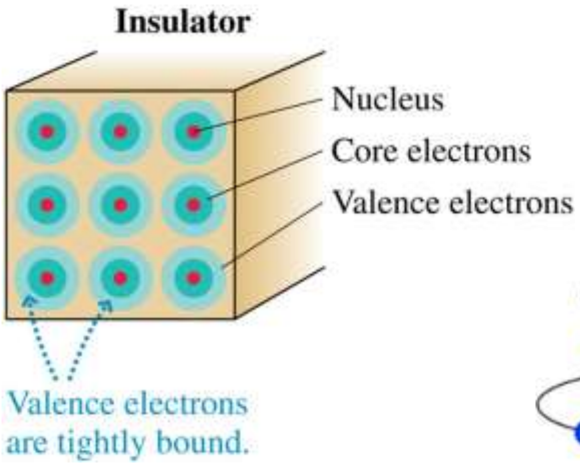
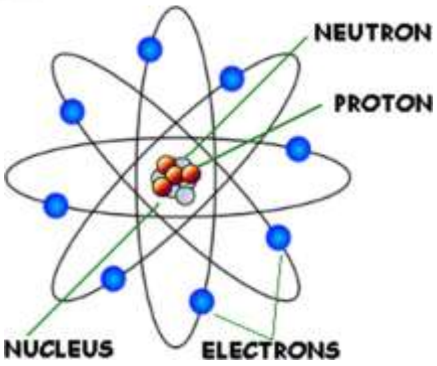
Conductors in Electrostatic Equilibrium

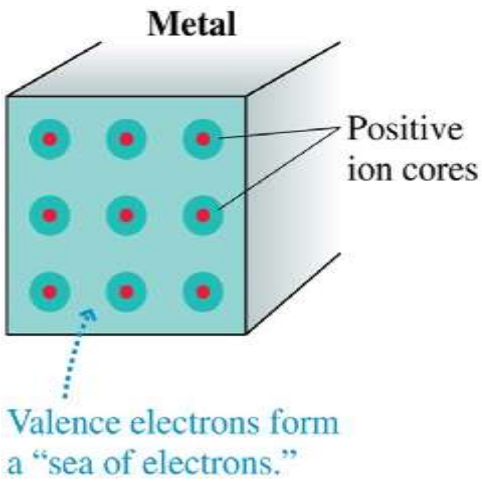
Insulators and conductors

- The electrons in an insulator are all tightly bound to the positive nuclei and not free to move around.





- In metals, the outer atomic electrons are only weakly bound to the nuclei.
- These outer electrons become detached from their parent nuclei and are free to wander about through the entire solid.



1. Definition of electrostatic equilibrium

A good electrical conductor contains electrons that are not bound to any atom and therefore are *free to move* about within the material.When no net motion of charge occurs within a conductor, the conductor is said to be in ***electrostatic equilibrium.***

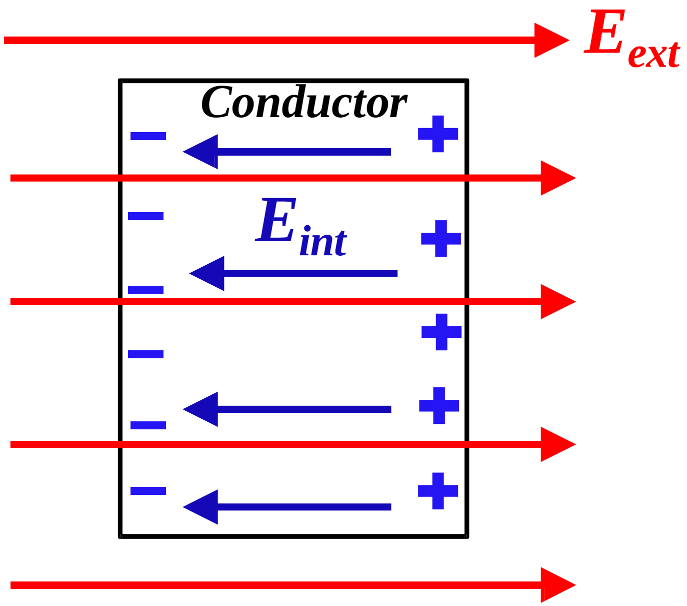
2. Properties of conductors in equilibrium

A conductor in electrostatic equilibrium has the following properties:

1. The electric field is zero everywhere inside the conductor.
2. Any net charge on an isolated conductor must reside entirely on its surface.
3. The E-field just outside a charged conductor is perpendicular to the conductor's surface and has a magnitude , where is the surface charge density at that point.
4. The Electric potential is constant in all the conductor V=Cte.

1st Property

✓ **Electric field inside a conductor**



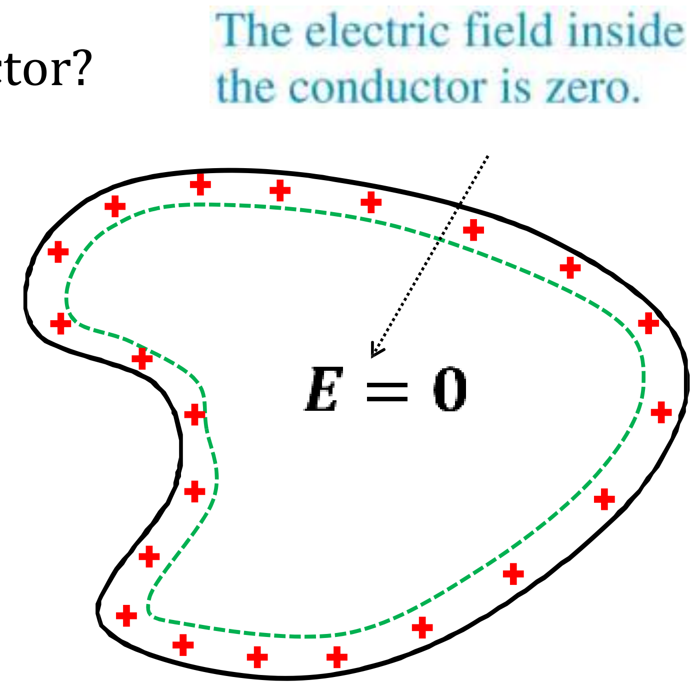
- Consider a conductor and apply an external electric field.
- Conductor has tons of free electrons and under the influence of E_{ext} they will run to the left surface leaving positive charges near the right surface and creating $E_{internal}$
- How many of them will move?

