Introduction to Physics 2: electrostatics and electrokinetics and magnetism

This course is intended for first-year students in Mathematical Sciences

Pr M.S. Benlatreche

2024- 2025

Contents

List of Figures

List of Tables

1. Introduction: Mathematics Revision

This lesson provides a mathematical revision covering fundamental concepts essential for physics and engineering applications. We will review the elements of length, surface area, and volume in different coordinate systems, as well as important mathematical operators and calculus techniques.

2. Elements of Length, Surface Area, and Volume in Different Coordinate Systems

We will analyze the fundamental differential elements in the three primary coordinate systems: Cartesian, cylindrical, and spherical. Understanding these elements is crucial for evaluating integrals in physics and engineering.

2.1 Cartesian Coordinate System

A Cartesian coordinate system is defined by an origin point O and three mutually perpendicular axes (Ox, Oy, Oz). The unit vectors along these axes are $\mathbf{i}, \mathbf{j}, \mathbf{k}$. Any point M in space is represented by the position vector:

$$\vec{R} = \overrightarrow{OM} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

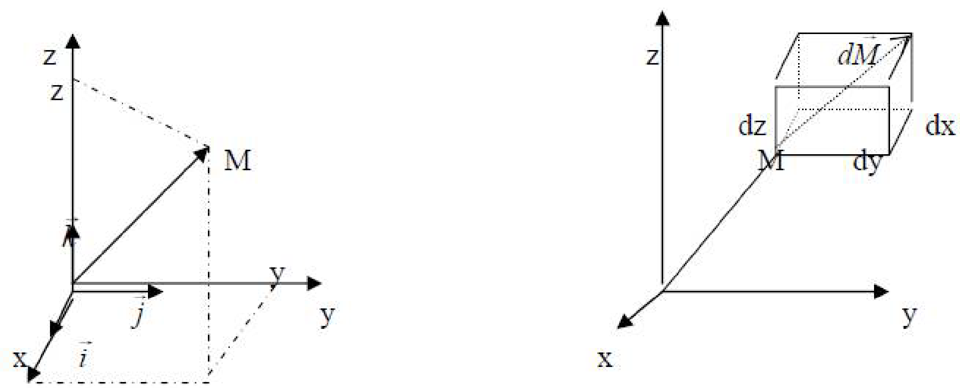

Figure 1: Cartesian basis (a) Position vector and (b) elementary displacement and volume

Example: Consider a straight-line motion along the x-axis where $x = 2t$, $y = 3$, and $z = 0$. The velocity vector is given by:

$$\mathbf{v} = \frac{d\vec{R}}{dt} = \frac{d}{dt}(2t\mathbf{i} + 3\mathbf{j} + 0\mathbf{k}) = 2\mathbf{i}$$

Differential Length Element: The differential displacement is given by:

$$d\overrightarrow{OM} = d\,\vec{l} = dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k}$$

Differential Surface Element: The surface element depends on the plane of integration:

$$dS_x = dydz, \quad dS_y = dxdz, \quad dS_z = dxdy$$

Differential Volume Element: The elementary volume is given by:

$$dV = dx\,dy\,dz$$

2.1.1 Cylindrical Coordinate System

In the cylindrical coordinate system (r, θ, z), a point is represented as:

$$x = r\cos\theta, \quad y = r\sin\theta, \quad z = z$$

It should also be noted that we can write:

$$\mathbf{u}_p = \cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j}$$

and derive this vector with respect to θ:

We obtain:

$$d\mathbf{u}_p = d\theta(-\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j}),$$

knowing that:

$$\cos\left(\theta + \frac{\pi}{2}\right) = -\sin\theta \quad \text{and} \quad \sin\left(\theta + \frac{\pi}{2}\right) = \cos\theta.$$

Thus:

$\dfrac{d\mathbf{u}_p}{d\theta}$ can be obtained by rotating $\mathbf{u}_p$ by an angle of $\dfrac{\pi}{2}$, and we can write:

$$\frac{d\mathbf{u}_p}{d\theta} = \mathbf{u}_\theta.$$

The position vector **DM** is written as:

$$\mathbf{DM} = \rho\mathbf{u}_p + z\mathbf{k} = (x\mathbf{i} + y\mathbf{j}) + z\mathbf{k},$$

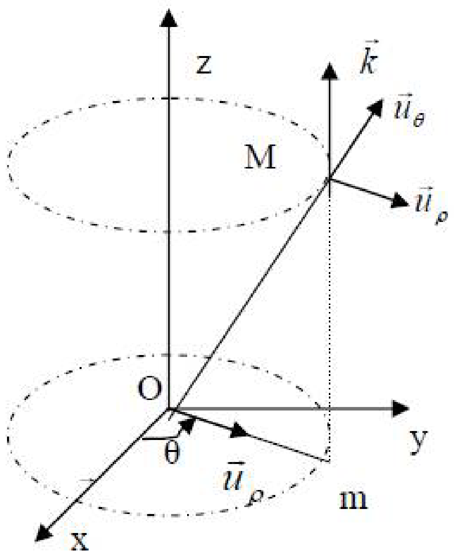

Figure 2: Cylindrical base

where x and y are the Cartesian coordinates of the point M in the Oxy plane, given by:

$$x = \rho \cos\theta, \quad y = \rho \sin\theta, \quad \text{and} \quad z = z.$$

The expression for the elementary displacement is:

$$d\mathbf{DM} = d\rho\, \mathbf{u}_p + \rho\, d\theta\, \mathbf{u}_\theta + dz\, \mathbf{k}.$$

The expression for the elementary surface is:

$$ds = \rho\, d\rho\, d\theta.$$

Example: Find the velocity vector for a particle moving in a circular path where $r = 2$, $\theta = t^2$, and $z = 4t$. The velocity components are:

$$v_r = \frac{dr}{dt} = 0, \quad v_\theta = r\frac{d\theta}{dt} = 2(2t), \quad v_z = \frac{dz}{dt} = 4$$

Thus, the velocity vector is:

$$\mathbf{v} = 0\mathbf{e_r} + 4t\mathbf{e}_\theta + 4\mathbf{e_z}$$

Differential Length Element:

$$d\overrightarrow{l} = dr\mathbf{e_r} + rd\theta\mathbf{e}_\theta + dz\mathbf{e_z}$$

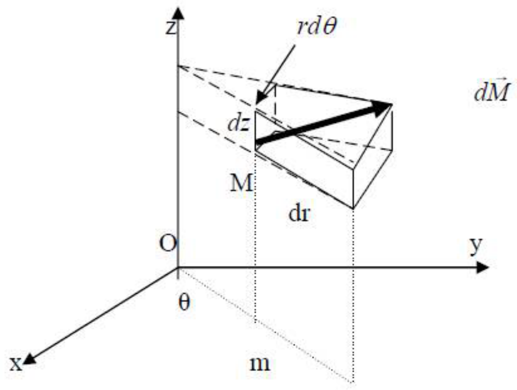

Figure 3: Cylindrical coordinates

Differential Surface Element:

$$dS_r = r d\theta dz, \quad dS_\theta = dr dz, \quad dS_z = r dr d\theta$$

Differential Volume Element:

$$dV = r dr d\theta dz$$

2.2 Spherical Coordinate System

In the spherical coordinate system (r, θ, ϕ), a point is represented as:

$$x = r \sin\theta \cos\phi, \quad y = r \sin\theta \sin\phi, \quad z = r \cos\theta$$

The position vector of point M in spherical coordinates, meaning in the spherical basis, is written as:

$$\overrightarrow{OM} = r\overrightarrow{u_r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}.$$

From the figure, we can express x, y, z in terms of r, θ, ϕ:

$$X = OM \cos\varphi = r \sin\theta \cos\varphi,$$

$$Y = OM \sin\varphi = r \sin\theta \sin\varphi,$$

$$Z = OM \cos\theta = r \cos\theta.$$

Thus, we deduce:

$$\overrightarrow{u_r} = \sin\theta \cos\varphi \,\mathbf{i} + \sin\theta \sin\varphi \,\mathbf{j} + \cos\theta \,\mathbf{k}.$$

The unit vector $\overrightarrow{u_\varphi}$ at OM is written as:

$$\overrightarrow{u_\varphi} = \cos\varphi\,\mathbf{i} + \sin\varphi\,\mathbf{j}.$$

This vector $\overrightarrow{u_\varphi}$ can be obtained either by replacing φ with $\varphi + 2\pi$ or by differentiating $\overrightarrow{u_r}$ with respect to φ:

$$\overrightarrow{u_\varphi} = -\sin\varphi\,\mathbf{i} + \cos\varphi\,\mathbf{j}.$$

This basis vector can also be expressed as the derivative of $\overrightarrow{u_r}$ with respect to φ:

$$\overrightarrow{u_\varphi} = \frac{1}{\sin\theta}\frac{\partial \overrightarrow{u_r}}{\partial \varphi}.$$

The third basis vector in the spherical coordinate system is given by:

$$\overrightarrow{u_\theta} = \frac{\partial \overrightarrow{u_r}}{\partial \theta}.$$

2.2.1 Elementary Displacement:

$$d\overrightarrow{M} = d(r\overrightarrow{u_r}) = dr\overrightarrow{u_r} + rd\overrightarrow{u_r} + r\frac{\partial \overrightarrow{u_r}}{\partial \theta}d\theta + r\frac{\partial \overrightarrow{u_r}}{\partial \varphi}d\varphi.$$

$$= dr\overrightarrow{u_r} + rd\theta\overrightarrow{u_\theta} + r(\sin\theta d\varphi)\overrightarrow{u_\varphi}.$$

2.2.2 Elementary Surface and Volume:

$$dS = r^2 \sin\theta\, d\theta\, d\varphi.$$

$$dV = r^2 \sin\theta\, dr\, d\theta\, d\varphi.$$

Example: Find the length of an infinitesimal arc in spherical coordinates for a small change in θ while keeping r and ϕ constant.

$$dl = rd\theta$$

Differential Length Element:

$$d\vec{l} = dr\mathbf{e_r} + rd\theta\mathbf{e}_\theta + r\sin\theta d\phi\mathbf{e}_\phi$$

Differential Surface Element:

$$dS_r = r^2\sin\theta d\theta d\phi, \quad dS_\theta = r\sin\theta dr d\phi, \quad dS_\phi = rdrd\theta$$

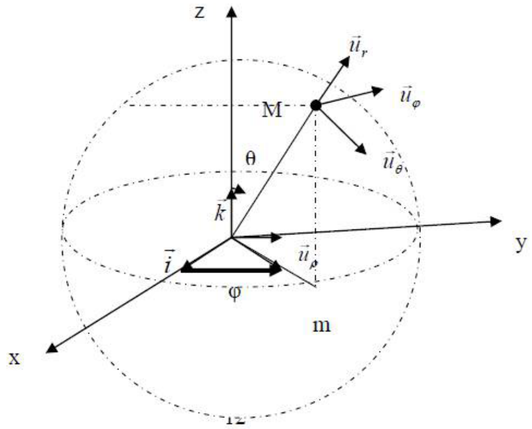

Figure 4: Spherical base

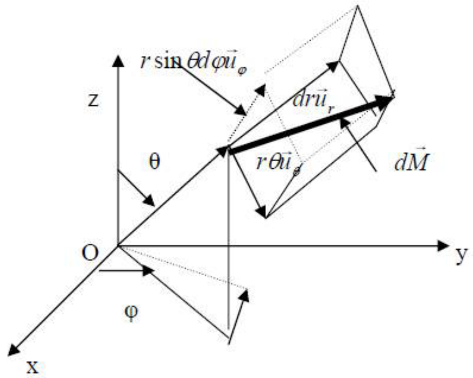

Figure 5: elementary volumes in spherical coordinates

Differential Volume Element:

$$dV = r^2 \sin\theta dr d\theta d\phi$$

2.2.3 Transformational relationships between different coordinates

From	To	Transformation Equations
Cartesian(x,y,z)	Spherical(r,θ,ϕ)	$r=\sqrt{x^2+y^2+z^2}$ $\theta=\arccos\left(\frac{z}{r}\right)$ $\phi=\arctan 2(y,x)$
Spherical(r,θ,ϕ)	Cartesian(x,y,z)	$x=r\sin\theta\cos\phi$ $y=r\sin\theta\sin\phi$ $z=r\cos\theta$
Cartesian(x,y,z)	Cylindrical(ρ,φ,z)	$\rho=\sqrt{x^2+y^2}$ $\varphi=\arctan 2(y,x)$ $z=z$
Cylindrical(ρ,φ,z)	Cartesian(x,y,z)	$x=\rho\cos\varphi$ $y=\rho\sin\varphi$ $z=z$
Spherical(r,θ,ϕ)	Cylindrical(ρ,φ,z)	$\rho=r\sin\theta$ $\varphi=\phi$ $z=r\cos\theta$
Cylindrical(ρ,φ,z)	Spherical(r,θ,ϕ)	$r=\sqrt{\rho^2+z^2}$ $\theta=\arctan\left(\frac{\rho}{z}\right)$ $\phi=\varphi$

