# The Electric Field

A lot of different fields are used in science and engineering. For example, a ***temperature field*** for an auditorium is the distribution of temperatures we would find by measuring the temperature at many points within the auditorium. Similarly, we could define a ***pressure field*** in a swimming pool. Such fields are examples of *scalar fields* because temperature and pressure are scalar quantities, having only magnitudes and not directions.

In contrast, an electric field is a ***vector field*** because it is responsible for conveying the information for a force, which involves both magnitude and direction. This field consists of a distribution of electric field vectors , one for each point in the space around a charged object.

Electric field strength (E) is the amount of electrostatic force observed per unit charge.

# Electric Fields

You  should be able to.

**01-**Identify that at every point in the space surrounding a charged particle, the particle sets up an electric field, which is a vector quantity and thus has both magnitude and direction.

**02-**Identify how an electric field can be used to explain how a charged particle can exert an electrostatic force ***F*** : on a second charged particle even though there is no contact between the particles.

**03-**Explain how a small positive test charge is used (in principle) to measure the electric field at any given point.

**04-**Explain electric field lines, including where they originate and terminate and what their spacing represents.

In principle, we can define at some point near the **charged object**, such as point ***P*** in Fig.*a*, with this procedure: At ***P***, we place a particle with a small positive charge $q_0$, called a ***test charge*** because we use it to test the field. (We want the charge to be small so that it does not disturb the object's charge distribution.).We then measure the electrostatic force that acts on the test charge.

The electric field at that point is then

$$\vec{E} = \frac{\vec{F}}{q_0} \quad \text{(electric field).}$$

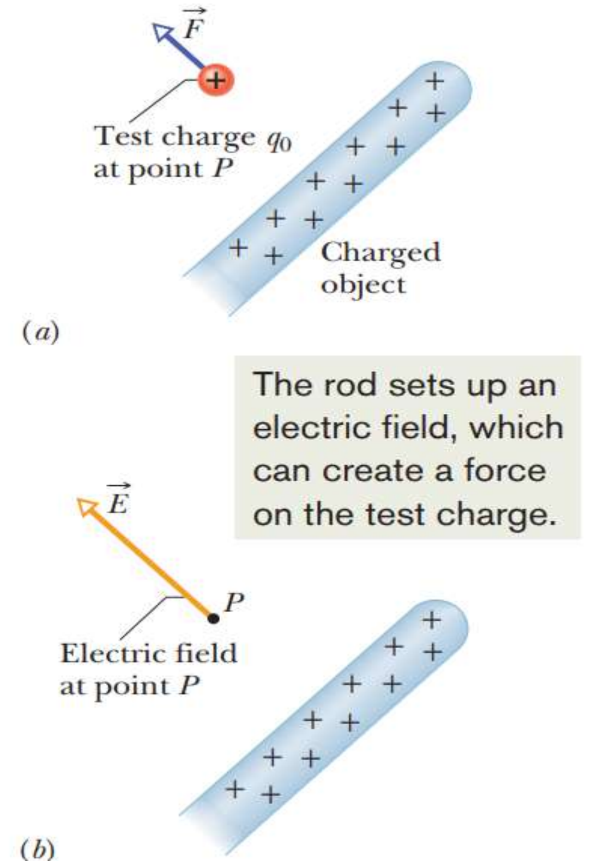



**Figure** (a) A positive test charge $q_0$ placed at point *P* near a charged object. An electrostatic force acts on the test charge. (*b*) The electric field at point *P* produced by the charged object.

- Because the test charge is positive, the two vectors in Eq. are in the same direction, so the direction of is the direction $\vec{F}$ we measure fo $\vec{E}$ The magnitude of at point ***P*** is $F/q_0$.
- The units in SI: Newton per Coulomb (N/C=V/m

# Electric Field Lines

Figure gives an example in which a sphere is uniformly covered with negative charge. If we place a **positive test charge** at any point near the sphere (Fig. *a*), we find that an electrostatic force pulls on it toward the center of the sphere. Thus at every point around the sphere, an electric field vector points radially inward toward the sphere. We can represent this electric field with electric field lines as in Fig. *b*. At any point, such as the one shown, the direction of the field line through the point matches the direction of the electric vector at that point electric field lines as in figure b.

