

Exercise Series No 2

Exercise 1

A student estimates their chances of passing their statistics exam at 65%, their chances of passing their chemistry exam at 80%, and their chances of passing both subjects at 50%.

- ❶ What is the probability that the student passes Statistics but not Chemistry?
- ❷ What is the probability that the student passes Chemistry but not Statistics?
- ❸ What is the probability that the student passes either Statistics or Chemistry?
- ❹ What is the probability that the student fails both Chemistry and Statistics?
- ❺ What is the probability that the student passes Statistics given that they passed Chemistry?

Exercise 2

Let the random experiment be: "a six-sided die is rolled, and the result is observed."

The following game is considered:

If the result is even, you win 2 DA.

If the result is 1, you win 3 DA.

If the result is 3 or 5, you lose 4 DA.

We define a random variable X that represents the gain in this game.

- ❶ Define the sample space Ω .
- ❷ Determine $X(\Omega)$.
- ❸ Determine the probability distribution of X .
- ❹ Determine the cumulative distribution function (CDF) of X .
- ❺ Calculate the expected value $E(X)$, the variance $V(X)$ and the standard deviation δ_x .

Exercise 3

We consider the function defined by

$$f(x) = \begin{cases} \frac{1}{2}(2-x) & \text{si } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

- ❶ Show that f is a probability density function of a random variable X .
- ❷ Determine its cumulative distribution function (CDF) .
- ❸ Calculate the expected value of X
- ❹ Calculate the variance and the standard deviation of X

Exercise 4

Let X be a random variable and the associated function is

$$f(x) = \begin{cases} ax(1-x) & \text{si } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- ❶ Determine a so that f is a probability density function.
- ❷ Determine its cumulative distribution function (CDF).
- ❸ Calculate $P(0 \leq X \leq \frac{1}{2})$