2.2.4 Solid Angles

A solid angle $d\Omega$ in spherical coordinates is given by:

$$d\Omega=\sin\theta d\theta d\phi$$

The total solid angle in three-dimensional space is:

$$\Omega=\int_0^{2\pi}\int_0^{\pi}\sin\theta d\theta d\phi=4\pi$$

3. Operators in Vector Calculus

Example: Compute the gradient of the scalar function $f(x,y,z)=x^2+y^2+z^2$:

$$\nabla f=(2x,2y,2z)$$

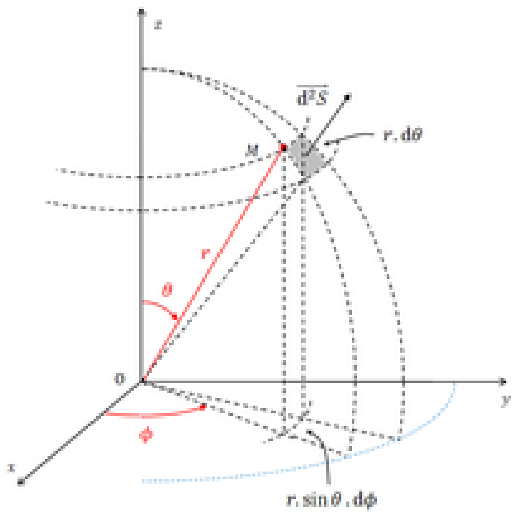

Figure 6: Solid Angles

Example: Compute the divergence of the vector field $\mathbf{A} = (x^2, y^2, z^2)$:

$$\nabla \cdot \mathbf{A} = \frac{\partial}{\partial x}(x^2) + \frac{\partial}{\partial y}(y^2) + \frac{\partial}{\partial z}(z^2) = 2x + 2y + 2z$$

These examples reinforce the mathematical concepts necessary for physics applications.

3.1 Applications:

3.1.1 Calculate the perimeter of a circle C with radius R (simple integral).

Solution:

We have $dl = R\,d\theta$, hence:

$$C = \int_0^{2\pi} R\,d\theta = 2\pi R.$$

3.1.2 Calculate the area of a disk D with radius R (double surface integral).

We use the differential surface element $dS = dp\,p\,d\theta$, hence:

Solution:

$$D = \iint_S dp\,d\theta = \int_0^R \int_0^{2\pi} \rho\,d\rho\,d\theta.$$

Evaluating the integral:

$$D = \int_0^{2\pi} d\theta \int_0^R \rho\,d\rho = 2\pi \times \frac{R^2}{2} = \pi R^2.$$

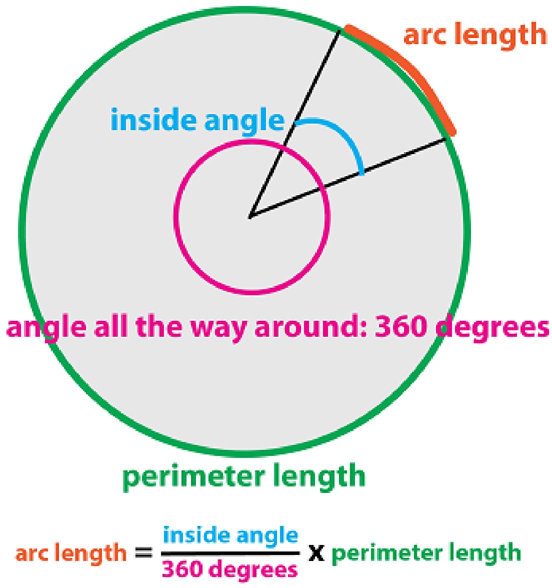

Figure 7: Perimeter of a circle

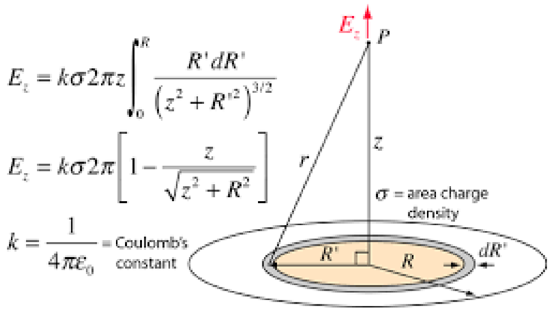

Figure 8: Area of a disk

3.1.3 Calculate the volume of a cylinder V with radius R and height H (triple volume integral).

We use the differential volume element $dV = dp\, p\, d\theta\, dz$, hence:

Solution:

$$V = \iiint_V dp\, d\theta\, dz = \int_0^R \rho\, d\rho \int_0^{2\pi} d\theta \int_0^H dz.$$

Evaluating the integral:

$$V = \int_0^H dz \int_0^{2\pi} d\theta \int_0^R \rho\, d\rho.$$

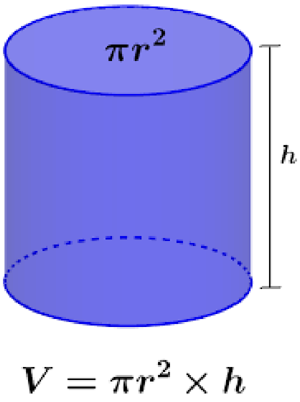

Figure 9: Volume of a cylinder

$$V = H \times 2\pi \times \frac{R^2}{2} = \pi R^2 H.$$

3.1.4 Calculate the surface area of a hemisphere D with radius R (excluding the horizontal disk) (double surface integral).

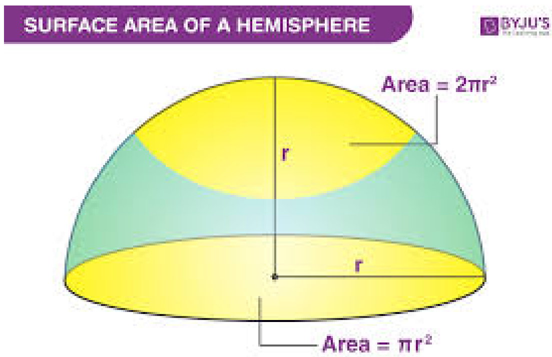

Figure 10: Surface area of a hemisphere

We use the differential surface element $dS = R^2 \sin\theta \, d\theta \, d\phi$, hence:
Solution:

$$D = \iint_S R^2 \sin\theta \, d\theta \, d\phi.$$

Evaluating the integral:

$$D = R^2 \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\phi.$$

$$D = R^2(-\cos\theta\Big|_0^\pi) \times (2\pi) = R^2(1+1) \times 2\pi = 2\pi R^2.$$

3.1.5 Calculate the volume of a sphere V with radius R (triple volume integral).

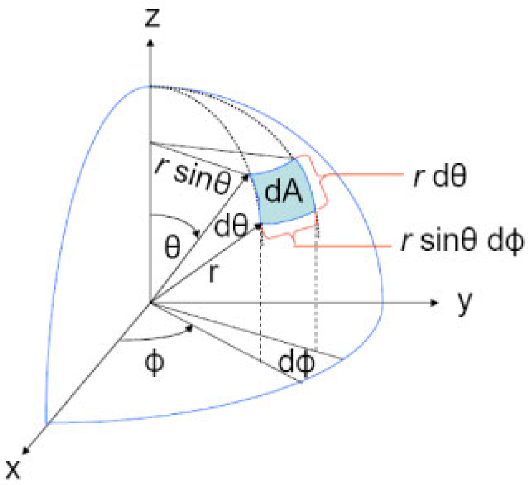

Figure 11: Volume of a sphere

We use the differential volume element $dV = r^2 \sin\theta \, dr \, d\theta \, d\phi$, hence:
Solution:

$$V = \iiint_V r^2 \sin\theta \, dr \, d\theta \, d\phi.$$

Evaluating the integral:

$$V = \int_0^R r^2 \, dr \int_0^\pi \sin\theta \, d\theta \int_0^{2\pi} d\phi.$$

$$V = \left(\frac{R^3}{3}\right) \times (-\cos\theta\Big|_0^\pi) \times 2\pi.$$

$$V = \frac{R^3}{3} \times (1+1) \times 2\pi = \frac{R^3}{3} \times 2 \times 2\pi = \frac{4}{3}\pi R^3.$$

4. Elementary Electric Charges

The electrical properties of matter originate at the atomic level. Matter is composed of atoms, each consisting of a nucleus around which a cloud of electrons orbits. These electrons repel each other but remain positioned around the nucleus. The nucleus consists of protons, which carry positive charges, and neutrons, which are neutral. The set of particles forming the nucleus is called nucleons.

Electrons and protons carry the same electric charge in absolute value, denoted by e. This electric charge, known as the elementary charge, has a value of:

$$e = 1.602 \times 10^{-19}\ \mathrm{C} \tag{4.1}$$

The electric force acting between positively charged protons and negatively charged electrons is responsible for the cohesion of atoms and molecules. The total charge of non-ionized atoms (i.e., those that have neither lost nor gained electrons) is zero.

An electric charge cannot take arbitrary values; it is always an integer multiple of the elementary charge:

$$Q = \pm ne \quad (\mathrm{C}) \tag{4.2}$$

This expresses the fundamental principle of charge quantization.

5. Electrification Experiment

When a glass rod is rubbed with a piece of silk and brought close to small pieces of paper, the paper pieces are attracted to the rod, indicating that electrons have been removed from the rod.

5.0.1 First Experiment

A small ball made of elderberry wood or polystyrene is suspended by a thread. A glass or amber rod, previously rubbed, is brought near the ball. Each rod first attracts and then repels the ball after contact (Figure 2.1a). However, if both rods are brought close to the ball simultaneously, nothing happens (Figure 2.1b).

5.0.2 Second Experiment

If two balls are electrified by contact with a rubbed glass rod, they repel each other. However, if each ball has touched different rubbed rods made of different materials, they attract each other.

These experiments demonstrate the existence of two states of electrification, corresponding to two types of electric charges: positive and negative. We recall the fundamental rule:

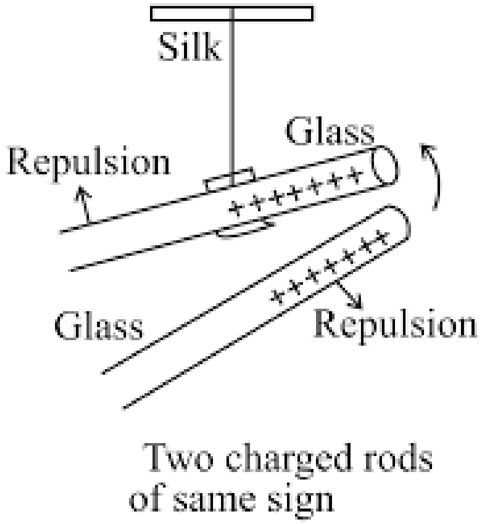

Figure 12: Electrification experiment

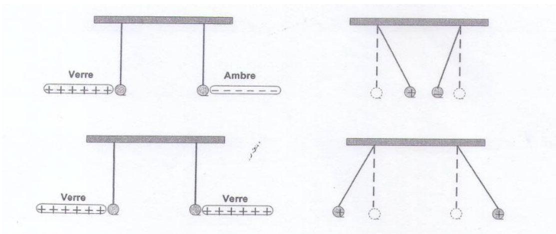

Figure 13: Electrification experience

Two bodies with the same type of charge repel each other, while bodies with opposite charges attract each other.

6. Coulomb's Law

Consider two point charges q_1 and q_2 placed in a vacuum. The first exerts a force proportional to q_1 on the second, and vice versa. The force between the two charges, known as electrostatic force, is proportional to the product of their charges:

$$\mathbf{F_e} = K\frac{q_1 q_2}{r^2}\mathbf{U_{12}} \tag{6.3}$$

where r is the distance between the two charges, and K is given by:

$$K = \frac{1}{4\pi\epsilon_0}, \quad \text{with} \quad \epsilon_0 = 8.854 \times 10^{-12} \text{ F/m} \tag{6.4}$$

Application: Calculate the force exerted by charge $q_1 = 3 \times 10^{-3}$ C on charge $q_2 = -5 \times 10^{-4}$ C separated by a distance of 20 mm.