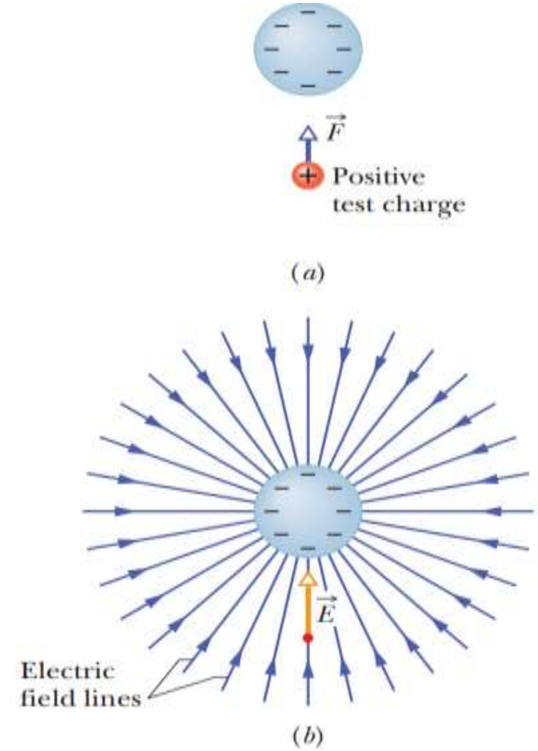



**Figure** (*a*) The electrostatic force acting on a positive test charge near a sphere of uniform negative charge. (*b*) The electric field vector
at the location of the test charge, and the electric field lines in the space near the sphere. The field lines extend *toward* the negatively charged sphere. (They originate on distant positive charges.)

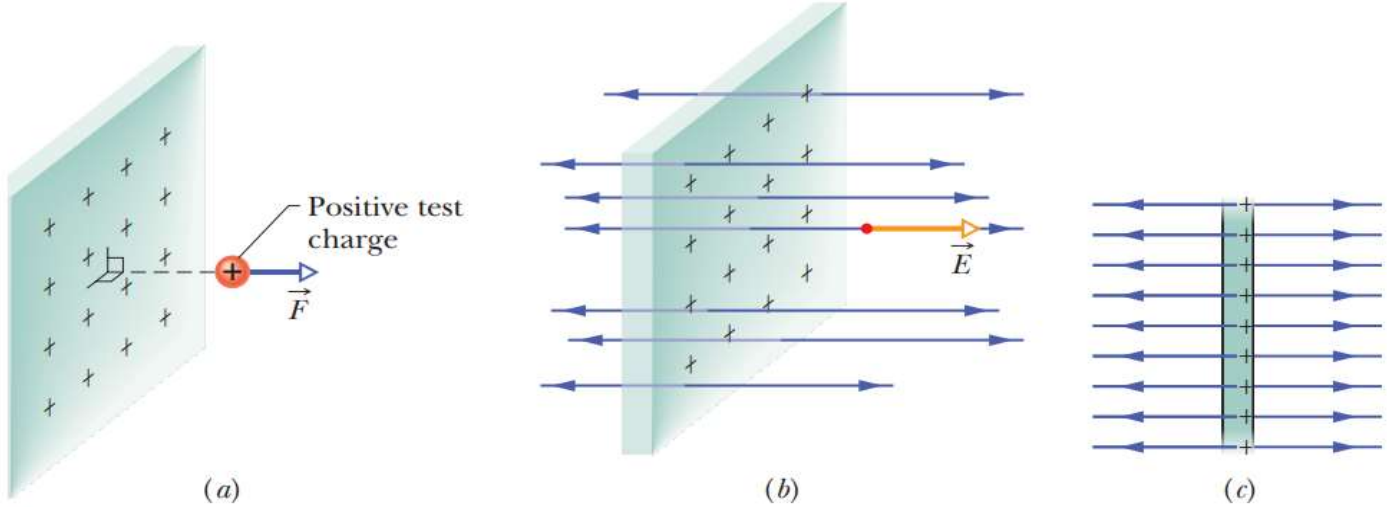



**Figure** (*a*) The force on a positive test charge near a very large, nonconducting sheet with uniform positive charge on one side. (*b*) The electric field vector at the test charge's location, and the nearby electric field lines, extending away from the sheet. (*c*) Side view.

- Electric field lines **extend away** from positive charge (where they originate) **and toward** negative charge (where they terminate).