-The electrons will keep moving until the internal field cancels out the external field inside the conductor

Thus, the electric field inside a conductor is zero in electrostatic situation

- ***Consider a positively charged conductor (in electrostatic equilibrium)***

Where does this excess charge reside in the conductor?



Let's apply Gauss's law

$$\Phi_E = \vec{E}\cdot d\vec{A} = \frac{Q_{in}}{s_0}$$

0

So Q_{in} =0 (inside the Gaussian surface)

Thus, the positive excess charge resides on the external surface of the conductor.

In simple words: They just repel each other

✓ 2nd Property

✓ Consider a conductor with surface charge density σ. Construct a Gaussian surface in the shape of a small cylinder with the end faces parallel to the surface. Part of the cylinder is just outside the surface and the rest is inside.

$$\emptyset = \oiint \vec{E}.d\vec{S} = \frac{Q_{enclosed}}{\varepsilon_0}$$

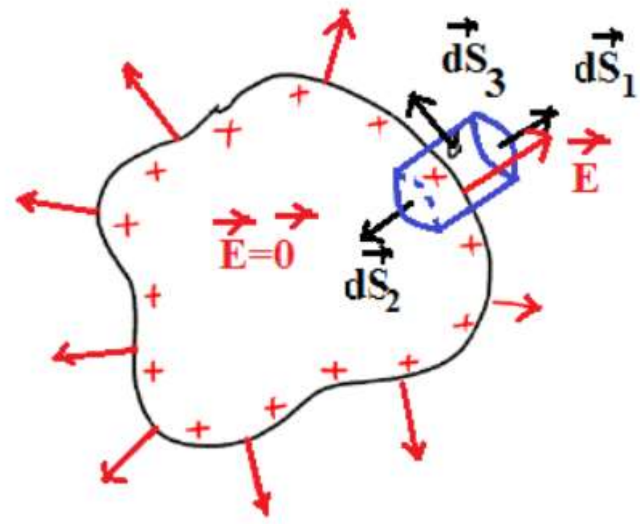
$$\Rightarrow \emptyset = \oiint \vec{E}.d\vec{S_1} + \oiint \vec{E}.d\vec{S_2} + \oiint \vec{E}.d\vec{S_3}$$

$$\oiint E.dS_1 = E.S_1 \ \ (E//dS_1 \text{ and in the same direction})$$

$$\oiint \vec{E}.d\vec{S_2} = 0 \ \ (\text{E} = 0 \text{ inside the conductor})$$

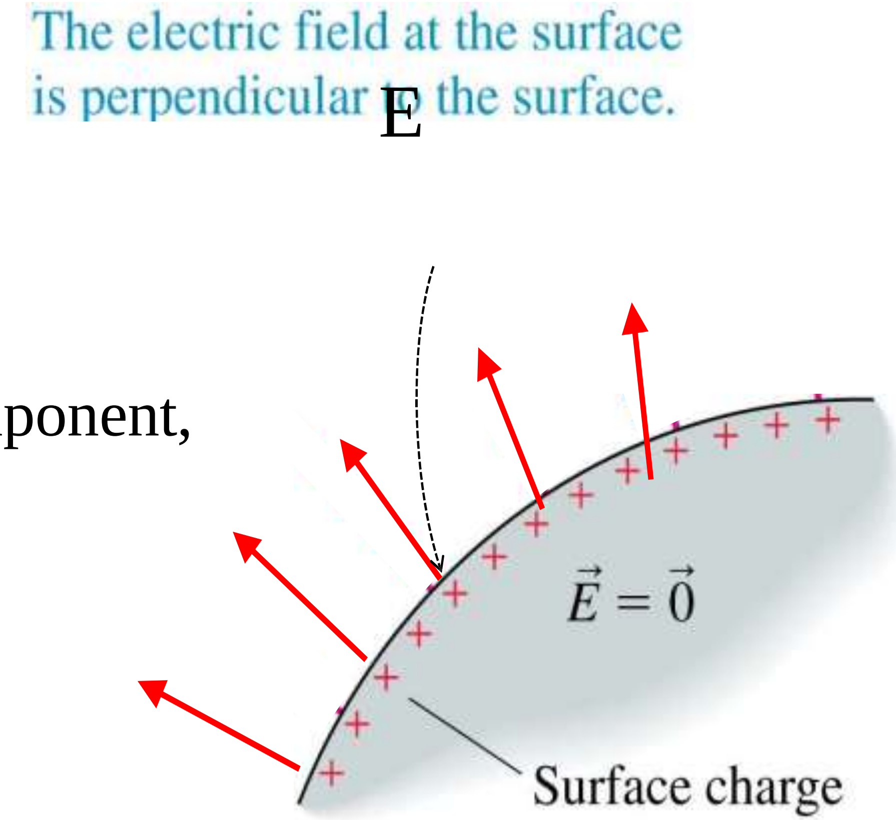
$$\oiint \vec{E}.d\vec{S_3} = 0 \ (\vec{E} \perp d\vec{S_3})$$

$$\Rightarrow \emptyset = E.S_1 = \frac{Q_{enclosed}}{\varepsilon_0} \Rightarrow \emptyset = E.S_1 = \frac{\sigma.S_1}{\varepsilon_0} \Rightarrow \boldsymbol{E} = \frac{\boldsymbol{\sigma}}{\boldsymbol{\varepsilon_0}}$$



✓ ***3rd Property*** E (outside) is ⊥ to the surface of a conductor

✓ The external electric field right at the surface of a conductor must be perpendicular to that surface.