Solution:

$$F = K\frac{q_1 q_2}{r^2} = 9 \times 10^9 \times \frac{(3 \times 10^{-3})(-5 \times 10^{-4})}{(20 \times 10^{-3})^2} \tag{6.5}$$

$$F = 33.75 \times 10^6 \text{ N} \tag{6.6}$$

7. Superposition Principle

Consider a charge q at point M in the presence of other charges q_i located at points M_i. The force $\mathbf{F}$ acting on charge q is:

$$\mathbf{F} = \sum_i K\frac{q q_i}{r_i^2}\mathbf{U_{iM}} \tag{7.7}$$

Application: Compute the resultant force acting on q_3 due to q_1 and q_2.

8. Electrostatic Field

An electric field exists at a point in space if a test charge q_0 at that point experiences an electrostatic force $\mathbf{F_e}$ such that:

$$\mathbf{E} = \frac{\mathbf{F_e}}{q_0} \tag{8.8}$$

8.1 Electric Field of a Point Charge

A charge Q at point O creates an electric field at any point M given by:

$$\mathbf{E}(M) = \frac{KQ}{r^2}\mathbf{U_{OM}} \tag{8.9}$$

9. Electrostatic Potential

9.1 Electric Potential

The work required to move a charge q_0 from point A to point B in an electric field is:

$$W_{AB} = q_0 \int_A^B \mathbf{E} \cdot d\mathbf{l} \tag{9.10}$$

The electric potential difference is defined as:

$$U_{AB} = V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{l} \tag{9.11}$$

9.2 Potential of a Point Charge

For a charge Q at point O, the electric potential at a distance r is:

$$V = K\frac{Q}{r} \tag{9.12}$$

assuming $V = 0$ at infinity.

10. Electrostatic Potential of continuous charge distribution

The potential at a distance r from a charge q is given by:

$$V(r) = \frac{Kq}{r} \tag{10.13}$$

The potential remains constant on spheres of radius r centered around the charge q, which are called equipotential surfaces.

10.1 Potential Created by Multiple Distinct Point Charges

We start from the relationship between the electric field $\mathbf{E}$ and the potential V, more precisely from the differential relation:

$$dV = \mathbf{E}(M) \cdot \overrightarrow{dl}$$

For a set of charges q_i, concentrated at point M, and using the superposition theorem:

$$dV = -\mathbf{E}(M) \cdot \overrightarrow{dl} = -\sum_{i=1}^{N} [\mathbf{E}_i(M)] \cdot \overrightarrow{dl} = \sum_{i=1}^{N} [-\mathbf{E}_i(M)] \cdot \overrightarrow{dl} = \sum_{i=1}^{N} dV_i$$

The sum of a set of differentials being the differential of the sum:

$$dV = \sum_{i=1}^{N} dV_i = d\left(\sum_{i=1}^{N} V_i\right)$$

$$V(M) = \sum_{i=1}^{N} V_i = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{N} \frac{q_i}{r_i} \tag{10.14}$$

Where r_i is the distance between q_i and point M. The charge q_i can be positive or negative, which is why it must be taken with its sign.

Proof. Using the relationship between the electric field $\mathbf{E}$ and potential V, we obtain:

$$V(M) = \sum_i \frac{Kq_i}{r_i} \tag{10.15}$$

where r_i is the distance between charge q_i and point M. The charges q_i can be positive or negative.

□

10.2 Potential Due to a Continuous Charge Distribution

For a continuous charge distribution, integration is used:

$$V(M) = K \int \frac{dq}{r} \tag{10.16}$$

10.2.1 Volume Distribution

$$V(M) = K \iiint \frac{\rho dV}{r} \tag{10.17}$$

where ρ is the volume charge density.

10.2.2 Surface Distribution

$$V(M) = K \iint \frac{\sigma dS}{r} \tag{10.18}$$

where σ is the surface charge density.

10.2.3 Linear Distribution

$$V(M) = K \int \frac{\lambda dl}{r} \tag{10.19}$$

where λ is the linear charge density.

Here is the combined and corrected translation of the text from the images into English, rewritten in LaTeX:

—

c) If the distribution is linear:

$$V(M) = \int_C \frac{\lambda\, dl}{4\pi\epsilon_0 r} \quad (II-26)$$

where λ is the linear charge density.

10.3 Applications:

10.3.1 Field and Potential Created by a Ring:

A ring with center O and radius R carries a charge q uniformly distributed with a linear charge density $\lambda > 0$.

1. Calculate the potential created at point M on the axis Oy located at a distance y from O. 2. Deduce the electric field vector at point M.

10.3.2 Solution:

For the given point M, the quantities r, y, and R are constant. Starting from Figure I.8 and setting $K = \frac{1}{4\pi\epsilon_0}$, we can write:

$$dV = K\frac{dq}{r}$$

Integrating over the entire charge distribution:

$$\int dV = \frac{K}{r}\int dq \implies V = \frac{Kq}{r} + C_\infty$$

From the figure, we can see that:

$$r = \sqrt{R^2 + y^2}$$

After substituting K and $q = \lambda \cdot 2\pi R$, we arrive at the expression:

$$V = \frac{\lambda}{2\epsilon_0} \cdot \frac{R}{\sqrt{R^2 + y^2}} + C_\infty$$

Now, to determine the magnitude of the electric field E, we differentiate the expression for V with respect to y, using the relation:

$$\vec{E} = -\frac{dV}{dy} \implies \vec{E} = \frac{\lambda R}{2\epsilon_0} \cdot \frac{y}{(R^2 + y^2)^{3/2}}\vec{u}$$

10.3.3 Field and Potential Created by a Disk:

Consider a disk with center O and radius R, uniformly charged on its surface. The surface charge density is σ ($\sigma > 0$) Figure 14.

1. Calculate the electric field and the potential created by this distribution at a point M on the axis (Oz).

To do this, we decompose the disk into rings of radius ρ and width $d\rho$. Let P be a point on the ring and P' the symmetric point of P with respect to O.

First, let's examine the symmetry of the problem: the distribution has a revolution symmetry around the axis OZ. Any plane containing the axis OZ is a plane of even symmetry for the distribution. Therefore, the electric field $\vec{E}$ at a point M on the axis OZ is directed along $\vec{k}$:

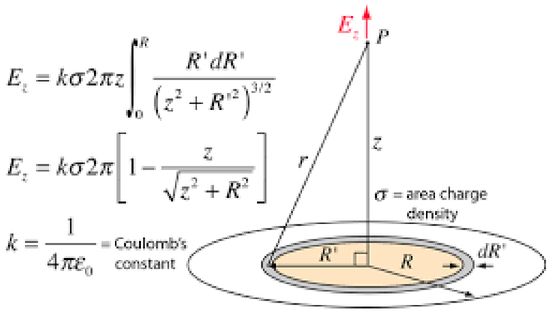

Figure 14: Potential Created by a Disk

$$\vec{E}(M) = E(0,0,Z) = E(Z)\vec{k}$$

A charge element $dq = \sigma\, ds$, centered at P (Figure II-9), creates at a point M on the axis of the disk an elementary field $d\vec{E}$ given by:

$$d\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}\vec{u}$$

where $ds = \rho\, d\rho\, d\theta$ and $r = \sqrt{\rho^2 + Z^2}$.

The charged disk has a revolution symmetry around its axis, for example, the axis ZZ, so the field is directed along this axis. We have:

$$d\vec{E} = \frac{\sigma}{4\pi\epsilon_0}\frac{\rho\, d\rho\, d\theta}{\rho^2 + Z^2}\vec{u}$$

$$d\vec{E}_Z = d\vec{E}\cos\alpha = \frac{\sigma}{4\pi\epsilon_0}\frac{\rho\, d\rho\, d\theta \cos\alpha}{\rho^2 + Z^2}\vec{k}$$

The total electric field at point M is obtained by integrating over the entire disk:

$$E(M) = \frac{\sigma}{4\pi\epsilon_0}\int_0^R \int_0^{2\pi} \frac{\rho\, d\rho\, d\theta}{\rho^2 + Z^2}\cos\alpha$$

Since $\cos\alpha = \frac{Z}{r}$, we have:

$$E(M) = \frac{\sigma}{4\pi\epsilon_0}\int_0^R \int_0^{2\pi} \frac{\rho\, d\rho\, d\theta}{\rho^2 + Z^2}\frac{Z}{\sqrt{\rho^2 + Z^2}}$$

$$E(M) = \frac{\sigma}{2\epsilon_0}\left(\frac{Z}{|Z|} - \frac{Z}{\sqrt{R^2 + Z^2}}\right)\vec{k}$$

When Z is large, the field weakens. However, when $R \gg Z$, and M is very close to the disk, the field becomes:

$$E(M) = \pm\frac{\sigma}{2\epsilon_0}\vec{k}$$

The potential at point M is derived from the field by integration:

$$\vec{E}(M) = -\nabla V(M) = -\frac{dV}{dZ}\vec{k}$$

Thus,

$$V = \frac{\sigma}{2\epsilon_0}\left(Z - \sqrt{R^2 + Z^2}\right)$$

11. Electrostatic Energy

11.1 Energy of a Point Charge in an Electric Field

The work done to move a charge q from A to B in an electric field **E** is:

$$W_{AB} = q(V_A - V_B) \tag{11.20}$$

11.2 Energy of a System of Point Charges

The total electrostatic energy W of a system of point charges is given by:

$$W = \frac{1}{2}\sum_i q_i V_i \tag{11.21}$$

11.3 Energy of a Continuous Charge Distribution

$$W = \frac{1}{2}\iiint \rho V \, dV \tag{11.22}$$

12. Electric Dipole

12.1 Definition

An electric dipole consists of two equal and opposite charges separated by a small distance. The dipole moment **p** is given by:

$$\mathbf{p} = q\mathbf{a} \tag{12.23}$$

12.2 Potential Created by a Dipole

The potential at a point P due to a dipole is:

$$V = \frac{Kp\cos\theta}{r^2} \tag{12.24}$$

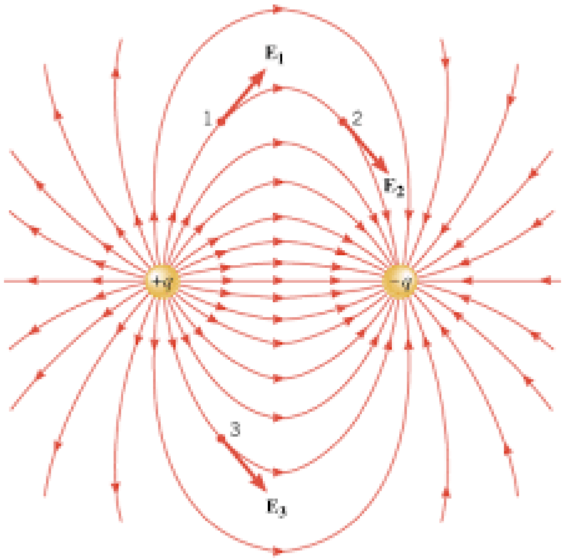

Figure 15: Electric Field of a Dipole

12.3 Electric Field of a Dipole

The radial and angular components of the electric field are:

$$E_r = \frac{Kp(2\cos\theta)}{r^3} \tag{12.25}$$

$$E_\theta = \frac{Kp\sin\theta}{r^3} \tag{12.26}$$

13. Gauss's Theorem

13.0.1 Objectives:

To be able to quickly provide the expression for the electrostatic field created by a source with a high degree of symmetry.

13.1 Prerequisites:

By drawing two networks of lines on any surface, the surface is decomposed into smaller areas bounded by these lines (see the figure).

If the lines are very numerous and evenly distributed, each of these areas has a very small surface. Consider a point P on the surface S. If the number of lines increases indefinitely, the small area around the point P decreases and tends to approach the portion of the tangent plane at P to the surface S. In the limit, its area dS becomes infinitely small and coincides with a portion of the plane. It is called the surface element surrounding the

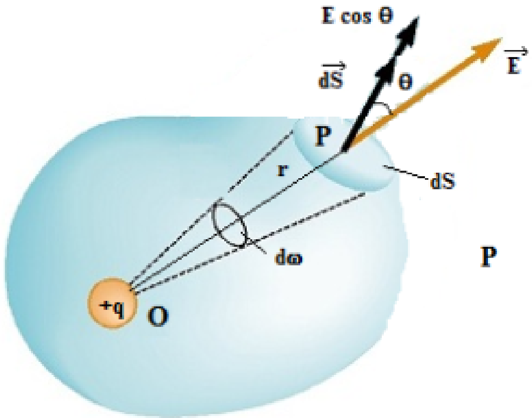

Figure 16: Gauss's Theorem

point P. Thus, any surface S can be considered as the juxtaposition of an infinite number of surface elements dS.

