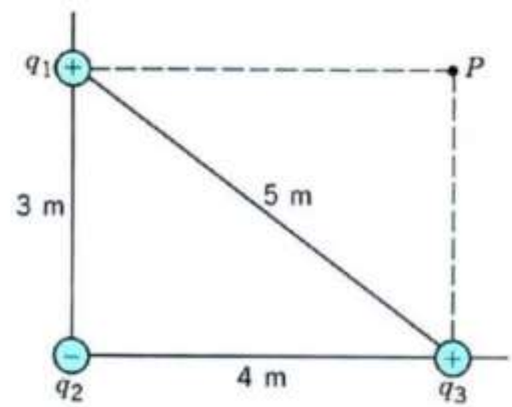
Three point charge, q1=1uC, q2=-2uC, and q3=3uC are fixed at the positions shown in Fig.25.13a. (a) What is the potential at point P at the corner of the rectangle? (b) What is the total potential energy of q1, q2, and q3?(c) How much work would be needed to bring a charge q4=2.5uC from infinity and to place it at P?

Solution:

$(a)\ V_p = V_1 + V_2 + V_3 = 7.65\times10^3\ V$

$(b)\ U = U_{12} + U_{13} + U_{23} = -1.41\times10^{-2}\ J$

$(c)\ W_{ext} = q_4 V_p = 0.19\ J$



Choosing V=0 at r=∞.

Continuous charge distributions

Q1. What is charge density?

The charge density is the measure of electric charge per unit length, per unit area of a surface, or per unit volume of a body or field. It tells us how much charge is stored in a particular field. Charge density can be determined in terms of volume, area, or length.

Q2. What is continuous charge distribution?

In a continuous charge distribution, all the charges are closely bound together i.e. having very less space between them. But this closely bound system doesn't mean that the electric charge is uninterrupted. It clears that the distribution of separate charges is continuous, having a minor space between them.

Q3. What are the types of continuous charged distributions?

Types of continuous charged distributions :

1. Linear Charge Distribution

2.Surface Charge Distribution

3.Volume Distribution of charge

Q4. What is symmetrical charge distribution?

A symmetrical charge distribution is when there is a couple of geometrical transformations such that these transformations do not affect any physical change.

Electric Field – Continuous Charge Distribution

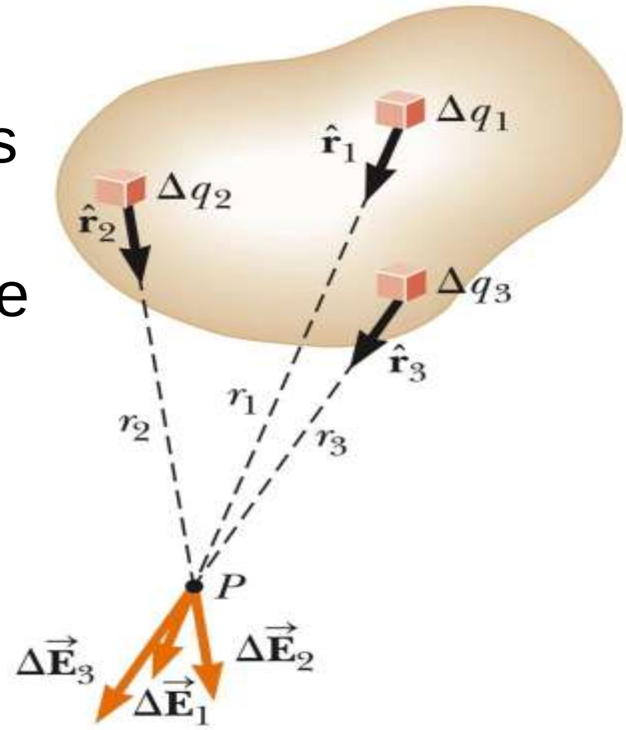
Procedure:

- Divide the charge distribution into small elements, each of which contains Δq.
- Calculate the electric field due to one of these elements at point *P*.
- Evaluate the total field by summing the contributions of all the charge elements.
- For the individual charge elements

$$\Delta \vec{\mathbf{E}} = k_e \frac{\Delta q}{r^2} \hat{\mathbf{r}}$$

- Because the charge distribution is continuous

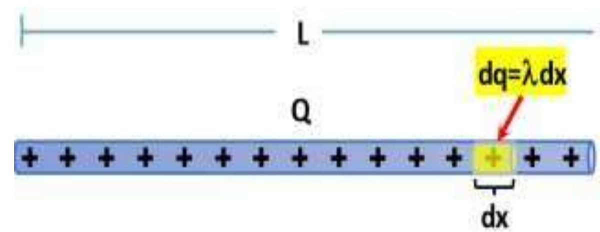
$$\vec{\mathbf{E}} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_i^2} \hat{\mathbf{r}}_i = k_e \int \frac{dq}{r^2} \hat{\mathbf{r}}$$



Continuous charge distributions

General charge distributions:

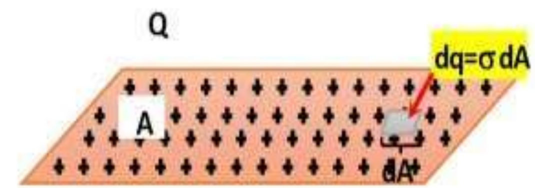
1- Linear charge distribution: $\lambda = \frac{Q}{L}$



- **Linear charge density:** λ (charge per unit length).
- **Differential charge element:** $dq = \lambda dx$ (where dx is a

differential element of length as shown) .

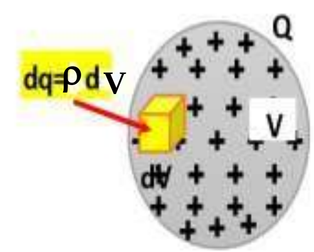
2- Surface charge distribution: $\sigma = \frac{Q}{A}$



- **Surface charge density: σ** (charge per unit surface).
- **Differential charge element:** $dq = \sigma dA$

(where dA is a differential element of surface area as shown).

3- Volume charge distribution: $\rho = \frac{Q}{V}$



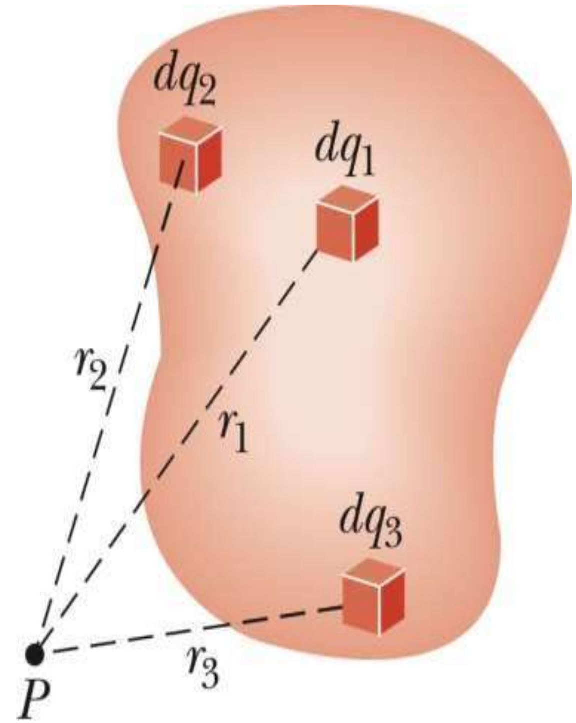
- **Volume charge density: ρ** (chargper unit Volume)
- **Differential charge element:** $dq = \rho dV$

(where dV is a differential element of volume as shown).