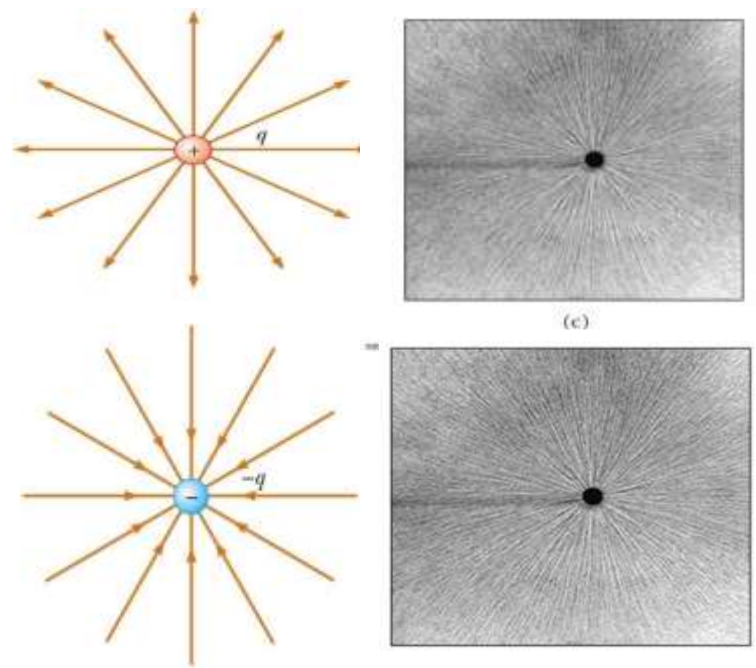
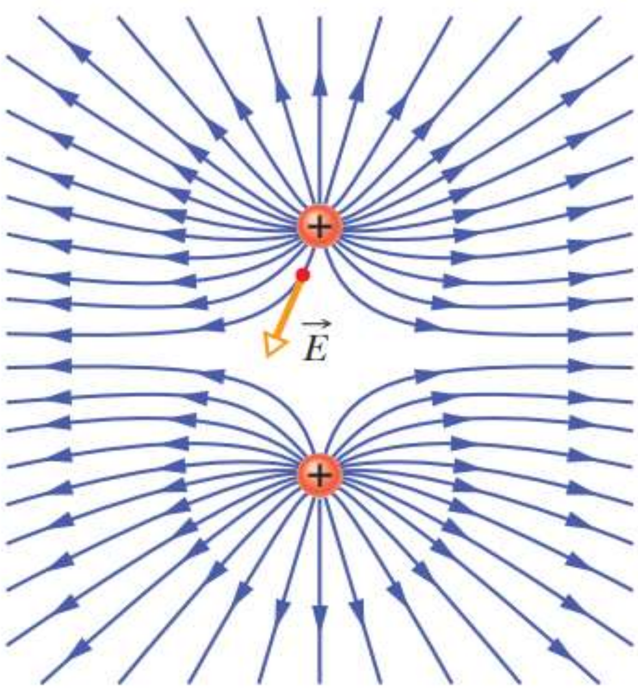
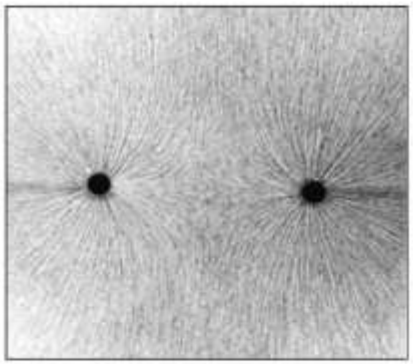


Figure  shows the field lines for **two particles with equal positive charges**. Now the field lines are curved, but the rules still hold: (1) the electric field vector at any given point must **be tangent to the field line at that point** and in the same direction, as shown for one vector, and (2) a closer spacing means a larger field magnitude. To imagine the full three-dimensional pattern of field lines around the particles, mentally rotate the pattern in Figure around the axis of symmetry, which is a vertical line through both particles.



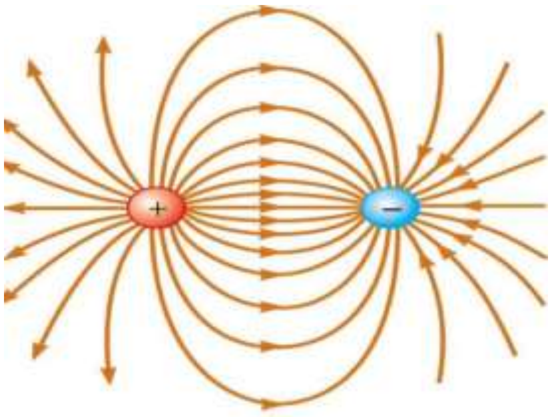
(b)

**Figure** Field lines for two particles with equal positive charge. Doesn't the pattern itself suggest that the particles repel each other?

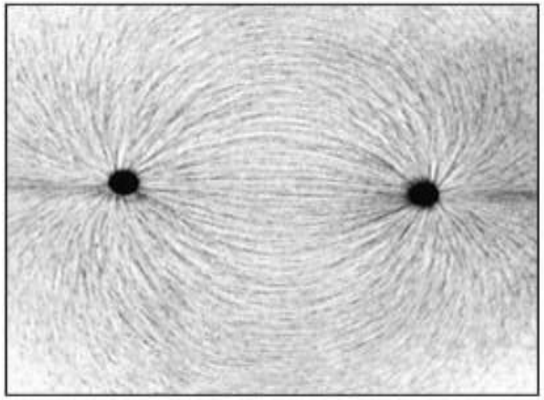
# Electric Field Line Patterns

- An electric *dipole* consists of two equal and opposite charges
- The charges are equal and opposite
- The number of field lines leaving the positive charge

