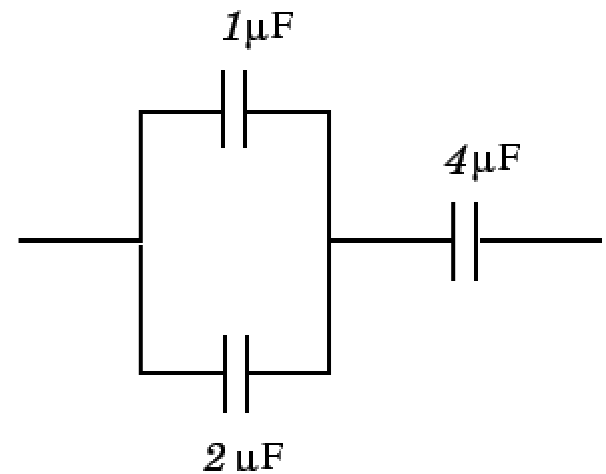
Question: A $1\,\mu\text{F}$ and a $2\,\mu\text{F}$ capacitor are connected in parallel, and this pair of capacitors is then connected in series with a $4\,\mu\text{F}$ capacitor, as shown in the diagram. What is the equivalent capacitance of the whole combination? What is the charge on the $4\,\mu\text{F}$ capacitor if the whole combination is connected across the terminals of a 6V battery? Likewise, what are the charges on the $1\,\mu\text{F}$ and $2\,\mu\text{F}$ capacitors?



Solution:

The equivalent capacitance of the $1\,\mu\mathrm{F}$ and $2\,\mu\mathrm{F}$ capacitors connected in parallel is

$$1 + 2 = 3\,\mu\mathrm{F}$$



When a $3\,\mu\mathrm{F}$ capacitor is combined in series with a $4\,\mu\mathrm{F}$ capacitor, the equivalent capacitance of the whole combination is given by

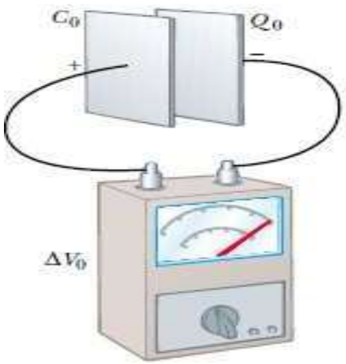
$$\frac{1}{C_{eq}} = \frac{1}{(3 \times 10^{-6})} + \frac{1}{(4 \times 10^{-6})} = \frac{(7)}{(12 \times 10^{-6})}\ \mathrm{F}^{-1},$$

Example:

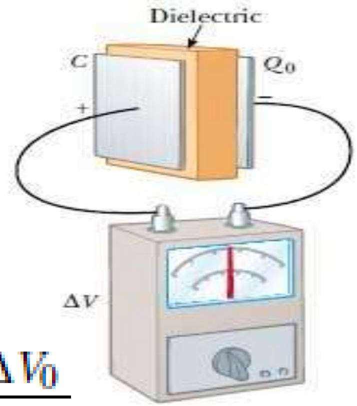
A 10 000μF capacitor is described as having a maximum working voltage of 25 V. Calculate the energy stored by the capacitor.

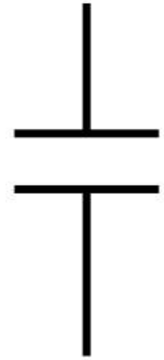
$E = ½\ CV^2 = ½ \times 10{,}000 \times 10^{-6} \times 25^2 = 3.125\ J$

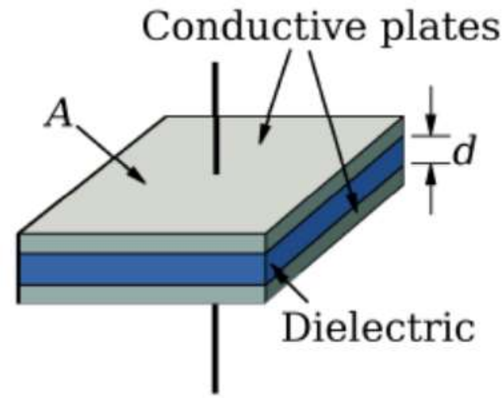
Capacitor with a Dielectric



$$\Delta V = \frac{\Delta V_0}{\kappa}$$







The dielectric constant

The surface charges on the dielectric reduce the electric field inside the dielectric. This reduction in the electric field is described by the *dielectric constant k*, which is the ratio of the field magnitude E_0 without the dielectric to the field magnitude E inside the dielectric:

$$\kappa = \frac{E_0}{E}$$

Every dielectric material has a characteristic *dielectric strength,* which is the maximum value of the electric field that it can tolerate without breakdown

Capacitors have many important applications. They are used, for example, in digital circuits so that information stored in large computer memories is not lost during a momentary electric power failure; the electric energy stored in such capacitors maintains the information during the temporary loss of power. Capacitors play an even more important role as filters to divert spurious electric signals and thereby prevent damage to sensitive components and circuits caused by electric surges.

me properties of di electrics

	Dielectric Constant	Dielectric Strength (kV/mm)
Air (1 atm)	1.00054	3
Polystyrene	2.6	24
Paper	3.5	16
Transformer oil	4.5	
Pyrex	4.7	14
Ruby mica	5.4	
Porcelain	6.5	
Silicon	12	
Germanium	16	
Ethanol	25	
Water (20°C)	80.4	
Water (25°C)	78.5	
Titania ceramic	130	
Strontium titanate	310	8

For a vacuum, $\kappa = \text{unity}$