13.2 Surface Element:

Consider a surface element of area dS.

We associate with this element a vector called the "normal" vector $\vec{dS}$, defined as follows:

- Its origin is a point P on the element. - Its direction is normal to the surface. - Its magnitude is equal to the area dS.

The vector $\vec{dS}$ is therefore infinitely small. Its orientation is chosen arbitrarily (outward for closed surfaces). To orient $\vec{dS}$, one can also use the "corkscrew" rule. The contour C bounding the surface is oriented by arbitrarily choosing a positive direction of traversal. The vector $\vec{dS}$ is oriented according to the progression of a corkscrew turning in the direction of C.

13.3 Gauss's Theorem:

Gauss's theorem relies on the concept of the flux of a vector. This new concept is introduced in what follows. However, a good mastery of elementary vector operations, particularly the dot product, is necessary.

14. Concept of Flux

Let $\vec{E}$ denote the electric field vector at point P. Let $d\vec{S}$ be the surface element surrounding this point and the corresponding vector.

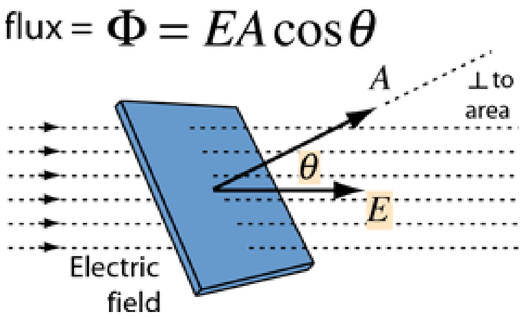

Figure 17: Concept of Flux

14.1 Definition:

By definition, the flux $d\Phi$ of the electric field $\vec{E}$ through the considered surface element $d\vec{S}$ is equal to the dot product:

$$d\Phi = \vec{E} \cdot d\vec{S}$$

This is called the elementary flux to indicate that it is relative to a surface element.

14.2 Sign of the Flux:

The sign of the flux depends on the direction of the vector $d\vec{S}$. Consider, for example, the two opposite vectors $d\vec{S}$ and $-d\vec{S}$, associated with a surface element.

If the vector $d\vec{S}$ makes an angle θ with the electric field $\vec{E}$, the vector $-d\vec{S}$ makes an angle $\pi-\theta$, and since $\cos(\pi-\theta) = -\cos(\theta)$, the dot products $\vec{E}\cdot d\vec{S}$ and $\vec{E}\cdot(-d\vec{S})$ have opposite values.

To calculate the algebraic flux of the electric field $\vec{E}$ through a surface element $d\vec{S}$, it is therefore necessary to choose, in accordance with the concept of positive flux, the direction of the vector $d\vec{S}$ associated with this element.

The flux of an electric field $\mathbf{E}$ through a closed surface S is given by:

$$\Phi = \oint \mathbf{E} \cdot d\mathbf{S} = \frac{Q_{\text{enc}}}{\epsilon_0} \tag{14.27}$$

where Q_{enc} is the total charge enclosed by the surface.

14.3 Flux Calculation

Consider the surface elements composing the surface S. For each of them, the elementary flux $d\Phi$ is calculated. The total flux Φ of the electric field through the surface S is obtained by summing the elementary fluxes. This sum is conventionally denoted by the notation:

$$\Phi = \iint_S \vec{E} \cdot d\vec{S}$$

To perform this calculation, the vectors $d\vec{S}$ associated with the surface elements are all oriented on the same side of the surface S.

14.4 Flux of a Point Charge

Let P be a point belonging to the surface element $d\vec{S}$. The field $\vec{E}$ created at P by the charge q is directed along $\vec{r}$ and oriented from q to P if $q > 0$; its magnitude is:

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

where r is the distance between q and P.

The elementary flux of this electric field through the surface element $d\vec{S}$ surrounding the point P is:

$$d\Phi = \vec{E} \cdot d\vec{S} = E\,dS\,\cos\theta$$

where θ is the angle between $\vec{E}$ and $d\vec{S}$.

However, $d\Omega = \frac{dS\cos\theta}{r^2}$ is the solid angle $d\Omega$ subtended by the contour of $d\vec{S}$ as seen from q (geometrically, it is a cone with vertex at q that is tangent to the surface element $d\vec{S}$).

Gauss's Theorem

Gauss's theorem is stated as follows:

14.5 Theorem:

The flux of the electric field through any closed surface S is equal to $\frac{1}{\epsilon_0}$ times the total algebraic charge contained within the volume bounded by this surface:

$$\Phi = \oint_S \vec{E} \cdot d\vec{S} = \frac{Q_{\text{int}}}{\epsilon_0}$$

14.5.1 Case of Charges Outside a Closed Surface S:

The elements $d\vec{S}_1$ and $d\vec{S}_2$ are seen under the same solid angle $d\Omega$ in absolute value. However, $\vec{E}_1$ and $d\vec{S}_1$ are collinear, while $\vec{E}_2$ and $d\vec{S}_2$ are opposite. Therefore, the fluxes $d\Phi_1$ and $d\Phi_2$ have opposite signs. The elementary fluxes cancel out in pairs, and the total flux of the field $\vec{E}$ created by the charge q outside the closed surface is zero.

14.5.2 Case of Charges Inside a Closed Surface S:

The sum of the elementary fluxes will not be zero because all the surface element vectors are, for example, all oriented outward from the surface. The total flux sent by q through S will be the sum of the elementary fluxes:

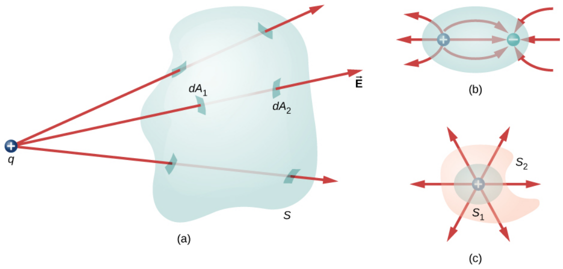

Figure 18: Case of Charges Inside a Closed Surface

$$\Phi = \oint_S \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0}$$

The unit of solid angle is the angle that subtends a unit area on a sphere of unit radius. Since the surface area of a unit sphere is 4π, the solid angle that subtends the entire space from a point is 4π. The sum extends over the entire space, i.e., 4π.

If there are N charges q_i inside S:

$$\Phi = \oint_S \vec{E} \cdot d\vec{S} = \frac{1}{\epsilon_0} \sum_{i=1}^{N} q_i$$

By defining:

$$Q_{\text{int}} = \sum_{i=1}^{N} q_i$$

The flux of $\vec{E}$ through a closed surface is equal to $\frac{1}{\epsilon_0}$ times the sum of the interior charges, regardless of the exterior charges.

14.6 Application of Gauss's Theorem:

The application of Gauss's theorem is very useful in problems that exhibit a high degree of symmetry. Verify this property with the simple example of the field $\vec{E}$ created by a point charge q.

The following two simulations will allow you to apply Gauss's theorem in the case of two uniformly charged structures with axes of symmetry. You can demonstrate the simplicity with which Gauss's theorem allows the calculation of the electrostatic field created by these two charge distributions, which exhibit a high degree of symmetry.

14.7 Methodology

Gauss's theorem is a valuable tool for determining the electric field $\vec{E}$ at any point P when the source charges exhibit high symmetry. The steps for calculating $\vec{E}$ are as follows:

1. Determine the orientation of the field using symmetry considerations.

2. Choose a "Gaussian surface" S (imaginary, with no physical reality): - Passing through the point of interest P. - Most suitable for simplifying the expression of the flux of $\vec{E}$ through it. - Possessing the same symmetry properties as the source. - Not coinciding with a charged material surface.

3. Express the flux Φ through the closed surface S.

4. Determine the total charge Q_{int} enclosed within the volume bounded by S.

5. Apply Gauss's theorem:

$$\Phi = \oint_S \vec{E} \cdot d\vec{S} = \frac{Q_{\text{int}}}{\epsilon_0}$$

If the Gaussian surface is well chosen, the left-hand side of the equation is a simple function of $\vec{E}$ and the distance r. Thus, the expression for the field $\vec{E}$ can be obtained as a function of the distance r and the source charges.

14.7.1 Case of axial symmetry

A source charge distribution has axial symmetry if the charge density at a point is a function only of the distance from an axis.

Cylindrical Charge Cloud with Volume Density $\rho = f(r)$:

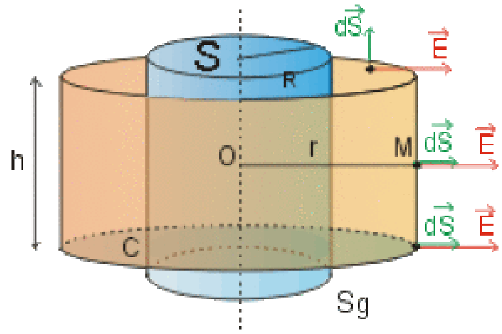

Figure 19: Case of axial symmetry

1. By symmetry, the electric field is radial (far from the edges of the source).

2. The most suitable Gaussian surface is a cylinder aligned with Δ and passing through the point of interest M (which can be inside or outside the source).

Point of Interest Outside the Source:

On the right sections S of the Gaussian cylinder S_g, the vectors $\vec{E}$ and $d\vec{S}$ are orthogonal, so the flux of $\vec{E}$ through S is zero. The flux of $\vec{E}$ through the closed Gaussian surface is reduced to the flux through the lateral surface.

$$\Phi = \iint_{S_g} \vec{E} \cdot d\vec{S} = \iint_{S_{\text{lat}}} \vec{E} \cdot d\vec{S} + \iint_{S} \vec{E} \cdot d\vec{S} = \iint_{S_{\text{lat}}} \vec{E} \cdot d\vec{S}$$

On the lateral surface, $\vec{E}$ and $d\vec{S}$ are collinear, so the flux reduces to:

$$\Phi = \iint_{S_{\text{lat}}} \vec{E} \cdot d\vec{S} = \iint_{S_{\text{lat}}} E \cdot dS$$

E is the same at every point on S_g and can therefore be taken out of the integral:

$$\Phi = \iint_{S_{\text{lat}}} E \cdot dS = E \iint_{S_{\text{lat}}} dS = ES_{\text{lat}}$$

The lateral surface area of the Gaussian surface is equal to $2\pi rh$:

$$\Phi = E \cdot 2\pi rh$$

Now, we only need to evaluate the charge Q_i inside the volume delimited by S_g according to the considered distribution. Gauss's theorem allows us to determine the field E by writing:

$$\Phi = E \cdot 2\pi rh = \frac{Q_i}{\epsilon_0}$$

15. Capacitance

The capacitance C of a conductor is defined as the ratio of the charge Q stored on the conductor to the electric potential V of the conductor:

$$C = \frac{Q}{V}$$

The unit of capacitance is the farad (F), where $1F = 1C/V$ (one farad equals one coulomb per volt).

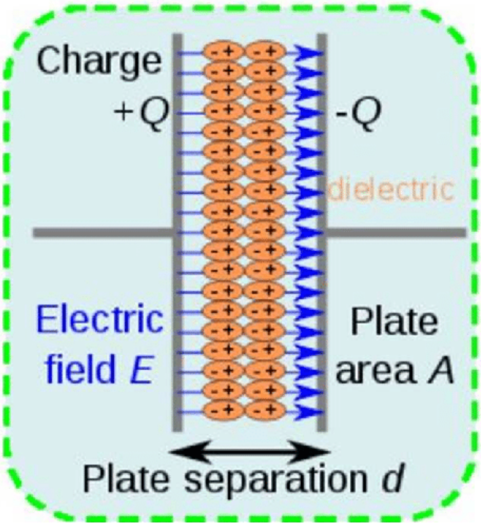

Figure 20: The Capacitance

$$C = \frac{Q}{V} \tag{15.28}$$

For a few common conductor geometries, the capacitance is given by:

1. Isolated Spherical Conductor of Radius R:

   $$C = 4\pi\varepsilon_0 R$$

   where ε_0 is the permittivity of free space.