✓ If it were to have a tangential and

cause a surface current, and the component,

it would exert a force on the surface

charges conductor would not be

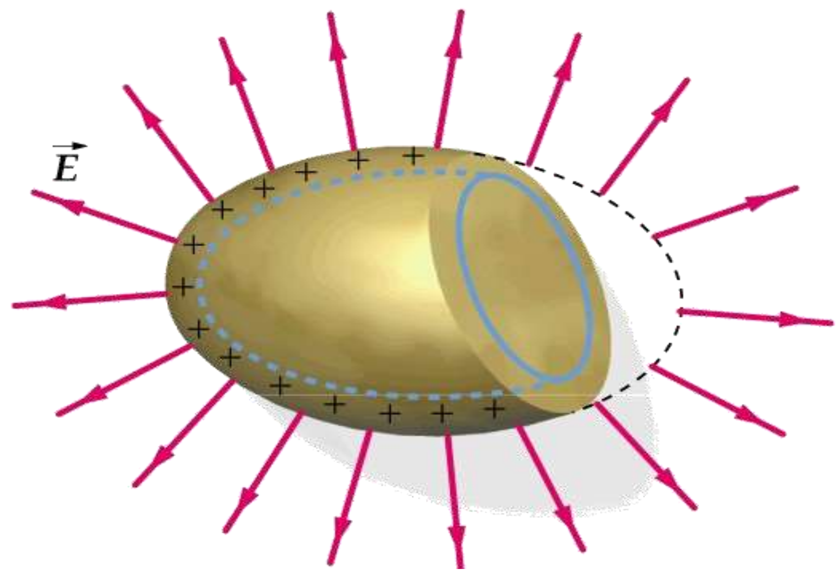
in electrostatic equilibrium (Proof by contradiction)

.

✓ *4th Property*

- Any conductor's point has equal electric potential:

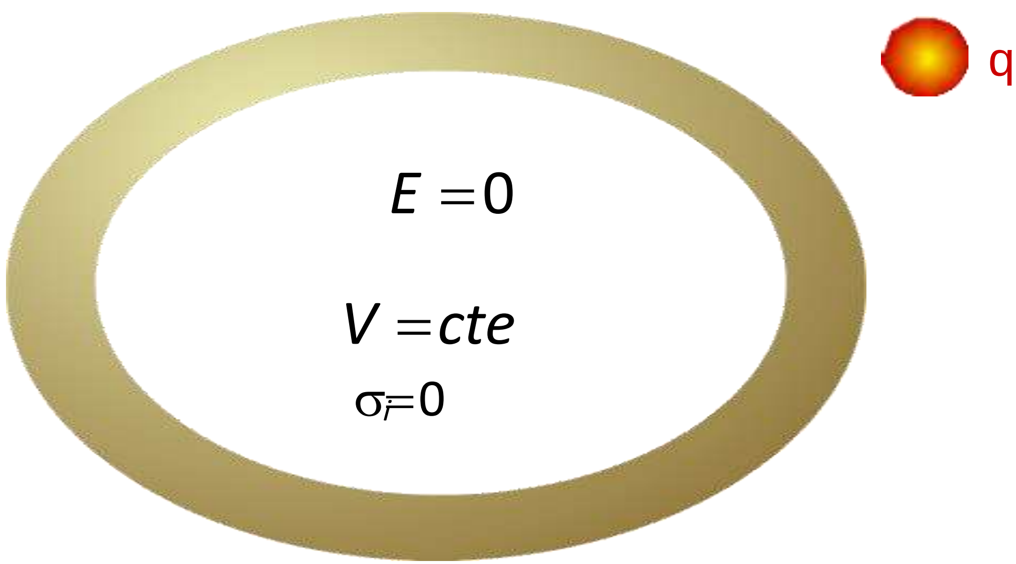
$$V_B - V_A = -\int_A^B \underbrace{\mathbf{E}}_{0} \cdot d\mathbf{l} = 0 \Rightarrow V_B = V_A$$



You did well on the questions on charge distributions on conductors

The Main Points

- Charges free to move
- E = 0 in a conductor
- Surface = Equipotential
- E at surface perpendicular to surface



❑ Two Spherical Conductors

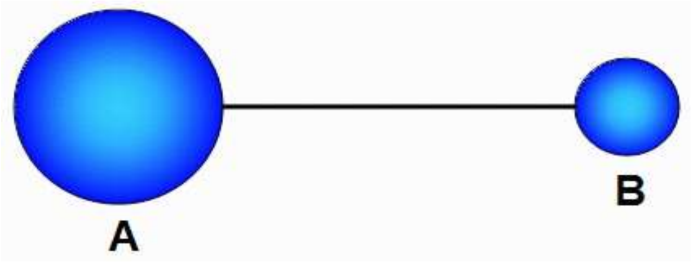
- Two spherical conductors are separated by a distance r. They each carry the same positive charge Q. Conductor A has a larger radius than conductor B.

- Compare the potential on surface A with the potential on surface B

A) $V_A > V_B$ B) $V_A = V_B$ C) $V_A < V_B$

They both replicate the electric field due to a point charge, and since the surface of A is farther away from its center than B, it will have a weaker electric field and therefore have less potential.

The two conductors are now attached by a conducting wire.



