

1 Words, Languages & Grammars

Exercise 1

I. Let the word $x = ((abc)^R \cdot baca)^R$ (α^R denotes the mirror reflection of α)

Give the string that x is equal to.

What is the value of $|x|$, $|x|_a$, $|x|_b$, and $|x|_c$?

Give a proper prefix of x containing at least two letters 'c'.

II. Let the alphabet $A = \{a, b\}$

1) Given the words $u = aa$ and $v = bab$, write the words uv , $(uv)^2$, and u^3v .

2) State all the words of length 2 defined on A .

3) Let the sets be

$$E1 = \{uv / u \in A^+; v \in A^+\}$$

$$E2 = \{uv / u \in A^+; v \in A^*\}$$

$$E3 = \{uv / u \in A^*; v \in A^*\}$$

What do these sets correspond to?

Exercise 2

a) What is the number of distinct factors of lengths 3, 4, and 5 of the following words of length 20:

$$f = abaababaabaababaabab$$

$$g = aaababbbbaabbabaaaab$$

b) Provide all the factors of the word $abbbaaa$.

c) Provide the list of left factors (prefixes) of $abbaa$.

d) Provide the list of right factors (suffixes) $abcd$.

e) How many prefixes does a word of length n have?

f) How many distinct factors does the word a^n have?

g) How many distinct factors does the word $a^m b^n$ have?

Exercise 3

Describe LM in the following cases:

1. $L = \{a, bb, ccc\}$ and $M = \{d, ee, fff\}$,

2. $L = M = \{\epsilon, a, aa\}$,

3. $L = \{a, b\}^*$ and $M = \emptyset$,

4. $L = \{aa\}$ and $M = \{a, b\}^*$,

5. $L = \{aa, aaa\}$ and $M = \{a, b\}^*$,

6. $L = \{\epsilon, a, aa, aaa\}$ and $M = \{a, b\}^*$,

Exercise 4

Find the languages: $L_2.L_3$, $L_2.L_1$, $L_1.L_3$, $L_5 \cap L_1$, $L_6 \cup L_5$, $L_1.(L_2 \cap L_4)$, $L_1.(L_2 \cap L_3)$, $(L_1.L_2)^R$, $(L_1)^R.(L_2)^R$ for the following formal languages :

$$L_1 = \{a^i b^j, i \geq j \geq 1\}$$

$$L_2 = \{a, aa, \epsilon\}$$

$$L_3 = \{b, ba\}$$

$$\left| \begin{array}{l} L_4 = \{\epsilon\} \\ L_5 = \{a^i b^j c^k / i, k \geq 0\} \\ L_6 = \{a^i b^i / i \geq 1\} \end{array} \right.$$

Exercise 5

Donner le type de G et déterminer L(G) dans les grammaires suivantes :

- | | |
|--|--|
| 1) $G_1: S \rightarrow a \mid b \mid aSb$ | 6) $G_6: S \rightarrow bA ; A \rightarrow aA \mid \varepsilon$ |
| 2) $G_2: S \rightarrow \varepsilon \mid bBa, Ba \rightarrow baT, baT \rightarrow baaS$ | 7) $G_7: S \rightarrow aSc \mid A, A \rightarrow bA \mid b$ |
| 3) $G_3: S \rightarrow aSa \mid bSb \mid U, U \rightarrow 0U \mid \varepsilon$ | 8) $G_8: S \rightarrow aSbS \mid \varepsilon$ |
| 4) $G_4: S \rightarrow aU \mid c, U \rightarrow Sb \mid d$ | 9) $G_9: S \rightarrow aRbc \mid abc, R \rightarrow aRTb \mid aTb ; Tb \rightarrow bT ; Tc \rightarrow cc$ |
| 5) $G_5: S \rightarrow aA \mid bB ; A \rightarrow a \mid ab ; B \rightarrow b \mid cb$ | 10) $G_{10}: S \rightarrow aAb \mid \varepsilon, A \rightarrow aSb, Ab \rightarrow \varepsilon$ |

Exercise 6

For each of the following languages, provide examples of words contained in each language, and the grammars that generate them:

$$L_1 = \{w \in A^* \mid 2|w|_a = |w|_b\}$$

$$L_2 = \{a^{2n}b^{3n} \mid n \geq 2\}$$

$$L_3 = \{ab^n a \mid n \in \mathbb{N}\}$$

$$L_4 = \{m \in \{a,b\}^*\}$$

$$L_5 = \{m \in \{a,b\}^* \mid m = xaaa \text{ with } x \in \{a,b\}^*\}$$

$$L_6 = \{a^n b^n \mid n \geq 0\}$$

$$L_7 = \{mm^R \mid m \in \{a,b\}^*\} \text{ (mirror language)}$$

$$L_8 = \{a^n b^n c^p \mid n > 0, p > 0\}$$

$$L_9 = \{a^p b^q c^r \mid p=q \text{ or } q=r, q > 0, r > 0, p > 0\}$$

Exercise 7

Let $V_1 = \{a,b\}$ and $V_2 = \{c,d\}$

Provide a Type 3 grammar generating V_1^+

Provide a Type 3 grammar generating V_2^*

Deduce a Type 3 grammar that generates $V_1^+ V_2^*$

Exercise 8

We consider the grammar $G = (V, N, S, R)$ such that:

$V = \{a,b\}, N = \{S, T, U\}, R = \{S \rightarrow TU, T \rightarrow ST / a, U \rightarrow US / b\}$

1. What is the type of G? Justify.
2. Are the words a, ababa, ababab, ba derived by the grammar G?
3. Determine the language generated by G

Exercise 9

Determine the type of G as well as the language it generates L(G).

$G = (\{a,b,c\}, \{S,A\}, S, \{S \rightarrow aSc / A, A \rightarrow bA / b\})$