2. Parallel Plate Capacitor with Plate Area A and Plate Separation d:

   $$C = \frac{\varepsilon A}{d}$$

   where $\varepsilon = \varepsilon_0\varepsilon_r$ is the permittivity of the dielectric medium.

3. Cylindrical Capacitor with Inner Radius R_1, Outer Radius R_2, and Length L:

   $$C = \frac{2\pi\varepsilon L}{\ln(R_2/R_1)}$$

The capacitance C of a conductor is defined as:
For a spherical conductor:

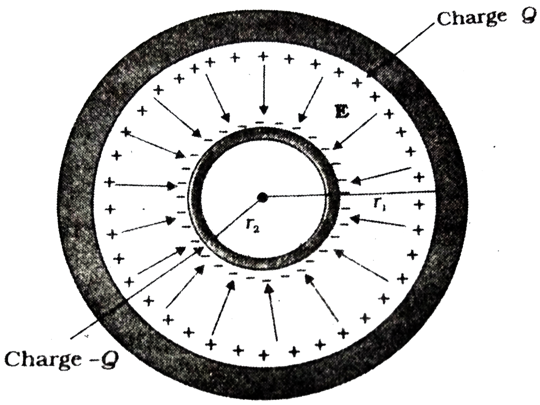

Figure 21: Spherical conductor

$$C = 4\pi\epsilon_0 R \tag{15.29}$$

For a cylindrical capacitor:

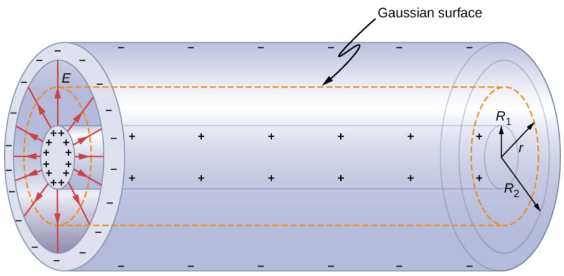

Figure 22: Cylindrical capacitor

$$C = \frac{2\pi\epsilon_0 h}{\ln(R_2/R_1)} \tag{15.30}$$

For a parallel plate capacitor:

$$C = \frac{\epsilon_0 S}{d} \tag{15.31}$$

15.1 Energy Stored in a Capacitor

The energy stored in a capacitor is given by:

$$W = \frac{1}{2}CV^2 \tag{15.32}$$

16. Chapter 3 "Electrokinetics"

Electrokinetics is the study of electric currents, that is, the study of the movement of electric charges in material mediums called conductors. In other words, it is the study of electric circuits and networks.

17. Electric Conductor

In electricity, a conductor is a material that contains electric charge carriers that can move easily. When this conductor is subjected to an electric field, the movement of charge carriers becomes globally ordered, which results in the observation of an electric current.

By extension, a conductor is an electrical or electronic component with low resistance, used to carry current from one point to another.

Among the conductive materials, we can mention metals, electrolytes (or ionic solutions), and plasmas.

Perfect conductors do not exist, so ohmic conductors are used, among which the best are silver, gold, and aluminum.

18. Electric Current

18.1 Definition

Electric current is a collective and organized displacement of charge carriers (electrons or ions). This flow of charges can occur in a vacuum (electron beams in cathode ray tubes...), or in a conductive material (electrons in metals, or ions in electrolytes). An electric current appears in a conductor when a potential difference is established between its terminals.

18.2 Intensity of Electric Current

The intensity of the electric current is a number describing the rate of flow of electric charge across a given surface, notably the cross-section of an electrical wire.

$$I(t) = \frac{dq(t)}{dt} \qquad \text{(III-1)}$$

Where:

- I is the intensity of the current.
- q is the electric charge.
- t is the time.

In the International System of Units, the intensity of the current is measured in amperes, a base unit whose normalized symbol is A. An ampere corresponds to a charge flow of one coulomb per second.

The intensity is measured using an ammeter which must be connected in series in the circuit.

18.3 Current Density

Current density is a vector describing the electric current at a local scale. Its direction indicates that of the displacement of charge carriers (but its sense can be opposite for negative charge carriers) and its norm corresponds to the intensity of the current per unit area. It is related to the electric current by:

$$I = \iint_S \vec{j} \cdot d\vec{S} \qquad \text{(III-2)}$$

Where:

- I is the intensity of the current.
- S is a surface.
- $\vec{j}$ is the current density.
- $d\vec{S}$ is the elementary surface vector.

In the International System of Units, the current density is measured in amperes per square meter ($\text{A}\cdot\text{m}^{-2}$).

19. Ohm's Law

The potential difference or voltage U (in volts) across the terminals of a resistor R (in ohms) is proportional to the intensity of the electric current I (in amperes) that flows through it (Figure III-1).

$$U = R \cdot I \qquad \text{(III-3)}$$

The resistance is the opposition exerted by a body to the passage of an electric current. The resistance is measured in ohms.

20. Joule Effect

The Joule effect is a heat production effect that occurs when an electric current passes through a conductor exhibiting resistance. It manifests as an increase in the thermal energy of the conductor and its temperature. In effect, this type of conductor transforms electrical energy into heat energy (energy dissipated as heat). The power dissipated by this conductor is equal to:

$$P = RI^2 \qquad \text{(III-4)}$$

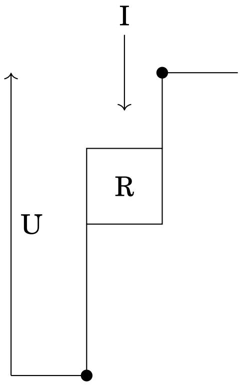

Figure 23: Resistor traversed by a current I under a voltage U

The unit of power is the watt (W).

R: the resistance of the conductor.

I: the intensity of the current that flows through the conductor.

From the definition of energy, we deduce that the energy consumed by a resistance during time t is equal to:

$$E = U.I.t = R.I^2 t = \frac{U^2}{R} t \qquad \text{(III-5)}$$

The unit of energy is the joule (J).

21. Grouping of Resistors

We distinguish two cases for the grouping of resistors:

21.1 Series Grouping

All resistors R_i are traversed by the same electric current I, and each of them has only one common end with another resistor (Figure III-2). The voltage $U_{AB} = U$ is equal to the sum of the voltages across the resistors.

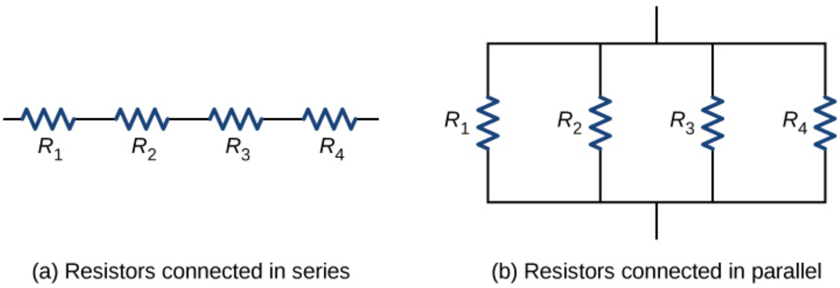

Figure 24: Series Grouping of Resistors

$$U = U_1 + U_2 + U_3 + \cdots + U_n = R.I \qquad \text{(III-6)}$$

$$U = R_1.I + R_2.I + R_3.I + \cdots + R_n.I = R.I \qquad \text{(III-7)}$$

Thus, we obtain the equivalent resistance of all the resistors grouped in series.

$$R = \sum_{i=1}^{n} R_i \qquad \text{(III-8)}$$

21.2 Parallel Grouping

This grouping is characterized by the fact that all the resistors have their terminals connected two by two (Figure III-3). The voltage is the same across the terminals of any resistor R_i .

The electric current that supplies the portion of the circuit is divided between the resistors, such that:

$$I = I_1 + I_2 + I_3 + \cdots + I_n \qquad \text{(III-9)}$$

$$\frac{U}{R} = \frac{U}{R_1} + \frac{U}{R_2} + \frac{U}{R_3} + \cdots + \frac{U}{R_n} \longrightarrow \frac{1}{R} = \left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}\right] \cdot U \qquad \text{(III-10)}$$

Thus, we obtain the equivalent resistance, in this case, which is always smaller than that of the smallest of the resistors connected in parallel.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n} \longrightarrow \frac{1}{R} = \sum_{i=1}^{n} \frac{1}{R_i} \qquad \text{(III-11)}$$

22. Electric Circuits

An electric circuit is a set of conductors (wires) and electrical (sockets, switches, ...) or electronic components (household appliances, ...) through which an electric current flows.

The electrokinetic study of an electric circuit consists of determining, at each point, the intensity of the current and the voltage.

23. Elements of an Electric Circuit

The electric circuit is composed essentially of the following elements (Figure III-4):

1. The node: is a point where two or more conductors meet.
2. The branch: is a portion of the circuit that is located between two nodes.
3. The mesh: is any closed loop, formed by a sequence of branches.

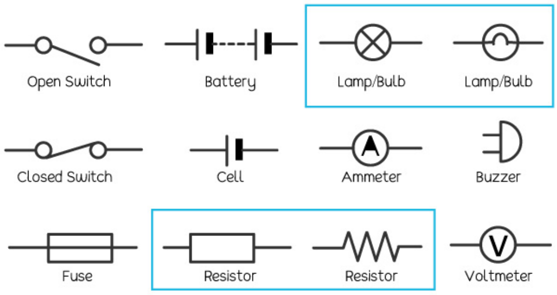

Figure 25: A general electric circuit

24. Generators

To obtain a continuous electric current in a closed circuit, it is essential to supply the circuit with energy. This is done by devices called generators. We can say that they are sources of electromotive forces to transport the charges.

Two types of generators are distinguished:

24.1 Generators or voltage sources

The voltage source, or voltage generator, is a dipole characterized by a constant voltage between its terminals, whatever the variable intensity it delivers. In what follows, we will be particularly interested in continuous voltage generators. This type of generator is characterized by an electromotive force e, and a low internal resistance r (Figure III-5).

It is possible to replace a voltage generator, whose characteristics are (e,r), with an ideal source, of electromotive force e, connected in series with an ohmic conductor, of resistance r as indicated in Figure III-5.

The electromotive force of a voltage generator is equal to the potential difference between its terminals when it does not deliver any current:

$$I = 0 \longrightarrow e = U_{AB}$$

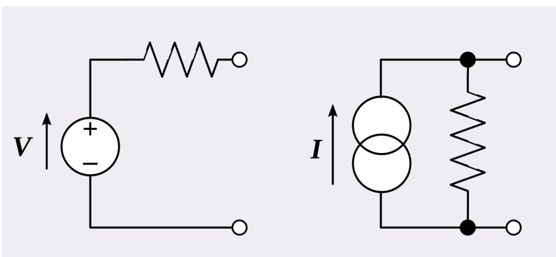

Figure 26: Representation of the voltage generator and current sources

24.2 Generators or current sources

The current source, or current generator, is a dipole characterized by the delivery of a constant current, whatever the variable potential difference between its terminals. In what follows, we will be essentially interested in continuous current generators. This type of generator is represented by the scheme of Figure III-6.

We can replace a current generator with an ideal current source, which delivers a constant current, and connected in parallel with an ohmic conductor, of resistance, as indicated in Figure III-6.

25. Kirchhoff's Laws

25.1 First Law (Node Law)

At a node in a circuit, the sum of the currents entering is equal to the sum of the currents leaving:

$$\sum I_e = \sum I_s \qquad \text{(III-12)}$$

This means that the charges do not accumulate, they flow at a node in the circuit, they obey the law of conservation of energy.

25.2 Second Law (Mesh Law)

In a mesh of an electric circuit, the algebraic sum of the products of resistance by the intensity of the current ($\sum_{k=1}^{n} R_k I_k$) is equal to the algebraic sum of the electromotive forces ($\sum_{k=1}^{n} e_k$).

$$\sum_{k=1}^{n} e_k = \sum_{k=1}^{n} R_k I_k \qquad \text{(III-13)}$$

When applying this law, one must choose a positive sense around the mesh: all electromotive forces and currents that have the same sense will be counted positively, those that are of opposite sense will be counted negatively. We consider the sense of e positive when we enter, after the positive pole, by the negative pole and leave by the positive pole (which results in an increase of potential), and the opposite in the contrary case.