equals the number of lines terminating on the negative charge



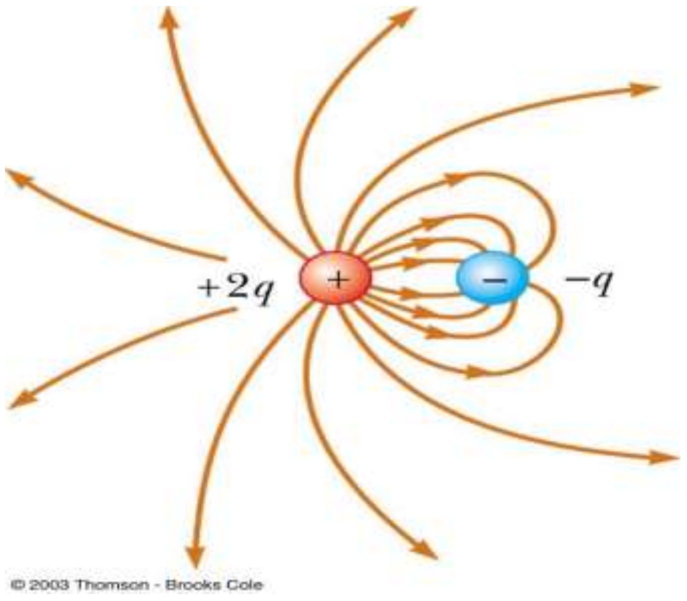

(a)



(b)

# Electric Field Patterns

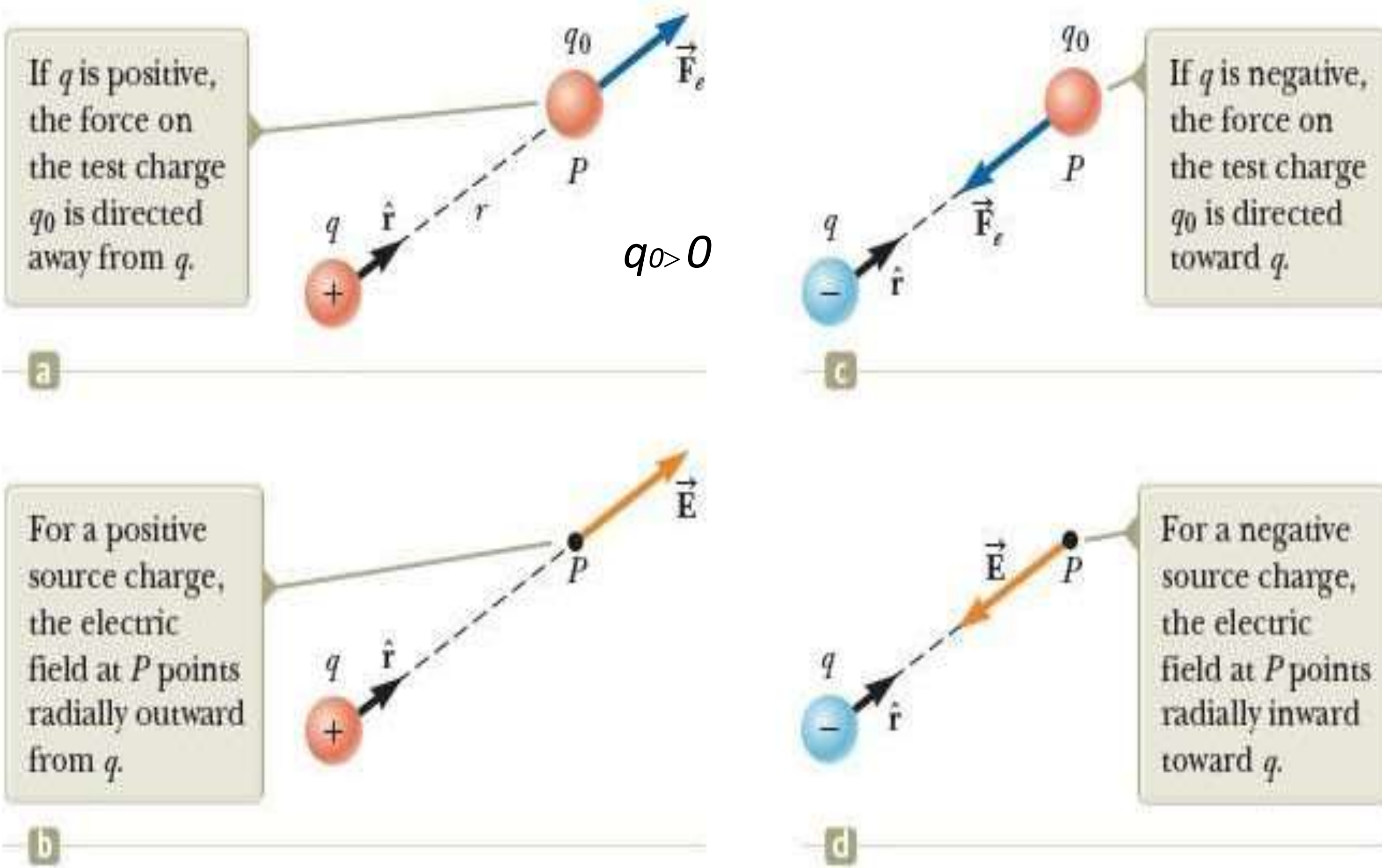
- Unequal and unlike point charges
- Note that two lines leave the +2q charge for each line that terminates on -q



