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Institute of Mathematics & and Computer Science

Mathematics Department

Series No. 3 Electromagnetic Exercises

1 Electromagnetic Exercise

1.1 Exercise 1

Two infinite wires, separated by a distance $d = 15\,\text{cm}$, carry currents of 1.1 A for the first and 2.2 A for the second.

a) On the diagram, draw the force that wire 1 exerts on wire 2.

b) Calculate the intensity of the magnetic field generated by wire 1 at the location of wire 2.

c) Calculate the magnitude of the force per unit length that wire 1 exerts on wire 2.

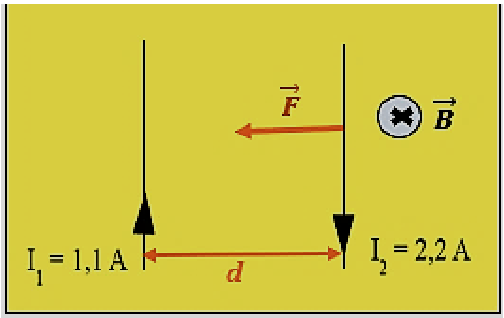


Figure 1: Two parallel wires carrying currents $I_1 = 1.1\,\text{A}$, $I_2 = 2.2\,\text{A}$, separated by $d = 15\,\text{cm}$

1.2 Exercise 2

A flat coil consisting of a single loop with a radius of 5 cm is carrying a current of intensity I.

1. How should this coil be oriented so that the total magnetic field at its center is zero? (Taking into account the Earth's magnetic field which is $B_{\text{earth}} = 2.2 \times 10^{-5}\,\text{T}$.)
2. What should be the current intensity I in this case?
3. Same question in the case where the coil consists of 50 loops?

1.3 Exercise 3

The circulation of an electron orbiting a proton in uniform circular motion produces a magnetic field in the vicinity of the proton. Consider one period of revolution of the electron around the nucleus.

- Calculate the intensity of this magnetic field, given that the radius of the orbit is $R = 0.529\,\text{Å}$ and the speed of the electron is $v = 2200\,\text{km/s}$.

1.4 Exercise 4

A rectangular loop $ACDE$ with side length $a = 20cm$ is made of a single turn of rigid conducting wire with a mass of $m = 16gram$. This loop, which is free to rotate without friction about its horizontal side AC, is placed in a uniform vertical magnetic field $\vec{B}$, directed upward, with an intensity $B = 0.1T$.

A current of intensity I flows through the loop, which assumes an equilibrium position defined by the angle θ, as illustrated in the figure below.

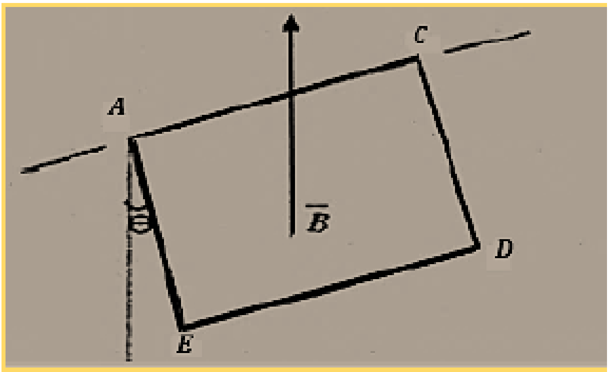


Figure 2: Frame in a uniform vertical magnetic field

1. Represent the direction of the current and the electromagnetic forces acting on each of the four sides.
2. Express the current intensity I as a function of: a, B, m, θ, and g, then calculate its value for:

$$\theta = 21°, \quad g = 9.8m/s^2$$

2 Solution of Exercise series 3

2.1 Exercise 1

Given:

- $I_1 = 1.1\,\mathrm{A}$
- $I_2 = 2.2\,\mathrm{A}$
- Distance between the wires: $d = 15\,\mathrm{cm} = 0.15\,\mathrm{m}$

a) **Direction of the force:**

If both currents flow in the same direction (assumed here to be out of the page), the magnetic force between the wires is **attractive**. Thus, wire 1 exerts a force on wire 2 **toward itself**.