1. What happens to the charge on sphere A when the wire is attached

A) Q_A increases B) Q_A decreases C) Q_A does not change

“The electric potential was lower at A , which means positive charges will travel towards A”

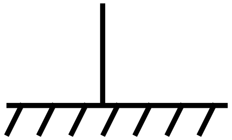
Ground

- Electric potential of a spheric conductor is given by:

$$V = \frac{Q}{4\pi\varepsilon_0 R}$$

- As Earth has a very big radius ($R \rightarrow \infty$) related to any object, electric potential of earth (ground) is zero for any charge Q. Ground can take or give any charge without change its electric potential (it's like the sea level)

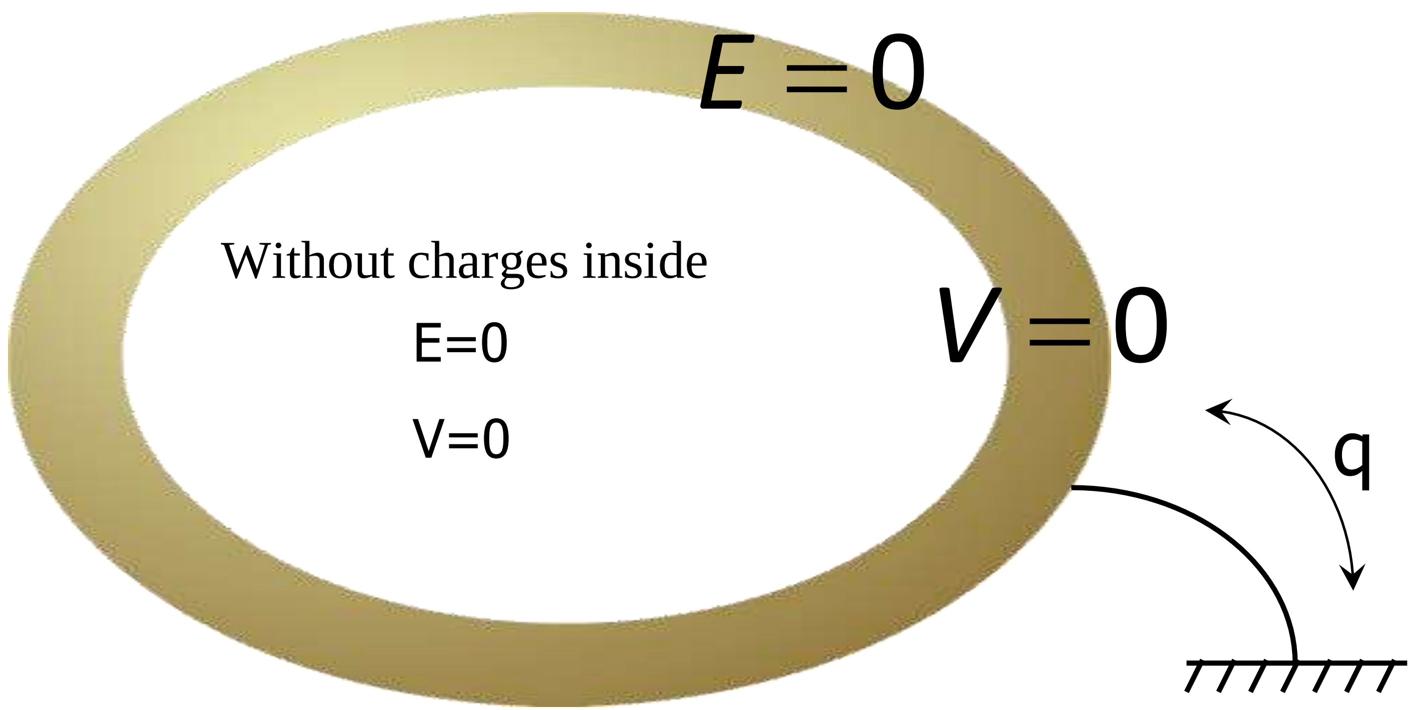
$$V_G = 0$$



Connecting a device to ground means safety for people

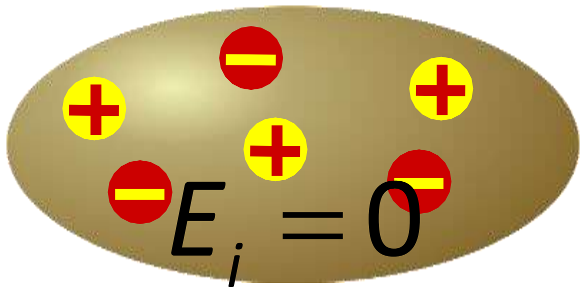
Linking a conductor to Ground

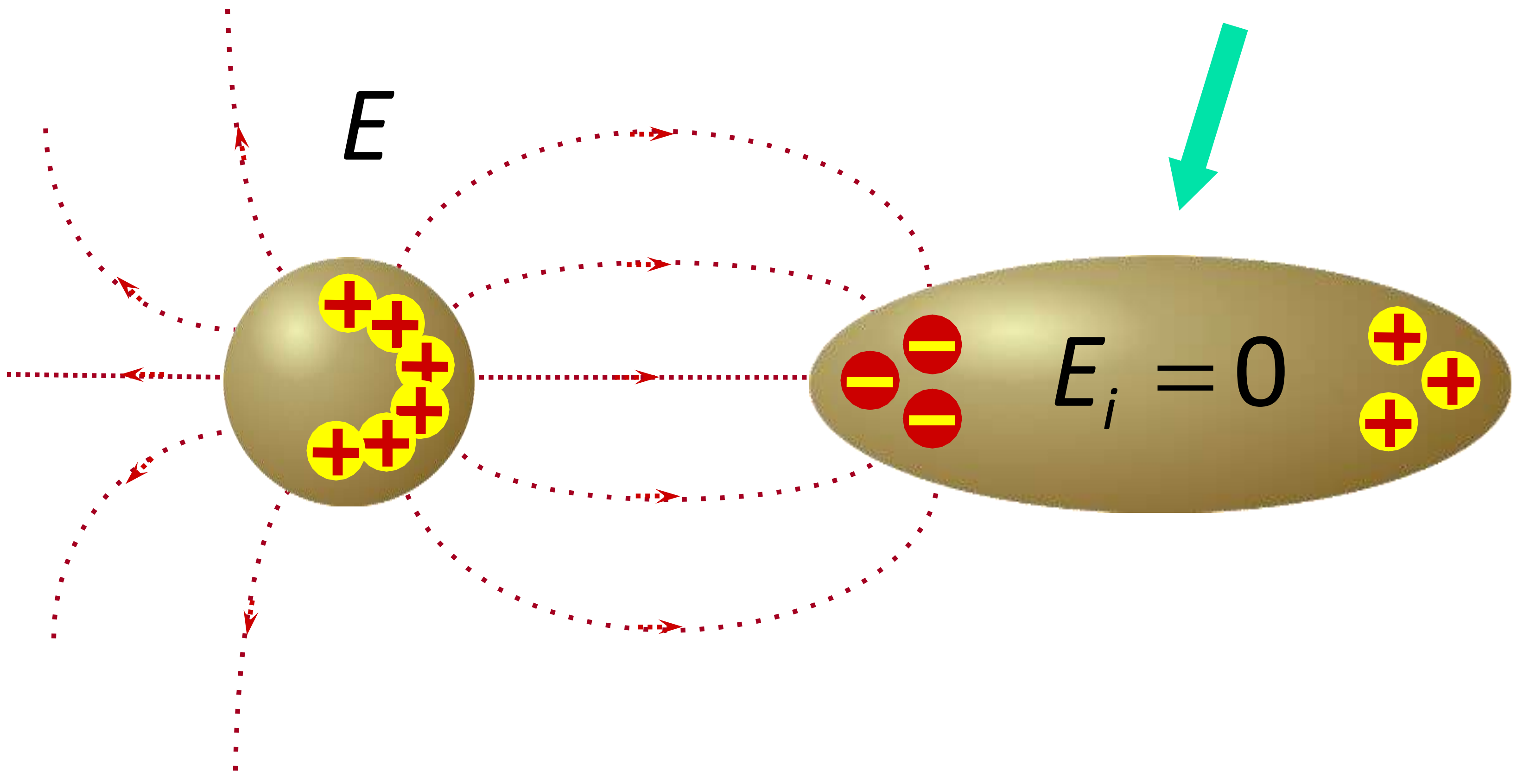
- Linking a conductor to Ground ( ) means:
  - 1. Electric potential is 0 (V=0)
  - 2. The conductor can change its charge by taking or giving electrons to Ground.