Electric Potential for a Continuous Charge Distribution

- Method 1: The charge distribution is known.
- Consider a small charge element
dq
- Treat it as a point charge.
- The potential at some point due to this charge element is

$$dV = k_e \frac{dq}{r}$$



Field due to arc of charge

Example 1:

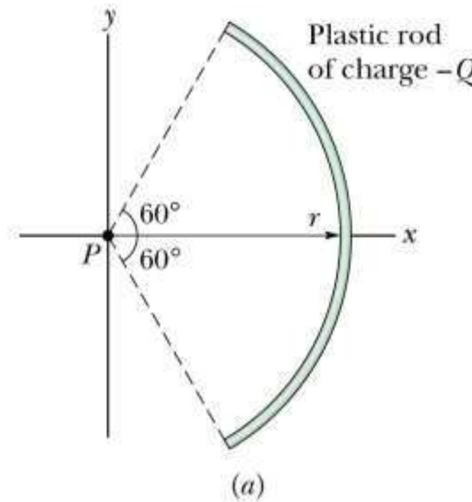
$$E_y = \int_{-L/2}^{L/2} dE_y = 0$$

$dE_x = k\, dq \cos\theta / r^2$

$dE_x = k\, \lambda\, ds \cos\theta / r^2$

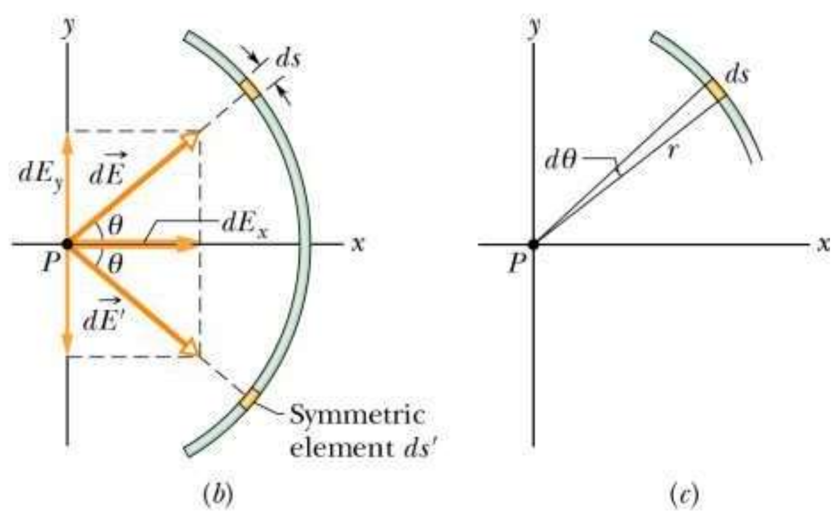
$$E_x = k\lambda \int_{-L/2}^{L/2} r\, d\theta \cos\theta / r^2 = k\lambda / r \int_{-\theta_0}^{\theta_0} d\theta \cos\theta$$

$$E_x = \frac{2k\lambda}{r} \sin\theta_0$$

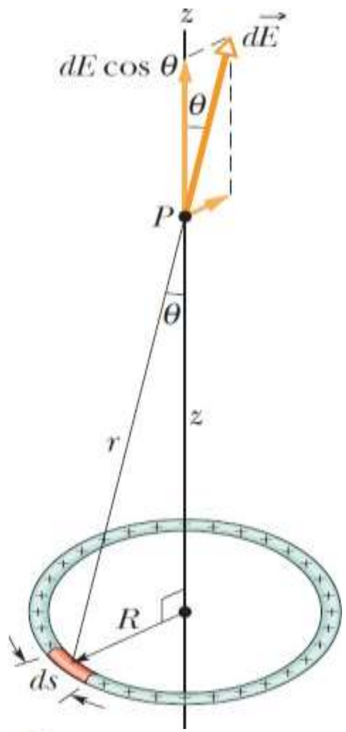


$s = r\,\theta$

$ds = r\, d\theta$



The Electric Field Due to a Line of Charge (continuous distributions)



Example 2:

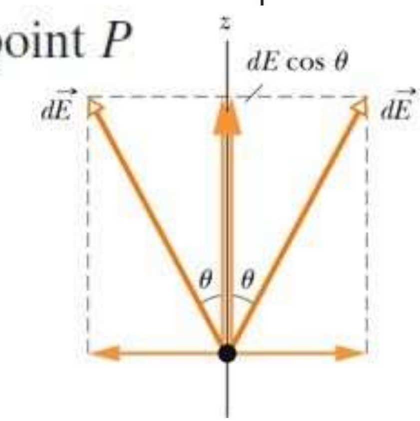
In the Figure (right), shows a thin ring of radius *R* with a uniform positive linear charge density λ around its circumference. We may imagine the ring to be made of plastic or some other insulator, so that charges can be regarded as fixed in place. What is the electric field at point *P*, a distance *z* from the plane of the ring along its central axis?

