

**Exercise 1** A student estimates a 65% chance of passing his statistics course, an 80% chance of passing his chemistry course, and a 50% chance of passing both subjects.

- (a) What is the probability that the student passes in Statistics but not in Chemistry.?
- (b) What is the probability that the student passes in Chemistry but not in Statistics.?
- (c) What is the probability that the student passes in Statistics or Chemistry.?
- (d) What is the probability that the student passes neither Chemistry nor Statistics.?
- (e) What is the probability that the student passes in Statistics given that he passed in Chemistry.?

**Exercise 2** Consider the random experiment "rolling a six-sided die and observing the result."

We consider the following game:

- If the result is even, we win 2 DA.
- If the result is 1, we win 3 DA.
- If the result is 3 or 5, we lose 4 DA.

We will define a random variable  $X$  that gives the gain in this game.

- (a) Give the fundamental set  $\Omega$ .
- (b) Determine  $X(\Omega)$ .
- (c) Determine the probability law of  $X$ .
- (d) Determine the distribution function of  $X$ .
- (e) Calculate the expected value  $E(X)$ , variance  $V(X)$ , and standard deviation  $\delta_X$ .

**Exercise 3** We perform the following random experiment: we roll a pair of fair dice, numbered from 1 to 6, and define the random variable  $X$  by:

$$X : \Omega \rightarrow \mathbb{R}$$

$$(a, b) \mapsto X(a, b) = \max(a, b)$$

- (a) Give the fundamental set  $\Omega$ .
- (b) Determine  $X(\Omega)$ .
- (c) Determine the probability law of  $X$ .
- (d) Determine the distribution function of  $X$ .
- (e) Calculate the expected value  $E(X)$ , variance  $V(X)$ , and standard deviation  $\delta_X$ .

**Exercise 4** Consider the function defined by:

$$f(x) = \begin{cases} \frac{1}{2}(2-x) & \text{if } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Show that  $f$  is a probability density function of a random variable  $X$ .
- (b) Determine its distribution function.
- (c) Calculate the expected value of  $X$ .
- (d) Calculate the variance and standard deviation of  $X$ .

**Exercise 5** Let  $X$  be a random variable with the associated function:

$$f(x) = \begin{cases} ax(1-x) & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Determine  $a$  such that  $f$  is a probability density function.
- (b) Determine its distribution function.
- (c) Calculate  $P(0 \leq X \leq \frac{1}{2})$ .