Electrostatic influence

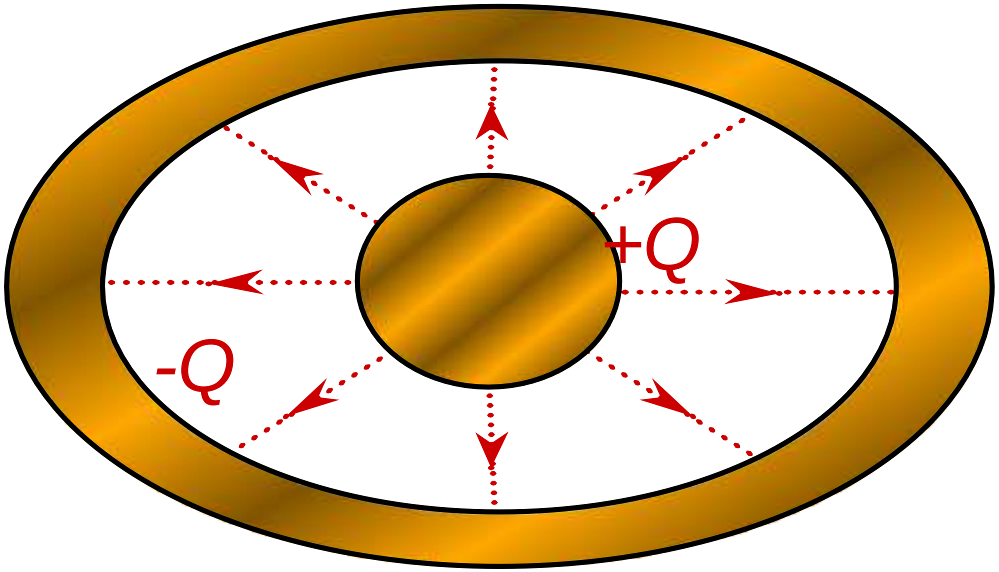
- When we put an electric charge near a conductor, electrostatic influence divides the charge inside the conductor.

