Practice Exercises

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Abstract

Exercises with solutions. Please review them carefully..

Exercise 1

1) Calculation of the electrostatic field produced by an infinite plane at any point in space using Gauss's theorem:

To calculate the electrostatic field produced by an infinite plane, we consider a Gaussian surface formed by a field tube perpendicular to the plane, closed by two surface elements ΔS parallel to the plane and symmetric with respect to it.

0.1 Solution

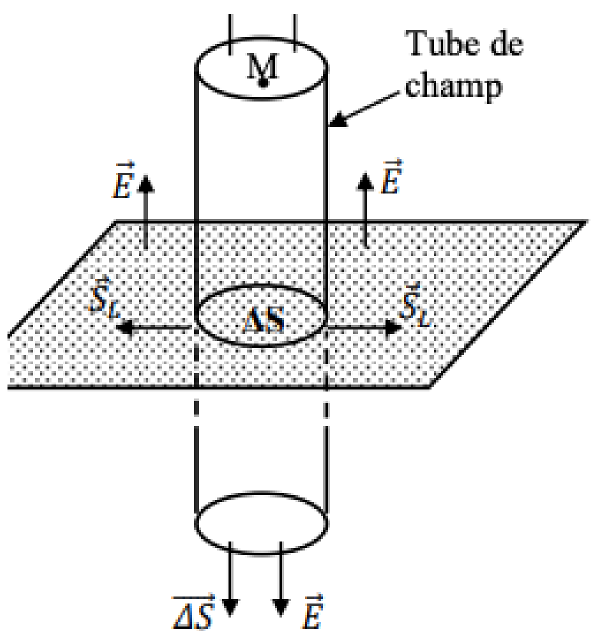

Figure 1: Illustration

Thus, the Gaussian surface area is:

$$S_{\text{Gauss}} = S_{\text{Lateral}} + 2\,\Delta S$$

The field lines are perpendicular to the plane. The direction of $\vec{E}$ changes when we cross the charged plane.

$$\Phi = \Phi_{S_L} + 2\Phi_{\Delta S} \Rightarrow \vec{E}\cdot\vec{S}_G = \vec{E}\,(\vec{S}_L + 2\Delta\vec{S}) = \vec{E}\cdot\vec{S}_L + \vec{E}\cdot 2\Delta\vec{S}$$

but

$$\Phi_{S_L} = \vec{E} \cdot \vec{S}_L = 0 \quad \text{since} \quad \vec{E} \perp \vec{S}_L \quad ; \quad \text{thus :} \quad \vec{E} \cdot \vec{S}_G = 2\vec{E} \cdot \Delta\vec{S}$$

Since $\vec{E} \parallel \Delta\vec{S}$, we have:

$$E\, S_G = 2\, E\, \Delta S$$

On the other hand:

$$\sum q_{\text{int}} = \sigma\, \Delta S$$

Therefore, Gauss's theorem states:

$$2\, E\, \Delta S = \frac{\sigma\, \Delta S}{\varepsilon_0} \quad \Rightarrow \quad \boxed{E = \frac{\sigma}{2\, \varepsilon_0}}$$

1 Exercise 2

1) Calculation of the electrostatic field created at a point M using Gauss's theorem:

a) When M is located inside the cylinder: $r < R$

By symmetry, the field is radial and can only depend on a point M by the distance r ($r < R$). Thus, we write: $\vec{E}_{\text{int}} = \vec{E}_{\text{int}}(r)$.

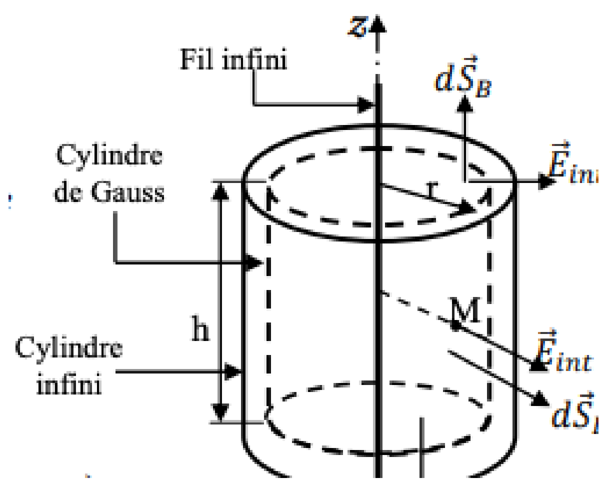

Figure 2: Illustration Exo 2

Applying Gauss's theorem to a cylinder of height h, axis Oz, and radius r, passing through M where we want to calculate the field. This cylinder closed at its ends by two circular bases forms the Gaussian surface.