# Rules for Drawing Electric Field Lines

- The lines must begin on positive charges and terminate on negative charges.
- In the case of an excess of charge, some lines will begin or end infinitely far away.
- The number of lines drawn leaving a positive charge or ending on a negative charge is proportional to the magnitude of the charge.
- No two field lines can cross each other.

# More About Electric Field Direction




- If $q$ is positive, the force and the field are in the same direction
- If $q$ is negative, the force and the field are in opposite directions

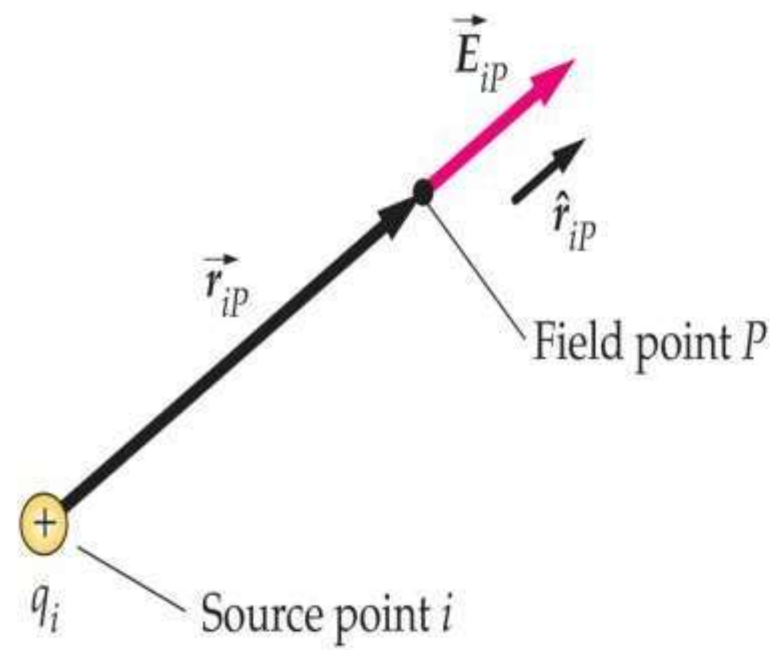
# The Electric Field Due to a Point Charge

- To find the electric field due to a charged particle (often called a ***point charge***), we place a positive test charge at any point near the particle, at distance ***r***. From Coulomb's law (Eq.),the force on the test charge due to the particle with charge $q_i$ is

$$\vec{\mathbf{F}}_{i0} = k_e \frac{q_i q_o}{r_{ip}^2} \hat{\mathbf{r}}_{ip}$$

- Then, the electric field will be

$$\vec{\mathbf{E}}_p = \frac{\vec{\mathbf{F}}_{i0}}{q_o} = k_e \frac{q_i}{r_{ip}^2} \hat{\mathbf{r}}_{ip}$$



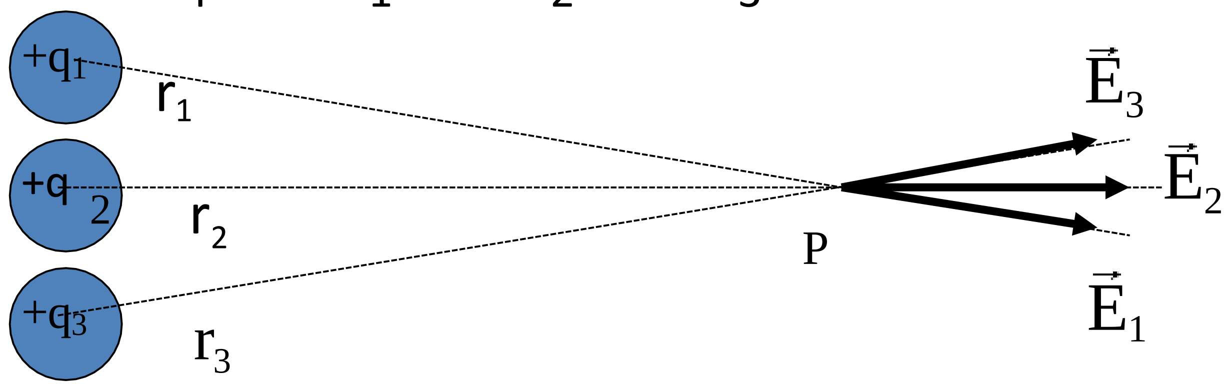
# Superposition with Electric Fields

- At any point *P*, the total electric field due to a  group of source charges equals the vector sumof the electric fields of all the charges.