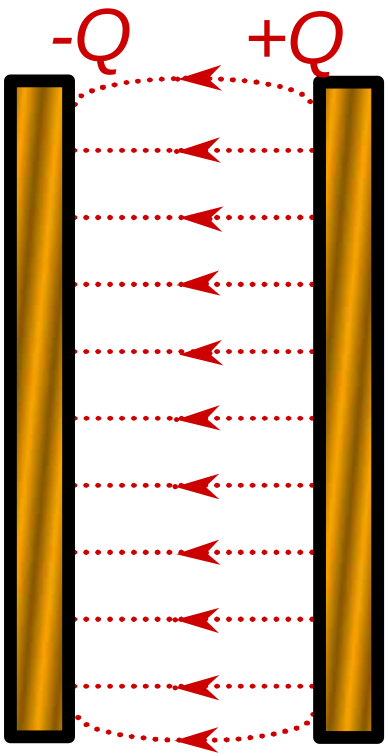




Total electrostatic influence

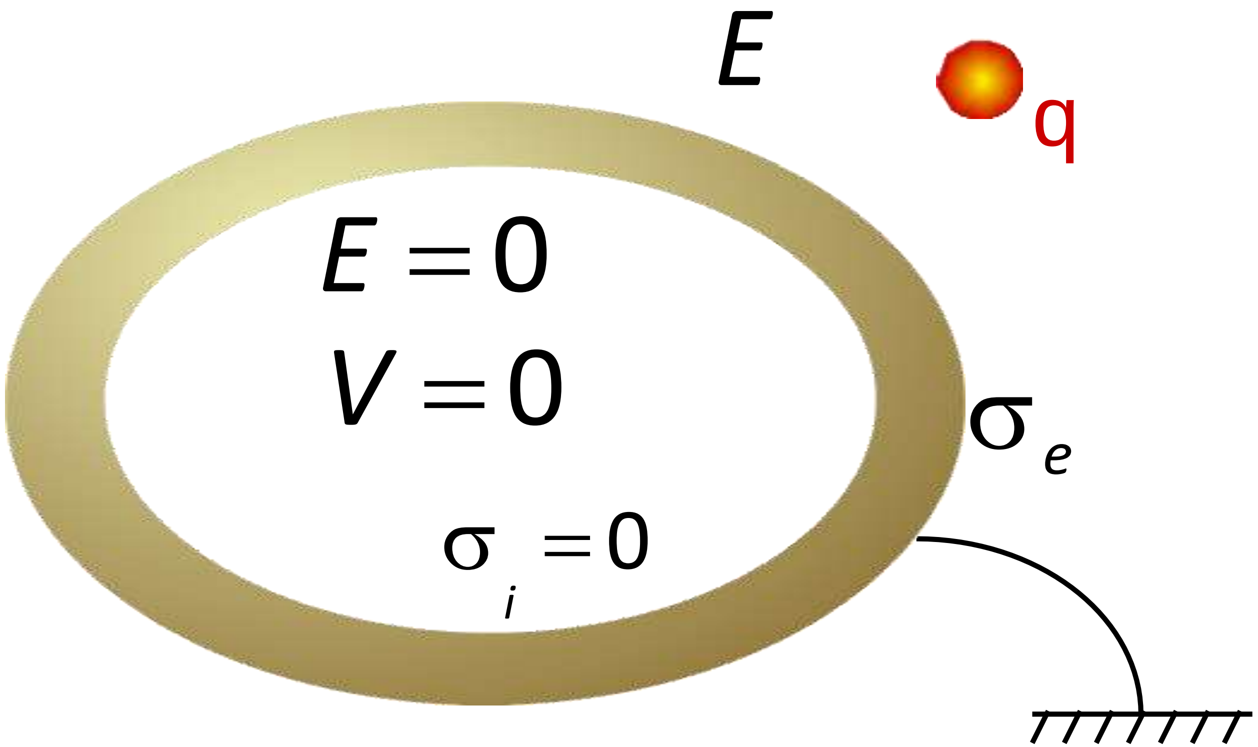
- Total Electrostatic influence between two conductors occurs when all the field lines starting from a conductor end in the other conductor.
  - Surfaces with total influence have the same charge but different sign.





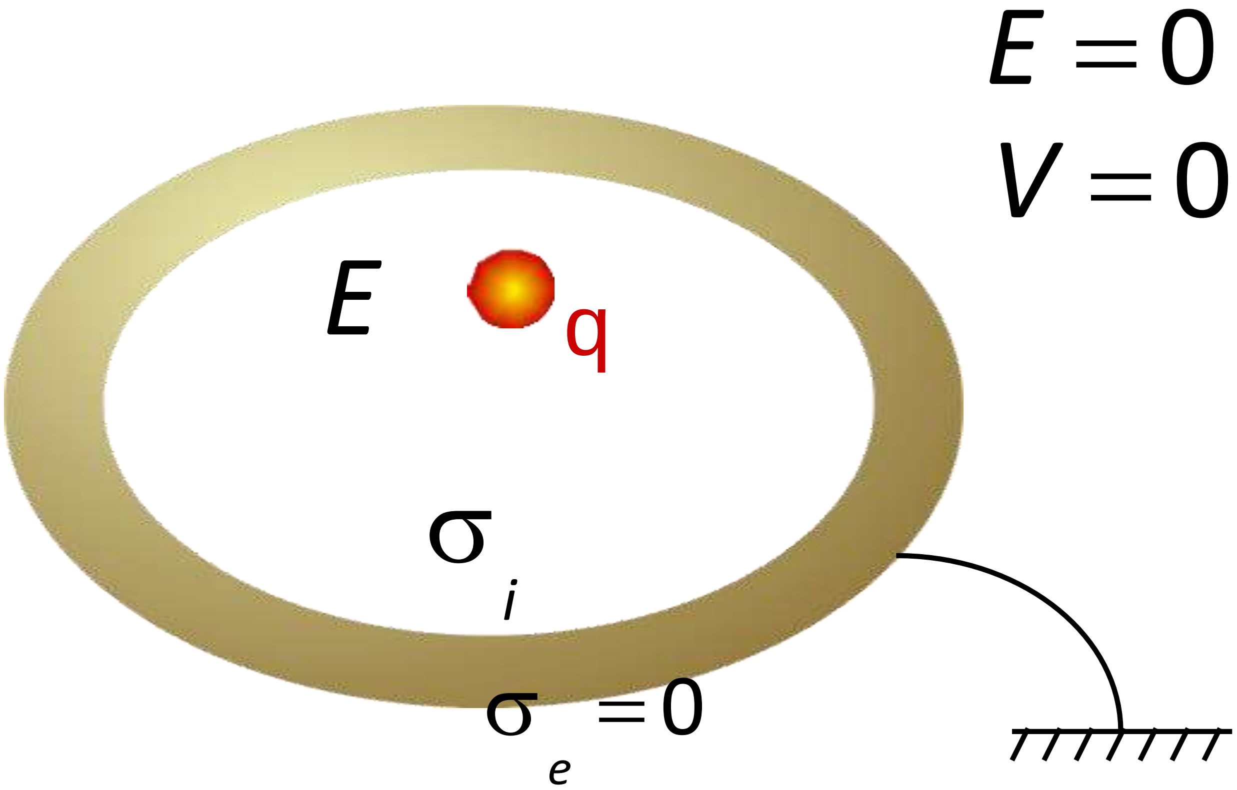
Electric shield or Faraday's cage

- A hollow conductor linked to ground divides electrically the inner and outer spaces. It's known as an electric shield. Outer charges don't influence inner space………