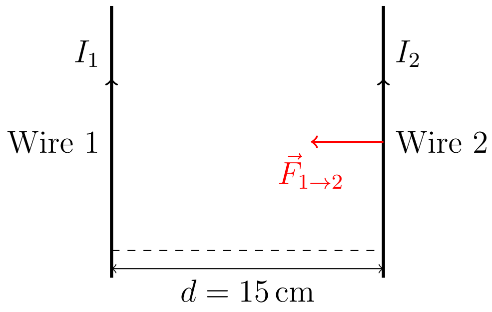


Figure 3: Attractive force between two parallel currents in the same direction

b) **Magnetic field at wire 2 due to wire 1:**

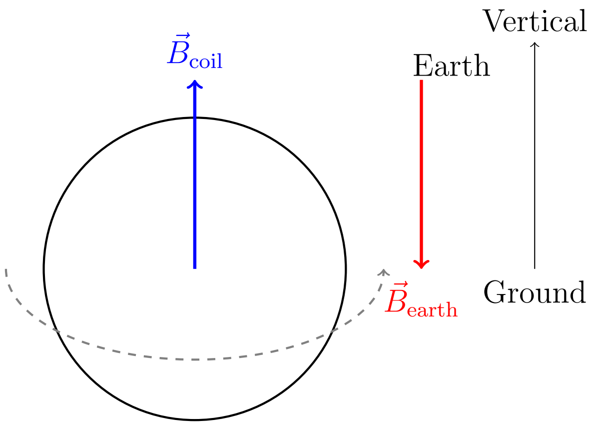
$$B_1 = \frac{\mu_0 I_1}{2\pi d} = \frac{4\pi \times 10^{-7} \times 1.1}{2\pi \times 0.15} = \frac{4.4 \times 10^{-7}}{0.3} = \boxed{1.47 \times 10^{-6}\,\mathrm{T}}$$

c) **Force per unit length on wire 2:**

$$\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d} = \frac{4\pi \times 10^{-7} \times 1.1 \times 2.2}{2\pi \times 0.15} = \frac{9.68 \times 10^{-7}}{0.3} = \boxed{3.23 \times 10^{-6}\,\mathrm{N/m}}$$

2.2 Exercise 2 – Compensating Earth's Magnetic Field with a Coil

2.2.1 a) Orientation of the Coil (Diagram)



Circular Coil

Current direction (right-hand rule)

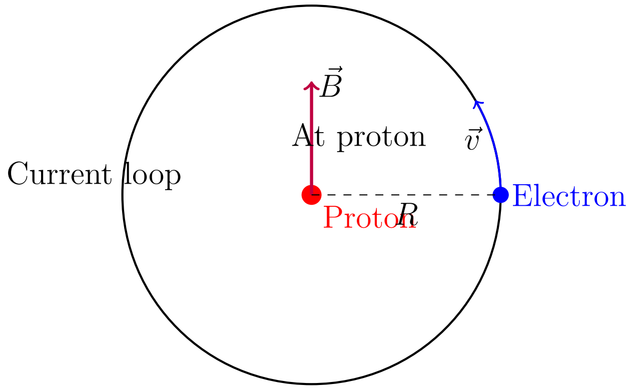
Explanation: The coil must generate a magnetic field $\vec{B}_{\text{coil}}$ upward to cancel the Earth's field $\vec{B}_{\text{earth}}$, which is directed downward. This requires a **clockwise current** when viewed from above, according to the **right-hand rule**.