$$\vec{\mathbf{E}} = k_e \sum_i \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i$$

# Superposition of Fields

$$\vec{E}_P = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots$$




$$\vec{E}_P = \frac{kq_1}{r_1^2}\hat{r}_{10} + \frac{kq_2}{r_2^2}\hat{r}_2 + \frac{kq_3}{r_3^2}\hat{r}_3 + \dots$$

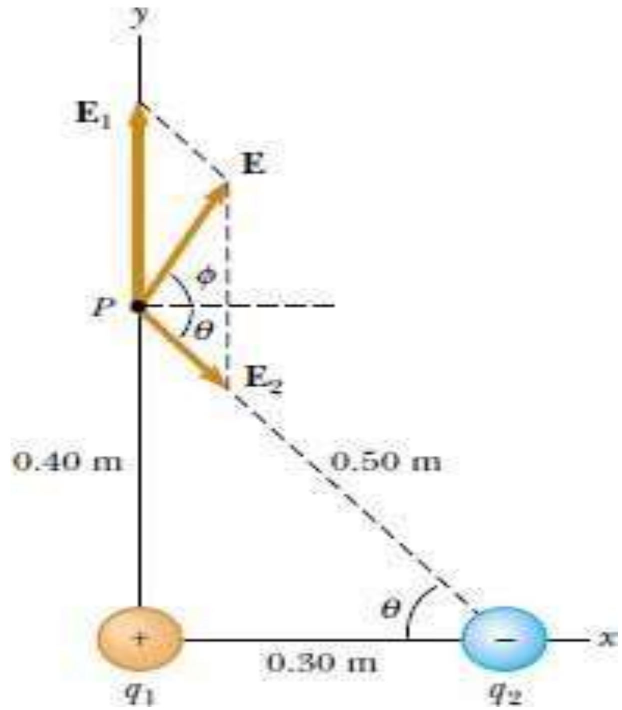
$$\vec{E}_P = k\left(\frac{q_1}{r_1^2}\hat{r}_1 + \frac{q_2}{r_2^2}\hat{r}_{20} + \frac{q_3}{r_3^2}\hat{r}_3 + \dots\right) = k\sum_i^N \frac{q_i}{r^2}\hat{r}_i$$

# Example 3. Electric Field Due to Two Charges

- Find the electric field at the point ***P***, which has coordinates (0,0,40)m.

  $q_1$= 7μC, $q_2$ = -5 μC.
  - Remember, the fields add as vectors
  - The direction of the individual fields is the direction of the force on a positive test.

- Find the electric field due to $q_1$, $\vec{E}_1$
- Find the electric field due to $q_2$, $\vec{E}_2$
- The total electric field due to two charges $q1$,$q2$.
- $\vec{E} = \vec{E}_1 + \vec{E}_2$



# Electric Potential

- A charge $q$ moving in a constant electric field $E$ experiences a force $F = q.E$ from that field.

Also, as we know from our study of work and energy**, the work done on the charge by the field as it moves from point $r_1$ to $r_2$ is;**

$$\mathbf{W} = \int_{r1}^{r2} \boldsymbol{F}.\, dr = q \int_{r1}^{r2} \boldsymbol{E}.dr$$

- The electric force is conservative and it allows us to calculate an **electric potential energy**, which as usual we will denote by ***U*** and **the change in potential energy** is the negative of the work done by the electric force:

$$\Delta \mathbf{U}_{elec} = -\mathbf{W} = -q \int_{r1}^{r2} \boldsymbol{E}.dr$$

- We usually want to discuss the potential energy of a charge at ***a particular point***, that is, we would like a function $U(r)$.
- Usually we will make the choice that the potential energy is zero when the charge is infinitely far away: $U(\infty) = 0$.