Let *ds* be the (arc) length of any differential element of the ring. Since λ is the charge per unit (arc) length, the element has a charge of magnitude

$$dq = \lambda\, ds$$

This differential charge sets up a differential electric field $d\vec{E}$ at point *P*

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{r^2}. = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{(z^2 + R^2)}.$$



The Electric Field Due to a Line of Charge (continuous distributions)

Thus, the perpendicular components cancel and we need not consider them further. This leaves the parallel components; they all have the same direction, so the net electric field at ***P*** is their sum

$$\cos\theta = \frac{z}{r} = \frac{z}{(z^2 + R^2)^{1/2}}. \qquad dE\cos\theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}\,ds.$$

To add the parallel components $dE\cos\theta$ produced by all the elements, we integrate this equation around the circumference of the ring, from s 0 to s $2\pi R$.

$$E = \int dE\cos\theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}\int_0^{2\pi R} ds = \frac{z\lambda(2\pi R)}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}.$$

$$\boxed{E = \frac{qz}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}}$$

If $z \gg R$ → $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{z^2}$

Indefinite Integrals:

$$\int \frac{a^2}{(x^2 + a^2)^{3/2}}dx = \frac{x}{\sqrt{x^2 + a^2}}$$

$$\int \frac{x}{(x^2 + a^2)^{3/2}}dx = \frac{-1}{\sqrt{x^2 + a^2}}$$

The Electric Field Due to a Charged Disk

Example 3:

The Figure shows a circular plastic disk of radius *R* that has a positive surface charge of uniform density σ on its upper surface . What is the electric field at point *P*, a distance *z* from the disk along its central axis?

We divide the disk into concentric flat rings and then to calculate the electric field at point *P* by adding up (that is, by integrating) the contributions of all the rings. Figure shows one such ring, with radius *r* and radial width *dr*. Since s is the charge per unit area, the charge on the ring is $dq = \sigma\, dA = \sigma(2\pi r\, dr)$,

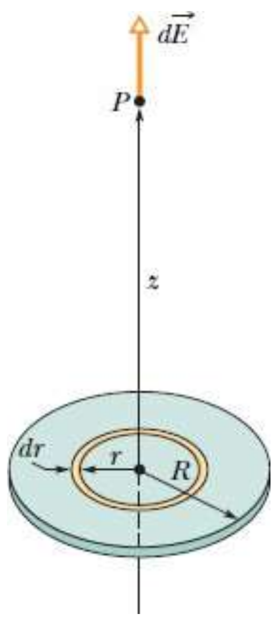
where dA is the differential area of the ring.

From The Electric Field Due to a Line of Charge we have

$$E = \frac{qz}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}} \quad \text{(charged ring).}$$

Then we get

$$dE = \frac{z\sigma 2\pi r\, dr}{4\pi\varepsilon_0(z^2 + r^2)^{3/2}},$$



Which we my be written as:

$$dE = \frac{\sigma z}{4\varepsilon_0} \frac{2r\,dr}{(z^2 + r^2)^{3/2}}. \quad \Rightarrow \quad E = \int dE = \frac{\sigma z}{4\varepsilon_0} \int_0^R (z^2 + r^2)^{-3/2}(2r)\,dr.$$

To solve this integral, we cast it in the form $\int X^m\,dX$ by setting $X = (z^2 + r^2)$, $m = -\frac{3}{2}$, and $dX = (2r)\,dr$. For the recast integral we have

$$\int X^m\,dX = \frac{X^{m+1}}{m+1},$$

$$E = \frac{\sigma z}{4\varepsilon_0} \left[\frac{(z^2 + r^2)^{-1/2}}{-\frac{1}{2}} \right]_0^R.$$