2.2.2 b) and c) Calculations

- For 1 loop: $\boxed{I = 3.50\,\text{A}}$
- For 50 loops: $\boxed{I = 70\,\text{mA}}$

2.3 Exercise 3 – Magnetic Field from an Orbiting Electron

2.3.1 Diagram of the Model



2.3.2 Summary of Calculation

- Radius of orbit: $R = 0.529\,\text{Å} = 0.529 \times 10^{-10}\,\text{m}$
- Speed of electron: $v = 2.2 \times 10^{6}\,\text{m/s}$

- Orbital period: $T = \dfrac{2\pi R}{v} \approx 1.51 \times 10^{-16}\,\mathrm{s}$
- Equivalent current: $I = \dfrac{e}{T} \approx 1.06 \times 10^{-3}\,\mathrm{A}$
- Magnetic field:

$$B = \frac{\mu_0 I}{2R} \approx \boxed{12.6\,\mathrm{T}}$$

2.4 Exercise 4

A rectangular loop $ACDE$ of side length $a = 20\,\mathrm{cm}$ is formed by a single turn of rigid conducting wire of mass $m = 16\,\mathrm{g}$. The loop is free to rotate without friction about the horizontal axis AC, and it is placed in a uniform vertical magnetic field $\vec{B}$ directed upward with magnitude $B = 0.1\,\mathrm{T}$.

A current of intensity I flows through the loop. In equilibrium, the loop makes an angle θ with the vertical, as shown in the figure. We are to:

1. Represent the direction of the current and the electromagnetic forces acting on the four sides.
2. Express the current intensity I in terms of a, B, m, θ, g, and compute its value for:

$$\theta = 21^\circ, \quad g = 9.8\,\mathrm{m/s^2}$$

2.4.1 Direction of Current and Forces

- Let the current enter from point A to C (axis of rotation), then flow along CD, then along DE, and finally from E to A.
- In the magnetic field $\vec{B}$ (vertical upward), the **Lorentz force** on each segment is given by:

$$\vec{F} = I\vec{l} \times \vec{B}$$

- The vertical segments AC and DE produce torques; only the segment DE contributes to net torque since AC lies along the axis.

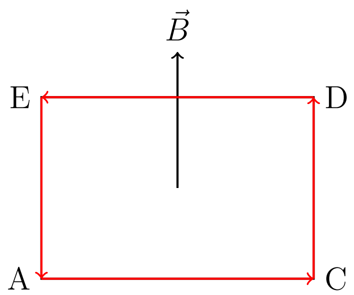


Figure 4: Current and magnetic field directions on the loop.

2.4.2 Expression of the Current

At equilibrium, the magnetic torque balances the gravitational torque:

2.4.3 Magnetic Torque

The force on segment DE is:

$$F = IaB$$

This force acts perpendicular to the plane of the loop at a distance $a\cos\theta$ from the axis. Hence, the magnetic torque is:

$$\tau_{\mathrm{mag}} = IaB \cdot a\cos\theta = Ia^2 B\cos\theta$$

Gravitational Torque

The gravitational torque acts on the center of mass of the loop, located at a distance $\frac{a}{2}\sin\theta$ from the axis:

$$\tau_{\mathrm{grav}} = mg \cdot \frac{a}{2} \cdot \sin\theta$$

Equilibrium Condition

Setting $\tau_{\mathrm{mag}} = \tau_{\mathrm{grav}}$, we obtain:

$$Ia^2 B\cos\theta = \frac{mga}{2}\sin\theta$$

$$\Rightarrow I = \frac{mg\tan\theta}{2aB}$$

2.4.4 Numerical Evaluation

$$m = 16\,\mathrm{g} = 0.016\,\mathrm{kg}, \quad a = 0.2\,\mathrm{m}, \quad B = 0.1\,\mathrm{T}$$
$$\theta = 21^\circ, \quad g = 9.8\,\mathrm{m\,s^{-2}}$$

$$\tan\theta = \tan(21^\circ) \approx 0.383864$$

$$I = \frac{0.016 \times 9.8 \times 0.383864}{2 \times 0.2 \times 0.1} \approx \frac{0.0602}{0.04} = 1.505\,\mathrm{A}$$

$$\boxed{I \approx 1.51\,\mathrm{A}}$$