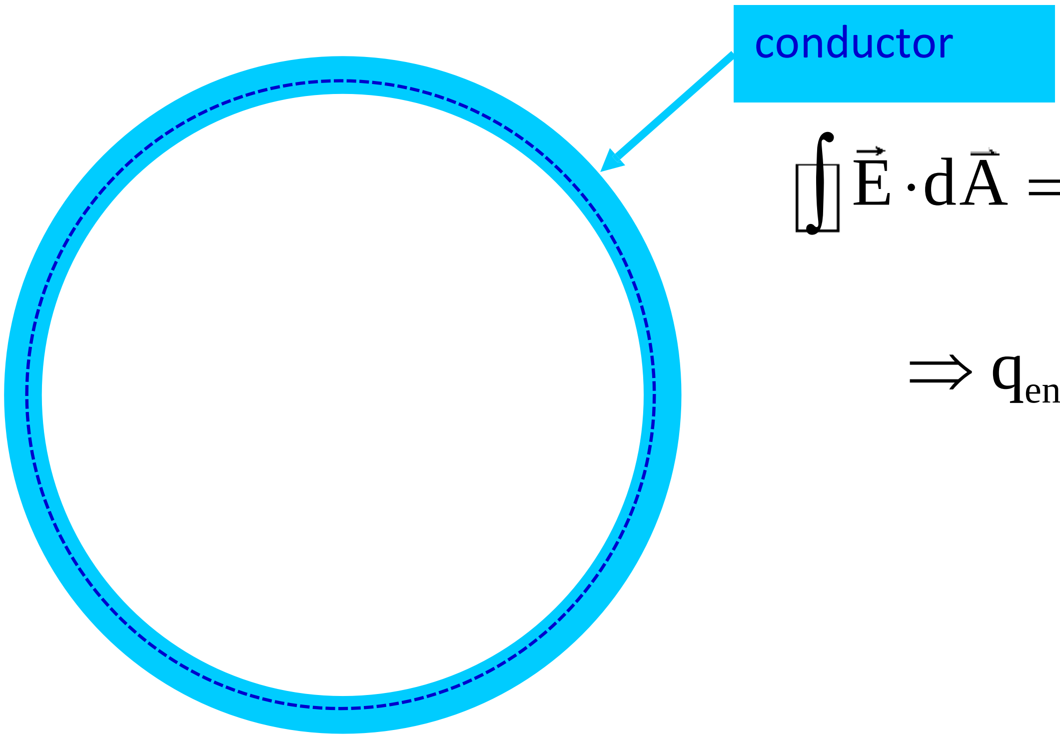
Electric shield or Faraday's cage

- And inner charges don't influence outer space.



Example 01:

If there is an empty nonconducting cavity inside a conductor, Gauss' Law tells us there is no net charge on the interior surface of the conductor.

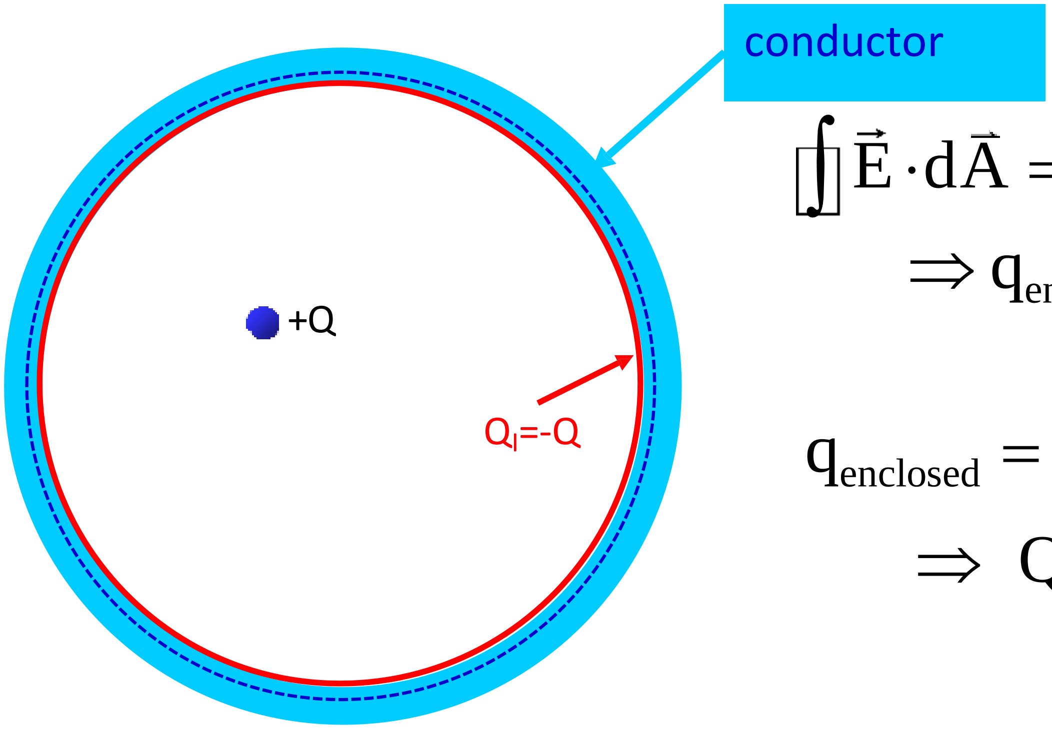


$$\oint \vec{E} \cdot d\vec{A} = \oint 0 \cdot dA = 0$$

$$\Rightarrow q_{enclosed} = 0$$

Example 02:

If there is a nonconducting cavity inside a conductor, with a charge inside the cavity, Gauss' Law tells us there is an equal and opposite induced charge on the interior surface of the conductor.



$$\oint \vec{E}\cdot d\vec{A} = \oint 0 \cdot dA = 0$$

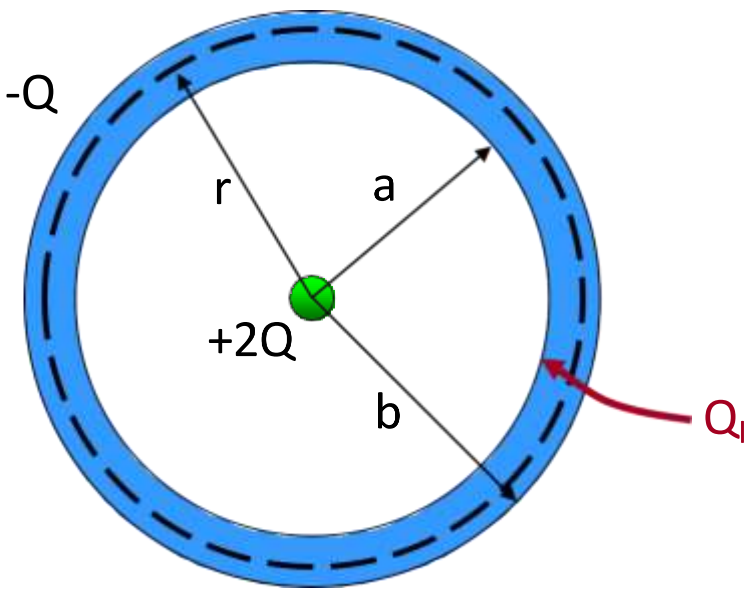
$$\Rightarrow q_{enclosed} = 0$$

$$q_{enclosed} = 0 = +Q + Q_I$$

$$\Rightarrow Q_I = -Q$$

Example 03:

a conducting spherical shell of inner radius ***a*** and outer radius ***b*** with a net charge ***-Q*** is centered on point charge ***+2Q***.
Use Gauss's law to show that there is a charge of ***-2Q*** on the inner surface of the shell, and a charge of ***+Q*** on the outer surface of the shell

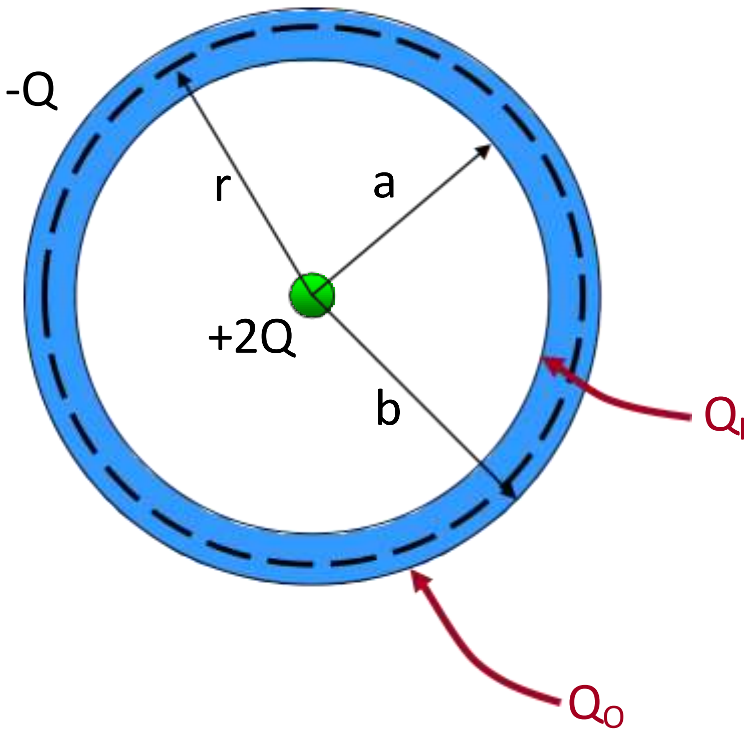


$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_0}$$

E=0 inside the conductor!

Let ***r*** be infinitesimally greater than ***a***.

$$0 = \frac{q_{enclosed}}{\varepsilon_0} = \frac{Q_I + 2Q}{\varepsilon_0} \Rightarrow Q_I = -2Q$$



$$Q_I = -2Q$$

From Gauss' Law we know that excess charge on a conductor lies on surfaces.

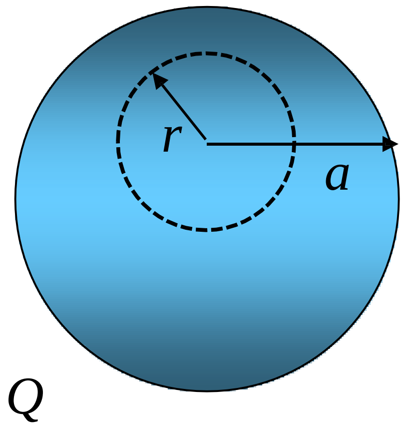
Electric charge is conserved:

$$Q_{shell} = -Q = Q_I + Q_O = -2Q + Q_O$$

$$-Q = -2Q + Q_O \Rightarrow Q_O = +Q$$

Example 04:

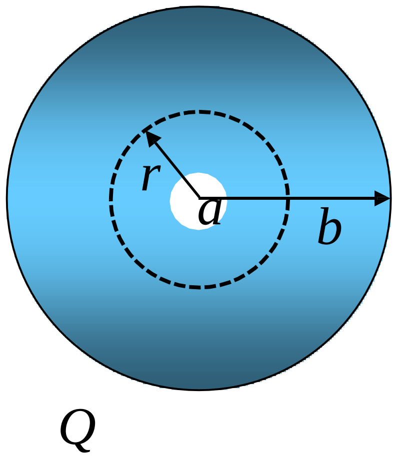
An insulating sphere of radius a has a uniform charge density $\boldsymbol{\rho}$ and a total positive charge $\boldsymbol{Q}$. Calculate the electric field at a point inside the sphere.



$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o} = \frac{\rho V_{enclosed}}{\varepsilon_o}$$

$$E\left(4\pi r^2\right) = \frac{\rho\left(\frac{4}{3}\pi r^3\right)}{\varepsilon_o}$$

Calculate the electric field at a point inside the sphere.



$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\varepsilon_o}$$

$$q_{enclosed} = \rho V_{enclosed} = \rho\left(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi a^3\right)$$

Calculate the electric field at a point outside the sphere.

$$q_{enclosed} = \rho V_{enclosed} = \rho\left(\frac{4}{3}\pi b^3 - \frac{4}{3}\pi a^3\right)$$

A conductor in **electrostatic equilibrium** has the following properties.

1. The electric field zero everywhere inside the conductor.

2. Any net charge on the conductor resides entirely on its surfaces.

3. The electric field just outside the conductor is perpendicular to its surface

and has a magnitude σ/ε_0 , σ is the surface charge density at that point.

4. On an irregularly shaped conductor, the surface charge density is greatest

where the radius of curvature of the surface is the smallest.