26. Applications

Consider, for example, the following circuit (Figure III-7):

We are looking for the values of the three currents I_1, I_2, and I_3, using Kirchhoff's laws. The conservation of current (first law of Kirchhoff) implies that $I_1 = I_2 + I_3$.

We then apply the potential conservation on the mesh of the circuit (second law of Kirchhoff) to the meshes ABEFA and BCDEB.

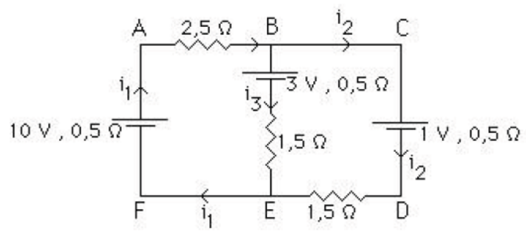

Figure 27: Electric circuit

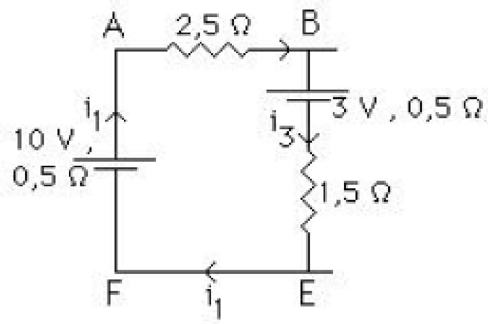

Figure 28: Mesh ABEFA

For the Mesh ABEFA:
Starting from point A where the potential V_A exists:

1. From A to B, a resistance of $2.5\,\Omega$ is traversed in the direction of the current i_1, which corresponds to a potential drop of $2.5\,i_1$ Volts.

2. From B to E,

   (a) We traverse a battery of $E = 3$ Volts from the highest potential to the lowest potential, which results in a potential drop of 3 Volts.

   (b) The internal resistance of the battery is traversed in the direction of the current i_3, which corresponds to a potential drop of $0.5\,i_3$ Volts.

   (c) Finally, the resistance of $1.5\,\Omega$ is also traversed in the direction of the current i_3, which leads to a potential drop of $1.5\,i_3$ Volts.

3. From E to F, there is no potential variation.

4. From F to A,

(a) The battery of $E = 10$ Volts is traversed from the lowest potential to the highest potential, raising the latter by 10 Volts.

(b) On the other hand, the traversal of the internal resistance of this battery in the direction of the current i_1 causes a potential drop of $0.5\,i_1$ Volts.

In total, we have done after one complete turn:

$$V_A - 2.5i_1 - 3 - 0.5i_3 - 1.5i_3 + 10 - 0.5i_1 = V_A$$

and therefore

$$U_{AA} = V_A - V_A = -2.5i_1 - 3 - 0.5i_3 - 1.5i_3 + 10 - 0.5i_1 = 0$$

For the mesh BCDEB:

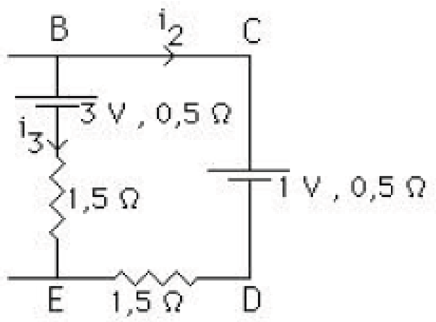

Figure 29: mesh BCDEB

Starting from point B where the potential V_B exists:

1. From B to D,

   (a) There is a loss of 1 Volt across the battery.

   (b) And a loss of $0.5i_2$ Volt across the internal resistance of this battery.

2. From D to E, there is a loss of $1.5i_2$ Volt.

3. From E to B,

   (a) There is a gain of $1.5i_3$ Volt.

   (b) A gain of $0.5i_3$ Volt across the internal resistance of the 3 Volt battery.

   (c) And a gain of 3 Volts due to the electromotive force of the battery.

In total, we have done after one complete turn:

$$V_B - 1 - 0.5i_2 - 1.5i_2 + 1.5i_3 + 0.5i_3 + 3 = V_B$$

and therefore

$$U_{BB} = V_B - V_B = -1 - 0.5i_2 - 1.5i_2 + 1.5i_3 + 0.5i_3 + 3 = 0$$

We obtain the following system of equations:

$$\begin{aligned} i_1 &= i_2 + i_3 \\ -2.5i_1 - 3 - 0.5i_3 - 1.5i_3 + 10 - 0.5i_1 &= 0 \\ -1 - 0.5i_2 - 1.5i_2 + 1.5i_3 + 0.5i_3 + 3 &= 0 \end{aligned}$$

Or even:

$$\begin{aligned} i_1 &= i_2 + i_3 \\ -3i_1 - 2i_3 + 7 &= 0 \\ -2i_2 + 2i_3 + 2 &= 0 \end{aligned}$$

This is a system of 3 equations with 3 unknowns: i_1, i_2, and i_3.
The resolution of this system gives:

$$\begin{aligned} i_1 &= 2\,\mathrm{A} \\ i_2 &= 1.5\,\mathrm{A} \\ i_3 &= 0.5\,\mathrm{A} \end{aligned}$$

The positive values of these currents indicate that the senses chosen at the beginning are correct.

27. Electromagnetism

Electromagnetism is a branch of physics that studies the interactions between electric currents and magnetic fields.

27.1 Introduction

In nature, there exist objects called permanent magnets that exert forces on each other as well as on iron, nickel, cobalt, and various alloys.

A magnet is a source that modifies the properties of the space around it, where this modification is attributed to the presence of a field called the magnetic field (Figure 1).

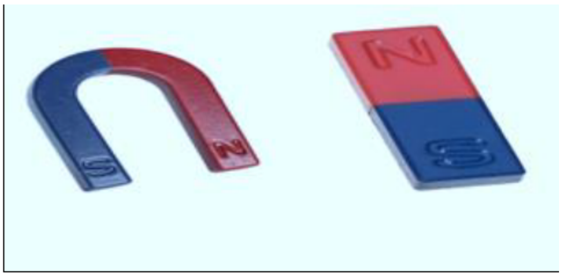

Figure 30: Magnet

27.2 Magnetic Field Created by a Magnet

A magnetic field $\vec{B}$ is a force field resulting from the movement of charges (an electric current). It represents a region of space subject to forces originating from either a magnet or an electric current.

The unit of magnetic field strength in the International System of Units (SI) is the **Tesla** (T), though it can also be measured in **Gauss** (G), where:

$$1\,\mathrm{G} = 10^{-4}\,\mathrm{T}$$

- $\vec{B}$: Magnetic field vector (T)
- Typical values:
  - Earth's magnetic field: $\approx 0.5\,\mathrm{G}$ ($5 \times 10^{-5}\,\mathrm{T}$)
  - Refrigerator magnet: $\approx 100\,\mathrm{G}$ ($0.01\,\mathrm{T}$)
  - MRI scanner: $\approx 1.5 - 3\,\mathrm{T}$

27.2.1 Earth's Magnetic Field

The Earth's magnetic field is generated by convective motions in the liquid metallic outer core (composed primarily of iron and nickel). This geodynamo effect creates a dipole field that:

- Aligns compass needles to magnetic north (deviation from geographic north is called **magnetic declination**)
- Provides crucial protection against solar wind (Van Allen radiation belts)

$$\vec{B}_{\text{earth}} \approx 0.25 - 0.65\,\text{G} \quad (25 - 65\,\mu\text{T})$$

Key characteristics:

- Inclination: Angle between field lines and surface (varies with latitude)
- Secular variation: Drifts ~0.1° annually
- Polarity reversals occur geologically (~300 kyr intervals)

Biospheric Significance of Earth's Magnetic Field The Earth's magnetic field plays a crucial role in enabling life by:

- Deflecting 99% of solar wind particles (typically $1 - 10$ keV protons/electrons)
- Creating the magnetopause boundary at $\sim 10R_E$ (Earth radii)
- Channeling charged particles toward polar regions, producing:
  - Aurorae borealis/australis (visible at $90 - 400$ km altitudes)
  - Ionospheric currents (~ 1 MA in auroral electrojets)

$$F_{\text{Lorentz}} = q(\vec{v} \times \vec{B}) \quad \text{(Deflection mechanism)}$$

Figure 31: Aurora

27.2.2 Magnetic Poles

Experimental observations show that when a small magnetized needle is placed freely at point M:

- Aligns along Earth's magnetic meridian (declination angle δ)
- Exhibits harmonic oscillations when disturbed:

$$\tau = -k\theta \quad \text{(Restoring torque)}$$

- Returns to equilibrium orientation with damping time $t_d \propto \eta/B$ (η: viscosity)

Theorem Magneto-static Equilibrium For a freely suspended magnet with dipole moment $\vec{m}$:

$$\vec{\tau} = \vec{m} \times \vec{B}_{\text{earth}} = 0 \implies \vec{m} \parallel \vec{B}_{\text{earth}}$$

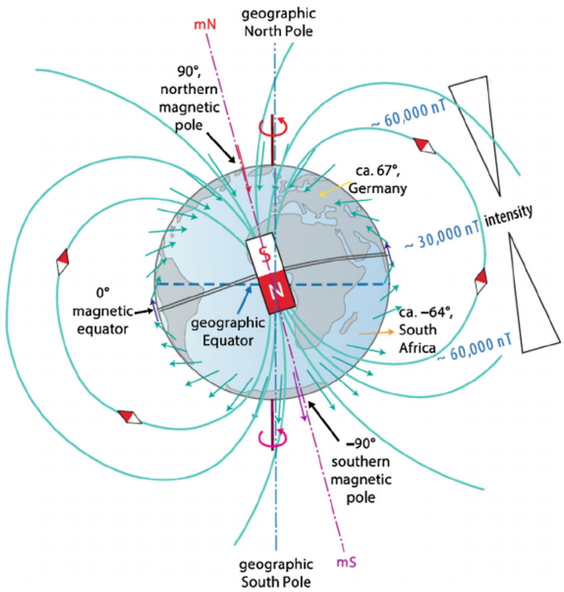

Figure 32: Alignment of magnetic dipoles with Earth's field showing: (a) True North, (b) Magnetic North, and (c) Declination

Magnetic Pole Nomenclature The needle's orientation defines two fundamental poles:

- **North Magnetic Pole** (N): Points toward Earth's *geographic north*
  - Actually corresponds to Earth's *south magnetic pole* (attraction between unlike poles)
  - Current position: 86.5°N, 164°E (2025 data)

- **South Magnetic Pole** (S): Points toward Earth's *geographic south*
  - Magnetic dipole moment: $\vec{m}$ directed from S to N
  - Field lines: $\vec{B}$ enters at S, exits at N

$$\vec{F} = \nabla(\vec{m} \cdot \vec{B}) \quad \text{(Force on magnetic dipole)}$$

27.3 Magnetic Field Characterization

At point M, the magnetic field vector $\vec{B}$ exhibits (Figure 3):

- **Direction**: Aligned with the magnetized needle's axis $x'x$
- **Orientation**: From south (s) to north (n) pole of the needle
- **Magnitude**: Quantified in tesla (T) via teslameter

$$B = \|\vec{B}\| = \frac{\mu_0}{4\pi}\left(\frac{3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}}{r^3}\right) \quad \text{(Dipole field)}$$

27.3.1 Magnetic Field Lines

Experimental Demonstration:

1. Place a bar magnet beneath paper
2. Sprinkle iron filings (ferromagnetic Fe_3O_4 particles)
3. Observe self-assembly into flux patterns

- High density at poles ($\nabla \cdot \vec{B} = 0$)
- Tangent condition: $\vec{B} \parallel d\vec{l}$

Key Properties:

- Lines never intersect (uniqueness theorem)
- Density $\propto$ field strength ($\Phi_B = \int \vec{B} \cdot d\vec{A}$)
- Form closed loops (solenoidal condition)

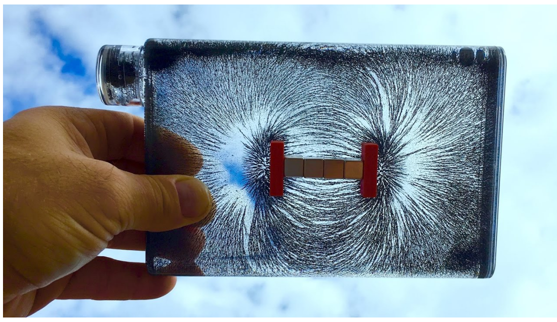

Figure 33: Visualization of magnetic flux lines showing:

27.4 Properties of Magnetic Field Lines

Tangency Condition: The magnetic field vector $\vec{B}$ is tangent to the field lines at every point:

$$\vec{B} \parallel d\vec{\ell} \quad \text{where } d\vec{\ell} \text{ is the line element}$$

Pole Behavior: Field lines exhibit specific geometry near poles:

- Emerge perpendicularly from the North magnetic pole surface
- Curve through space forming continuous loops
- Re-enter perpendicularly at the South magnetic pole surface

$$\oint_S \vec{B} \cdot d\vec{A} = 0 \quad \text{(Gauss's Law for Magnetism)}$$

Needle Alignment: The field direction matches a compass needle's orientation:

- Direction: Along the South-North axis of the magnetized needle
- Sense: From South to North pole of the test needle

$$\vec{\tau} = \vec{m} \times \vec{B} = 0 \quad \text{(Equilibrium condition)}$$

- Orthogonal emergence/entry at poles
- Curvature following right-hand rule

28. Magnetic Field Created by Electric Currents

28.1 Magnetic Field Around a Straight Conductor

Experimental Demonstration:

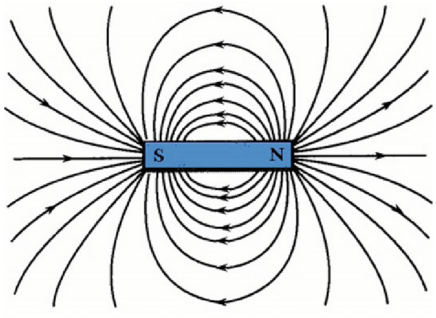

Figure 34: Characteristic dipole field line pattern showing

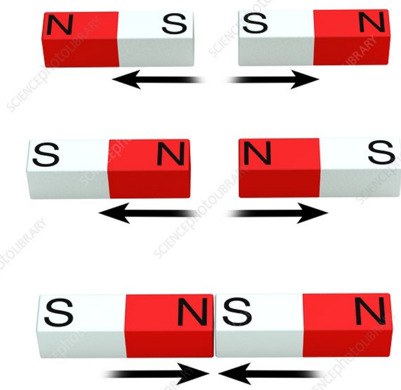

Figure 35: Interaction between the poles of a magnet

- Setup: Iron filings on plexiglass plate perpendicular to current-carrying wire
- Control case: Zero current $\Rightarrow$ random filing distribution
- Active case: Current $I \neq 0 \Rightarrow$ concentric circular patterns emerge

- Right-hand rule orientation (thumb $\parallel I$, fingers $\parallel \vec{B}$)
- Field strength $\propto 1/r$ (cylindrical symmetry)

28.2 Quantitative Description

The magnetic field $\vec{B}$ at distance r from an infinite straight conductor:

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\phi} \quad \text{(Biot-Savart Law)} \tag{28.33}$$

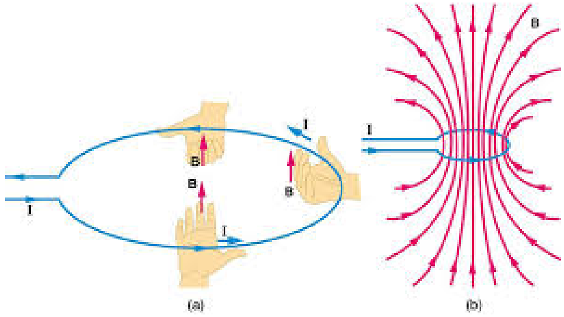

Figure 36: Circular magnetic field lines around current-carrying conductor showing

$\mu_0 = 4\pi \times 10^{-7}$ N/A^2	Permeability of free space
I	Current magnitude
$\hat{\phi}$	Azimuthal unit vector

Key Observations:

- Field forms closed loops (solenoidal property $\nabla \cdot \vec{B} = 0$)
- No monopole sources (divergence-free)
- Circular symmetry confirms Ampère's law: $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{enc}}$

28.3 Properties of the Magnetic Field Near a Straight Wire

Using compasses to analyze the magnetic field $\vec{B}$ at point M near a current-carrying conductor (Figure 8), we establish:

- **Planar Containment**: The field lies in planes perpendicular to the wire

$$\vec{B} \cdot d\vec{\ell} = 0 \quad \text{(Orthogonality condition)}$$

- **Current-Dependent Orientation**: Field direction follows the right-hand rule:
  - Thumb points with conventional current ($+I$)
  - Fingers curl in $\vec{B}$ direction
- **Field Strength Proportionality**:

$$B = \frac{\mu_0 I}{2\pi r} \quad \text{(Exact form of } k\text{)}$$

  where $k = \frac{\mu_0}{2\pi r}$ depends on:
  - Distance r from wire (inverse proportionality)
  - Medium's permeability $\mu = \mu_0 \mu_r$

Key Implications:

- Demonstrates axial symmetry of magnetic fields
- Verifies Ampère's circuital law in differential form: $\nabla \times \vec{B} = \mu_0 \vec{J}$
- Basis for current measurement devices (e.g., Hall probes)

28.4 Right-Hand Rule Determination

The direction of the magnetic field can be determined using the **right-hand rule**:
Mathematically, this corresponds to the cross product in the Biot-Savart Law:

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{\ell} \times \hat{r}}{r^2}$$

Configuration	Field Orientation
Current upward	Counterclockwise $\vec{B}$
Current downward	Clockwise $\vec{B}$

Advanced Applications:

- Solenoid winding direction determination
- Torque direction in electric motors (Lorentz force $\vec{F} = I\vec{L} \times \vec{B}$)
- Hall effect sensor polarity

29. Lorentz Magnetic Force

29.1 Definition

A charge q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ experiences the Lorentz force:

$$\vec{F}_m = q\vec{v} \times \vec{B}$$

29.2 Characteristics of the Lorentz Force

- **Direction**: Perpendicular to both $\vec{v}$ and $\vec{B}$

$$\vec{F}_m \perp \text{span}(\vec{v}, \vec{B})$$

- **Orientation**: Determined by the right-hand rule (Figure 9):
  1. Thumb: Direction of $q\vec{v}$ (positive q) or $-\vec{v}$ (negative q)
  2. Index finger: Direction of $\vec{B}$
  3. Middle finger: Direction of $\vec{F}_m$

- **Magnitude**:

$$F_m = |q|vB\sin\theta$$

q	Charge (C)
v	Velocity (m/s)
B	Magnetic field strength (T)
θ	Angle between $\vec{v}$ and $\vec{B}$

29.3 Special Cases

- Parallel motion ($\theta = 0°$): $F_m = 0$
- Perpendicular motion ($\theta = 90°$): $F_m = qvB$
- Helical motion: When $\vec{v}$ has both parallel and perpendicular components

29.4 Applications

- Particle accelerators (cyclotron motion)
- Mass spectrometry
- Hall effect sensors

29.5 Special Cases of Lorentz Force

[Extrema Conditions] The magnetic Lorentz force exhibits critical behaviors when:

- **Maximum Force** ($F_m^{\max}$): Occurs when $\vec{v} \perp \vec{B}$

$$F_m^{\max} = qvB \quad (\theta = 90°)$$

- **Null Force**: Arises in two scenarios:
  1. Parallel velocity and field ($\vec{v} \parallel \vec{B}$):

  $$F_m = 0 \quad (\theta = 0° \text{ or } 180°)$$

  2. Stationary charge ($\vec{v} = \vec{0}$):

  $$F_m = 0 \quad \text{(Electrostatic case)}$$

30. Application: Electron Motion in Earth's Magnetic Field

30.1 Problem Statement

An electron moves through Earth's magnetosphere perpendicular to magnetic field lines with:

- Velocity $v = 1000\,\text{km/s} = 10^6\,\text{m/s}$
- Magnetic field $B = 10^{-6}\,\text{T}$
- Electron charge $e = -1.6 \times 10^{-19}\,\text{C}$
- Electron mass $m_e = 9.11 \times 10^{-31}\,\text{kg}$

30.2 Solution

1. Lorentz Force Calculation For perpendicular motion ($\theta = 90^\circ$):

$$F = |q|vB\sin\theta = evB \quad (\text{since } \sin 90^\circ = 1)$$

$$F = (1.6 \times 10^{-19}\,\text{C})(10^6\,\text{m/s})(10^{-6}\,\text{T}) = 1.6 \times 10^{-19}\,\text{N}$$

The force direction is given by the left-hand rule for electrons (negative charge):

- Thumb: Opposite to $\vec{v}$ (since $q < 0$)
- Index: $\vec{B}$ direction
- Middle: $\vec{F}$ direction

2. Electron Acceleration Using Newton's Second Law:

$$a = \frac{F}{m_e} = \frac{1.6 \times 10^{-19}\,\text{N}}{9.11 \times 10^{-31}\,\text{kg}} = 1.76 \times 10^{11}\,\text{m/s}^2$$

30.3 Discussion

- The enormous acceleration ($\sim 10^{11}\,\text{m/s}^2$) explains rapid particle spiraling in magnetospheres
- Results in synchrotron radiation at relativistic velocities
- Practical implications for:
  - Van Allen radiation belts
  - Aurora formation mechanisms
  - Spacecraft charging effects

31. Electromagnetic Phenomena

31.1 Complete Lorentz Force

A charge q moving with velocity $\vec{v}$ through both electric ($\vec{E}$) and magnetic ($\vec{B}$) fields experiences the full electromagnetic Lorentz force:

$$\vec{F}_{\mathrm{EM}} = q(\vec{E} + \vec{v} \times \vec{B})$$

- **Electric Component**: $q\vec{E}$ (Independent of motion)
- **Magnetic Component**: $q\vec{v} \times \vec{B}$ (Velocity-dependent)

31.2 Laplace Force on Current-Carrying Conductors

31.2.1 Definition

For a straight conductor of length $\ell = PM$ carrying current I in perpendicular magnetic field $\vec{B}$:

$$\vec{F}_L = I\vec{\ell} \times \vec{B}$$

Microscopic Origin The N free electrons (charge density n) moving with drift velocity $\vec{v}_d$ experience individual Lorentz forces:

$$F_m = ev_d B \sin\alpha \quad \text{(Per electron)}$$

Macroscopic Formulation Converting to measurable quantities:

$$\begin{aligned} F_{\text{total}} &= N \cdot F_m = nA\ell \cdot ev_d B \\ &= (neAv_d) \cdot \ell B = I\ell B \end{aligned}$$

where $I = neAv_d$ is the conventional current.

Parameter	Description
A	Conductor cross-section
n	Electron density
v_d	Drift velocity (~mm/s)

31.2.2 Applications

- Electric motor operation
- Galvanometer mechanisms
- Current balance measurements

31.3 Laplace Force Formulation

Definition The Laplace force is the electromagnetic force exerted by a magnetic field on a current-carrying conductor:

$$\vec{F}_L = I\vec{\ell} \times \vec{B}$$

31.3.1 Derivation from Microscopic Principles

1. **Individual Electron Force**:

$$F_m = ev_d B \sin\alpha \quad \text{(Lorentz force per electron)}$$

2. **Total Charge Motion**: For N electrons moving through length ℓ:

$$\Delta t = \frac{\ell}{v_d} \quad \text{(Transit time)}$$

3. **Current Relationship**:

$$I = \frac{Ne}{\Delta t} = \frac{Nev_d}{\ell}$$

4. **Macroscopic Force**: Combining terms:

$$F_L = NF_m = N(ev_d B \sin\alpha) = \left(\frac{I\ell}{v_d}\right) ev_d B \sin\alpha = I\ell B \sin\alpha$$

31.3.2 Special Case: Perpendicular Fields ($\alpha = 90°$)

$$F_L^{\max} = I\ell B \quad \text{(Maximum force condition)}$$

Parameter	Typical Values
I	1A-100A (industrial applications)
ℓ	0.1-1m (motor windings)
B	0.1-1T (permanent magnets)

Engineering Applications

- **Motor Design**: F_L converts to torque $\tau = r \times F_L$
- **Circuit Breakers**: Magnetic trip mechanisms
- **Current Measurement**: Moving coil instruments

The microscopic derivation validates the macroscopic form while revealing:

- Force independence from charge sign (depends on conventional current)
- Proportionality to charge carrier density n

32. Application: Laplace Rail System

32.1 Problem Setup

A conducting bar M_1M_2 of length ℓ slides frictionlessly on Laplace rails, forming a closed circuit with:

- Total resistance R
- Applied EMF E
- Uniform magnetic field $\vec{B} = B\vec{e}_z$ (perpendicular to rail plane)
- Current direction as shown in figure