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right) \qquad \text{(charged disk)}$$

as the magnitude of the electric field produced by a flat, circular, charged disk at points on its central axis. (In carrying out the integration, we assumed that z ⌟ (.0

If we let $R \rightarrow \infty$ while keeping z finite, the second term in the parentheses in approaches zero, and this equation reduces to

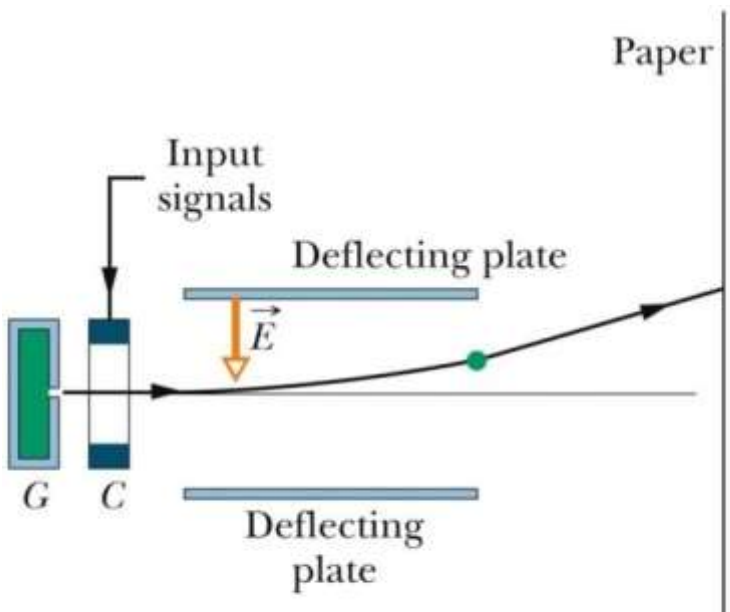
$$E = \frac{\sigma}{2\varepsilon_0} \qquad \text{(infinite sheet).}$$

A Point Charge in an Electric Field

If a particle with charge *q* is placed in an external electric field ***E***, an electrostatic force ***F*** acts on the particle:

$$\vec{F} = q\vec{E},$$

The electrostatic force $\vec{F}$ acting on a charged particle located in an external electric field $\vec{E}$ has the direction of $\vec{E}$ if the charge q of the particle is positive and has the opposite direction if q is negative.



Ink-jet printer. Drops shot from generator *G* receive a charge in charging unit *C*. An input signal from a computer controls the charge and thus the effect of field ***E*** on where the drop lands on the paper.

Problem

- The nucleus of a plutonium-239 atom contains 94 protons. Assume that the nucleus is a sphere with radius 6.64 fm and with the charge of the protons uniformly spread through the sphere. At the nucleus surface, what are the (a) magnitude and (b) direction (radially inward or outward) of the electric field produced by the protons?

Solution: Since the charge is uniformly distributed throughout a sphere, the electric field at the surface is exactly the same as it would be if the charge were all at the center. That is, the magnitude of the field is

$$E = \frac{q}{4\pi\varepsilon_0 R^2}$$

where *q* is the magnitude of the total charge and *R* is the sphere radius.
(a) The magnitude of the total charge is *Ze*, so

$$E = \frac{Ze}{4\pi\varepsilon_0 R^2} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(94)(1.60\times10^{-19}\ \mathrm{C})}{(6.64\times10^{-15}\ \mathrm{m})^2} = 3.07\times10^{21}\ \mathrm{N/C}.$$

(b) The field is normal to the surface and since the charge is positive, it points outward from the surface.

Problem

- What is the magnitude of a point charge that would create an electric field of 1.00 N/C at points 1.00 m away?

We find the charge magnitude $|q|$ from $E = |q|/4\pi\varepsilon_0 r^2$:

$$q = 4\pi\varepsilon_0 E r^2 = \frac{(1.00\,\text{N/C})(1.00\,\text{m})^2}{8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2} = 1.11\times10^{-10}\,\text{C}.$$