Gauss's theorem is written as:

$$\phi =_S \vec{E}_{\text{int}} \cdot d\vec{S} = \frac{\sum q_{\text{int}}}{\varepsilon_0}$$

Each surface element $d\vec{S}_B$ of each base is normal to the field $\vec{E}_{\text{int}}$, and the contribution of the bases to the outgoing flux is thus zero. At each point on the lateral surface, $\vec{E}_{\text{int}}$ and $d\vec{S}_L$ are collinear ($\vec{E}_{\text{int}}//d\vec{S}_L$), and E_{int} has a constant value.

Thus:

$$\phi = 2\phi_{S_B} + \phi_{S_L} = \phi_{S_L}$$

$$\phi_{S_L} = E_{\text{int}} \iint dS_L = E_{\text{int}} S_L = E_{\text{int}} \cdot 2\pi r h$$

$\sum q_{\text{int}}$ represents the sum of the internal charges contained in the lateral surface of the Gaussian cylinder, hence:

$$\sum q_{\text{int}} = \lambda \cdot h$$

Therefore:

$$\phi =_S \vec{E}_{\text{int}} \cdot d\vec{S} = \frac{\sum q_{\text{int}}}{\varepsilon_0} \Rightarrow E_{\text{int}} \cdot 2\pi r h = \frac{\lambda h}{\varepsilon_0} \Rightarrow \boxed{E_{\text{int}} = \frac{\lambda}{2\pi\varepsilon_0 r}}$$

2 Exercise 3

Three charges q_1, q_2, and q_3 are arranged according to Figure 1.5. Calculate the resultant force exerted on charge q_3.

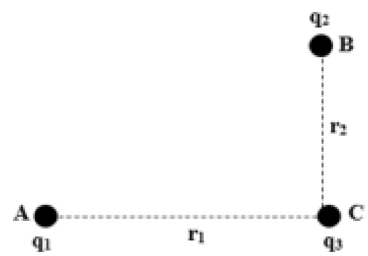

Figure 3: Illustration of the charges

Given:

$$q_1 = +1.5 \times 10^{-1}\,\text{C}, \quad q_2 = -0.5 \times 10^{-3}\,\text{C}, \quad q_3 = +0.2 \times 10^{-3}\,\text{C}$$

Distances:

$$AC = 1.2\,\text{m}, \quad BC = 0.5\,\text{m}$$

3 Solution of exo 3

Charges q_1 and q_3 have the same sign; thus, in this case, the force $\vec{F}_1$ is repulsive.

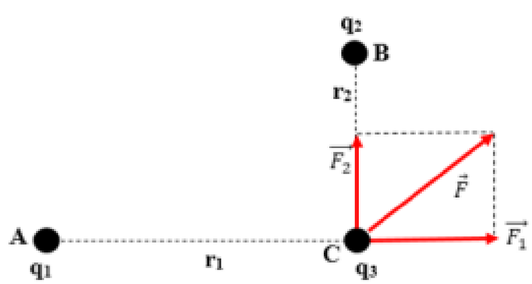

Figure 4: Illustration Exo 3

Charges q_2 and q_3 have opposite signs; therefore, the force $\vec{F}_2$ is attractive.

$$\vec{F}_1 = k\frac{q_1 q_3}{r_1^2}\,\vec{u}_{r_1} \quad \Rightarrow \quad F_1 = 1.8 \times 10^3\,\text{N}$$

$$\vec{F}_2 = k\frac{q_2 q_3}{r_2^2}\,\vec{u}_{r_2} \quad \Rightarrow \quad F_2 = 3.6 \times 10^3\,\text{N}$$

Consequently, the magnitude of the resultant force $\left|\vec{F}\right|$ is:

$$\left|\vec{F}\right| = \sqrt{F_1^2 + F_2^2} = 4.06 \times 10^3\,\text{N}$$