Exercise 1 A student estimates a 65% chance of passing his statistics course, an 80% chance of passing his chemistry course, and a 50% chance of passing both subjects.

   (a) What is the probability that the student passes in Statistics but not in Chemistry.?

   (b) What is the probability that the student passes in Chemistry but not in Statistics.?

   (c) What is the probability that the student passes in Statistics or Chemistry.?

   (d) What is the probability that the student passes neither Chemistry nor Statistics.?

   (e) What is the probability that the student passes in Statistics given that he passed in Chemistry.?

Exercise 2 Consider the random experiment "rolling a six-sided die and observing the result."

We consider the following game:

- If the result is even, we win 2 DA.

- If the result is 1, we win 3 DA.

- If the result is 3 or 5, we lose 4 DA.

We will define a random variable X that gives the gain in this game.

   (a) Give the fundamental set Ω.

   (b) Determine $X(\Omega)$.

   (c) Determine the probability law of X.

   (d) Determine the distribution function of X.

   (e) Calculate the expected value $E(X)$, variance $V(X)$, and standard deviation δ_X.

Exercise 3 We perform the following random experiment: we roll a pair of fair dice, numbered from 1 to 6, and define the random variable X by:

$$X : \Omega \to \mathbb{R}$$

$$(a, b) \mapsto X(a, b) = \max(a, b)$$

(a) Give the fundamental set Ω.

(b) Determine $X(\Omega)$.

(c) Determine the probability law of X.

(d) Determine the distribution function of X.

(e) Calculate the expected value $E(X)$, variance $V(X)$, and standard deviation δ_X.

Exercise 4 Consider the function defined by:

$$f(x) = \begin{cases} \frac{1}{2}(2-x) & \text{if } 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that f is a probability density function of a random variable X.

(b) Determine its distribution function.

(c) Calculate the expected value of X.

(d) Calculate the variance and standard deviation of X.

Exercise 5 Let X be a random variable with the associated function:

$$f(x) = \begin{cases} ax(1-x) & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Determine a such that f is a probability density function.

(b) Determine its distribution function.

(c) Calculate $P(0 \le X \le \frac{1}{2})$.