32.2 Solution

a. Current Expression Using Ohm's Law for the circuit:

$$i = \frac{E}{R} \quad \text{(Conventional current)}$$

b. Laplace Force Formulation The force on the bar derives from:

$$\vec{F}_L = i\vec{\ell} \times \vec{B}$$

where:

- $\vec{\ell} = \ell\vec{e}_y$ (bar vector)
- $\vec{B} = B\vec{e}_z$ (field direction)

$$\vec{F}_L = \left(\frac{E}{R}\right)\ell\vec{e}_y \times B\vec{e}_z = \frac{E\ell B}{R}\vec{e}_x$$

Parameter	Physical Meaning
$\vec{e}_y \times \vec{e}_z = \vec{e}_x$	Right-hand rule basis
$\frac{E\ell B}{R}$	Force magnitude

32.3 Energy Considerations

- **Power Input**: $P_{\text{in}} = Ei = \frac{E^2}{R}$
- **Mechanical Power**: $P_{\text{mech}} = \vec{F}_L \cdot \vec{v}$
- **Energy Balance**: $P_{\text{in}} = P_{\text{mech}} + I^2R$ (Joule heating)

[Numerical Application] For $E = 12\,V$, $R = 4\,\Omega$, $\ell = 0.5\,m$, $B = 0.8\,T$:

$$i = 3\,A, \quad F_L = 1.2\,N\,(\text{in } \vec{e}_x \text{ direction})$$

Solution

a. By applying the loop (mesh) rule to the closed circuit, we obtain:

$$\mathscr{E} = iR \quad \Rightarrow \quad i = \frac{\mathscr{E}}{R}$$

b. The Laplace force exerted on the bar M_1M_2 is:

$$\vec{F} = i\overrightarrow{M_2M_1} \wedge \vec{B} = -i\,\ell\,\vec{e}_y \wedge B\,\vec{e}_z$$

$$\vec{F} = -i\ell B\,\vec{e}_x \quad \Rightarrow \quad \vec{F} = -\frac{\mathscr{E}}{R}\ell B\,\vec{e}_x$$

Representation of the force $\vec{F}$:

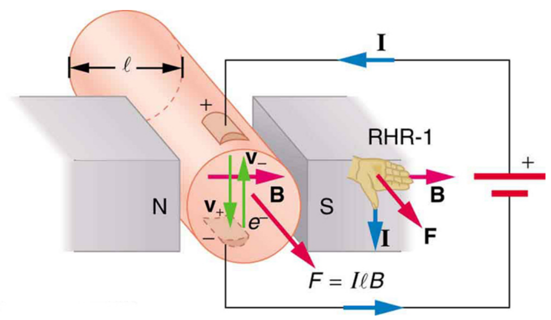

Figure 37: Diagram showing the direction of the current i and the Laplace force $\vec{F}$ on the bar in the magnetic field $\vec{B}$

32.4 Biot–Savart Law

Around 1820, Jean-Baptiste Biot and Felix Savart empirically established the law that governs the generation of a magnetic field by an electric current.

Let us consider a conducting wire describing a curve C, carrying a current of intensity I (see Figure 12).

We consider at a point P an elementary portion of the wire oriented as $d\vec{\ell}$.

The position vector of a point M relative to point P is $\vec{r} = \overrightarrow{PM}$.

The elementary magnetic field created at M according to the Biot–Savart law is given by:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell} \wedge \vec{r}}{r^3} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell} \wedge \vec{u}_{PM}}{r^2}$$

where μ_0 is the magnetic permeability of vacuum, $\mu_0 = 4\pi \times 10^{-7}$ S.I.

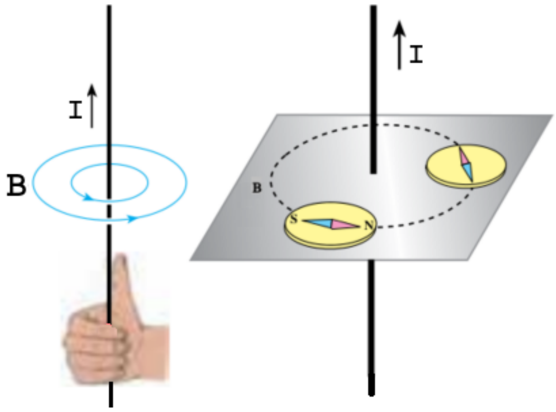

Figure 38: Element of conducting wire carrying a current I

32.5 Properties of a Magnetic Field Created by a Current

- The magnetic field produced is perpendicular to the plane defined by $\vec{d\ell}$ and $\vec{r}$.
- The direction of the magnetic field is determined using the right-hand (corkscrew) rule (see Figure 13).

> **NOTE:**
> *The direction of the magnetic field vector is given by the orientation of the thumb, with the index finger indicating the direction of the current.*

- To obtain the total magnetic field at a point M, one must sum all the elementary magnetic fields produced by each element of wire:

$$\vec{B}(M) = \int_C d\vec{B} = \int_C \frac{\mu_0}{4\pi} \frac{I d\vec{\ell} \wedge \vec{u}_r}{r^2}$$

32.5.1 Infinite Straight Wire

We consider an infinite straight wire carrying a constant current I. We aim to calculate the magnetic field created at point M by an element $d\vec{OP}$ observed at an angle α as shown in Figure 14.

According to the Biot–Savart Law:

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{OP} \wedge \overrightarrow{PM}}{||\overrightarrow{PM}||^3}$$

From Figure 14, we have:

$$\overrightarrow{PM} = \overrightarrow{PO} + \overrightarrow{OM}$$

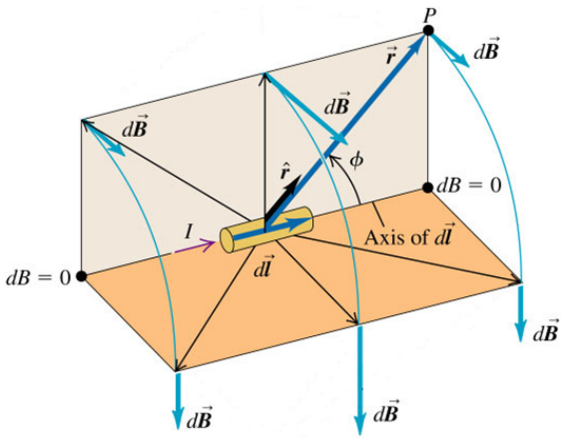

Figure 39: Magnetic field at point M due to a current element $d\vec{OP}$

$$PM = \frac{a}{\cos\alpha}$$

$$OP = PM\sin\alpha = a\tan\alpha \Rightarrow$$

$$dOP = \frac{a\,d\alpha}{\cos^2\alpha}$$

In the Cartesian coordinate system, this magnetic field is written as:

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{(d\vec{OP}\wedge(\vec{PO}+\vec{OM}))}{\|\vec{PM}\|^3} \Rightarrow d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{(d\vec{OP}\wedge\vec{PO}+d\vec{OP}\wedge\vec{OM})}{\|\vec{PM}\|^3}$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{(d\vec{OP}\wedge a\vec{u}_x)}{\|\vec{PM}\|^3} = \frac{\mu_0 I}{4\pi}\frac{a(d\vec{OP}\wedge\vec{u}_x)}{\|\vec{PM}\|^3}$$

By symmetry, only the component along the vector $\vec{k}$ is non-zero, so:

$$d\vec{B} = dB_z\,\vec{k} = \frac{\mu_0 I}{4\pi}\frac{a\,dOP}{\|\vec{PM}\|^3}\vec{k} = \frac{\mu_0 I}{4\pi}\frac{\cos\alpha\,d\alpha}{a}\vec{k}$$

The intensity of the total field created by the infinite wire is:

$$B = \int dB_z = \int_{-\pi/2}^{\pi/2}\frac{\mu_0 I}{4\pi}\frac{\cos\alpha}{a}\,d\alpha$$

$$B = \frac{\mu_0 I}{2\pi a}$$

32.5.2 Circular Loop

Let us consider a circular loop of radius R carrying a constant current I. We aim to calculate the magnetic field created at point M, which lies on the z-axis of the loop (Figure 15).

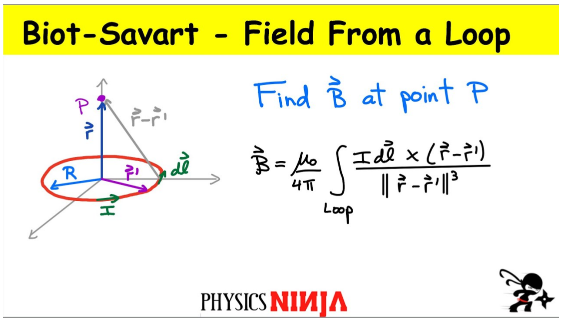

Figure 40: Magnetic field at point M created by a current-carrying circular loop

From Figure 40, we observe that:

$$\overrightarrow{PM} = \overrightarrow{PO} + \overrightarrow{OM} \quad \text{and} \quad PM = \frac{R}{\sin\alpha}$$

In cylindrical coordinates, we have:

$$d\vec{OP} = dOP\,\vec{u}_\theta = R\,d\theta\,\vec{u}_\theta$$

Substituting these equations into the Biot–Savart law, we get:

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{OP} \wedge \overrightarrow{PM}}{\|\overrightarrow{PM}\|^3} = \frac{\mu_0 I R\,d\theta}{4\pi} \frac{\vec{u}_\theta \wedge (-R\vec{u}_r + z\vec{k})}{\|\overrightarrow{PM}\|^3}$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{(R^2 d\theta \vec{k} + Rz\,d\theta \vec{u}_r)}{R^3/\sin^3\alpha}$$

By symmetry, diametrically opposite points on the loop produce magnetic fields whose radial components cancel out. Thus, only the z-component remains:

$$d\vec{B} = dB_z\,\vec{k} = \frac{\mu_0 I}{4\pi R} \sin^2\alpha\,d\theta\,\vec{k}$$

The total magnetic field created by the loop at point M is:

$$B = \oint dB_z = \frac{\mu_0 I}{4\pi R} \sin^2\alpha \int_0^{2\pi} d\theta$$

$$B = \frac{\mu_0 I}{2} \frac{R^2}{(R^2 + z^2)^{3/2}}$$

33. Application

An infinite straight wire is carrying a constant current of 10 A.

1. What is the intensity of the magnetic field produced by this current at a distance of 5 cm from the wire?
2. Could this magnetic field disturb a compass? Knowing that the Earth's magnetic field intensity is 2.2×10^{-5} T.

Solution

1. The intensity of the magnetic field created at a distance r from an infinite wire is given by:

$$B = \frac{\mu_0}{2\pi} \cdot \frac{I}{r}$$

Numerical application:

$$B = 4 \times 10^{-5} \text{ T}$$

2. This magnetic field would disturb a compass, as it is twice as intense as the Earth's magnetic field.

Magnetic Dipole

A magnetic dipole for the magnetic field is equivalent to the electrostatic dipole for the electric field. It is characterized by the magnetic moment vector (or magnetic dipole moment).

33.1 4.7.1 Magnetic Moment

In physics, the magnetic moment is a vector quantity that characterizes the strength of a magnetic source. This source may be an electric current, or a magnetized object.

The magnetic moment of a body is manifested by the tendency of that body to align in the direction of a magnetic field $\vec{B}$, as does a compass needle.

Consider a loop of surface S carrying a current of intensity I (Figure 16). Its magnetic moment $\vec{\mu}$ is defined as:

$$\vec{\mu} = I \cdot S \cdot \vec{n}$$

It is expressed in ampere-square meters (A·m^2).
$\vec{n}$ is the unit vector normal to the surface S.

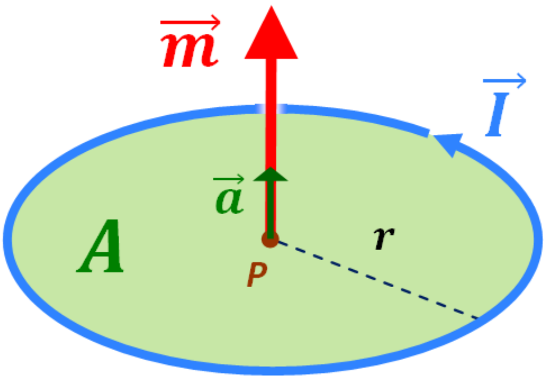

Figure 41: Magnetic moment of a loop carrying current