

Work 1 (Markov chain)**Exercise 1.**

Let's consider a markov chain with transition matrix as follows:

$$\mathbf{A} = \begin{bmatrix} 0,6 & 0,4 & 0 \\ 0,2 & 0,5 & 0,3 \\ 0 & 0,4 & 0,6 \end{bmatrix}$$

- 1- Make the graphical representation of the chain.
- 2- Check that the chain is irreducible and calculate the peridicity of states.
- 3- Calculate the stationary distribution of states.

Exercise 2.

Consider the following transition matrix:

$$\mathbf{P} = \begin{array}{c} \begin{array}{ccc} & \text{h} & \text{a} & \text{f} \end{array} \\ \begin{bmatrix} 0,5 & 0,45 & 0,05 \\ 0,1 & 0,5 & 0,4 \\ 0 & 0,1 & 0,9 \end{bmatrix} \end{array}$$

- 1- Calculate the probability of the following trajectories (h, a, f, h), (h, a, f, a), (a, a, a).
- 2- Calculate the distribution at $t = 1$ if we assume $\pi^0 = (1, 0, 0)$.
- 3- Show that a uniform distribution $X_0 = (1/3, 1/3, 1/3)$ is not a stationary distribution for this Markov chain.
- 4- Calculate the stationary Matrix, if it exists.

3. Exercise 3

Let's consider the markov chain,viewed in the course, that represents changes in weather, with the transition matrix as follows.

$$\mathbf{P} = \begin{bmatrix} 0,6 & 0,2 & 0,2 \\ 0,05 & 0,8 & 0,15 \\ 0,3 & 0,0,2 & 0,5 \end{bmatrix}$$

- Simulate the chain using the Monte-Carlo method method and Calculate the stationary distribution of states.
- find the stationary matrix.
- find the stationary distribution by calculating the left eigenvector of the matrix.