# Potential Difference $\Delta V$

- Potential energy difference per unit charge:

$$\Delta V = \frac{\Delta U}{q_0} = -\int_{r1}^{r2} E \,.dr$$

- Potential of a Point Charge and Groups of Points Charges

The potential due to a point charge *q* is :

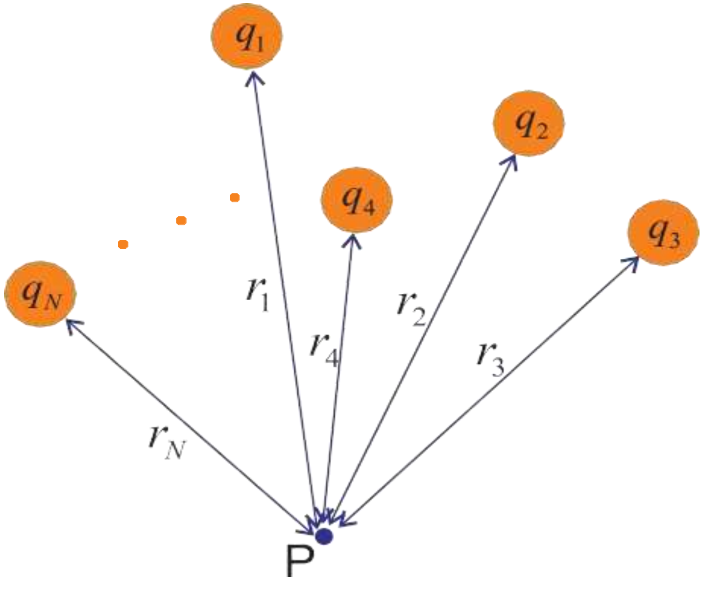
$$V(r) = K\frac{q}{r} = \frac{1}{4\pi\varepsilon 0} \cdot \frac{q}{r}$$

• Similarly, we take $V(r = \infty) = 0$.

$$V(r) = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

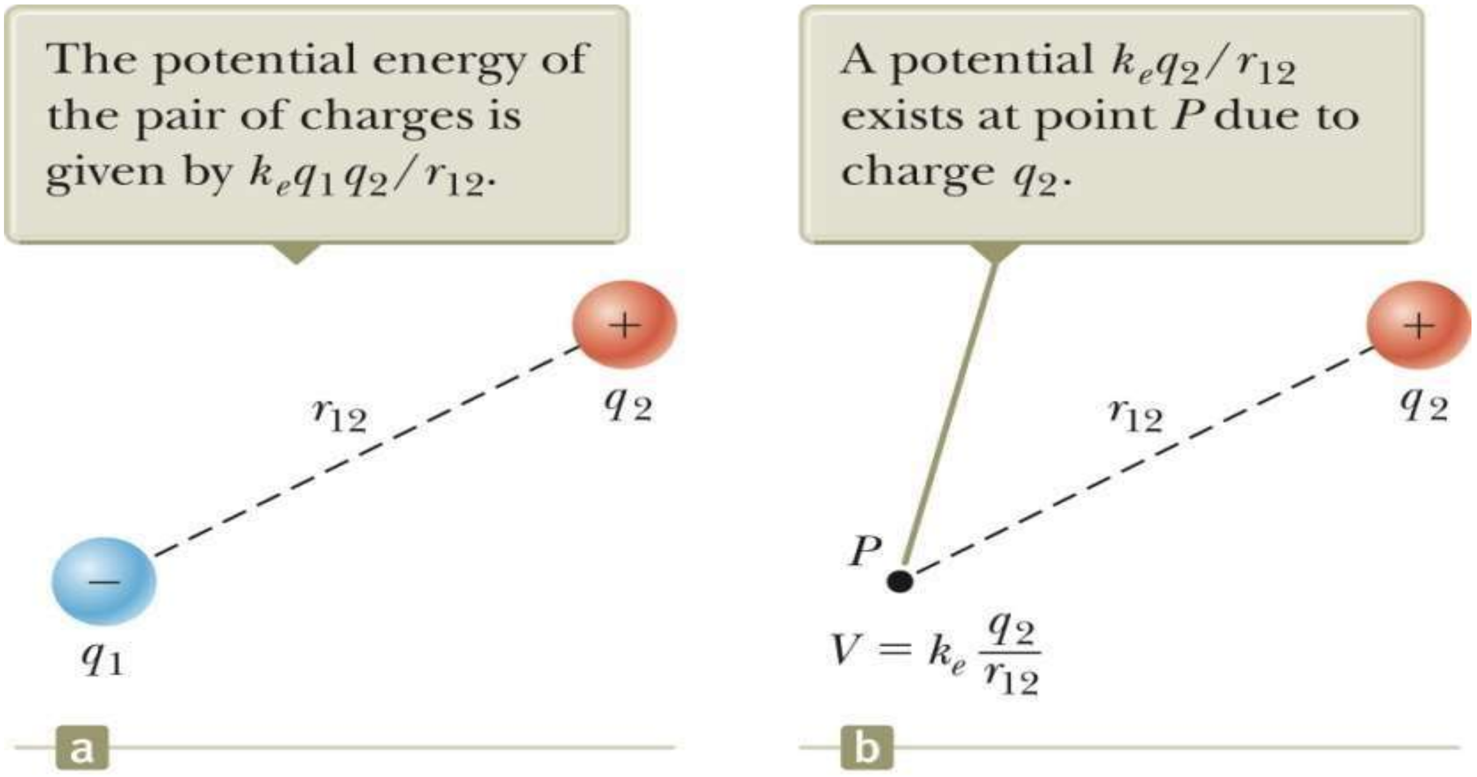
- Total potential at point ***P*** due to ***N*** charges : The sum is the algebraic sum
- $V=0,\ r = \infty$
- $V = V_1 + V_2 + \cdots + V_N$ (The superposition principle)
- $V = q_1/4\pi\epsilon_0 r_1 + q_2/4\pi\epsilon_0 r_2 + \ldots + q_N/4\pi\epsilon_0 r_N$

$$V(r) = \sum_{i=1}^{i=N} \frac{1}{4\pi\varepsilon_0} \frac{q_i}{r_i}$$



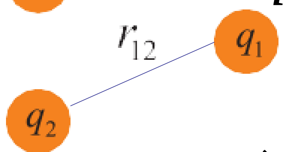
# Note: It is the difference in potential energy that is important.

- Reference point: $U(r = \infty) = 0$
- If $q_1$, $q_2$ same sign, then U (r) > 0 for all r, work must be done to bring the charges together.
- If $q_1$, $q_2$ opposite sign, then U (r) < 0 for all r, work is done to keep the charges apart.
- $U(r) = q_1 q_2 / 4\pi\epsilon_0 r_{12}$



- Potential Energy of A System of Charges

- Example 04: P.E. of 3 charges $q_1$, $q_2$, $q_3$
- **Start:** $q_1$, $q_2$, $q_3$ all at $r = \infty$, $U = 0$
- **Step1:** 

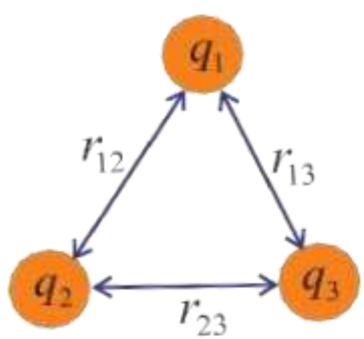
Move $q_1$ from $\infty$ to its position $\Rightarrow U = 0$
- **Step2:** 

Move $q_2$ from $\infty$ to new position

$$\Rightarrow U = q_1 q_2 / 4\pi\epsilon_0 r_{12}$$

- **Step3:** Move $q_3$ from $\infty$ to new position $\Rightarrow$ Total P.E.

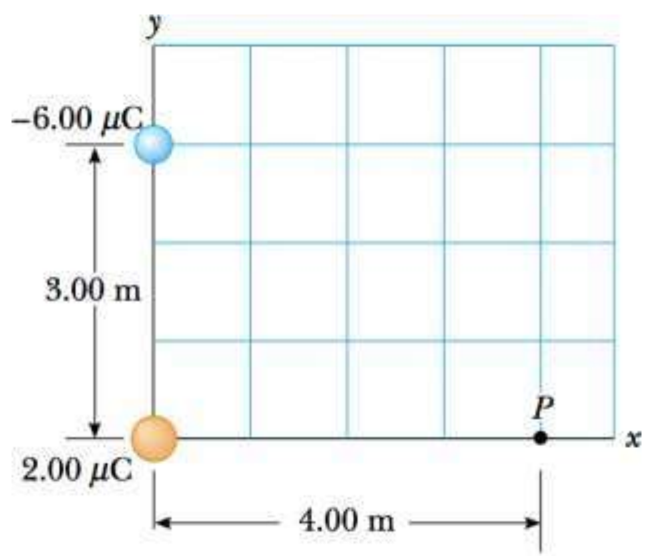
$$U = q_1 q_2 / 4\pi\epsilon_0 r_{12} + q_1 q_3 / 4\pi\epsilon_0 r_{13} + q_2 q_3 / 4\pi\epsilon_0 r_{23}$$



# Example 5: The Electric Potential Due to Two Point Charges

A charge $q_1$ =2μC is located at the origin, and a charge $q_2$ = -6 μC is located at (0,3.00) m, as shown in Figure.

➢ **Find** the total electric potenial due to these charges at the point ***P***, whose coordinates are (4.00, 0) m.




(a)

## Example 6: The Electric Potential Due to Two Point Charges

**B) Find** the change in potential energy of the system pf two charges plus

a charge $q_3$= 2μC as the latter charge moves from infinity to point ***P***.