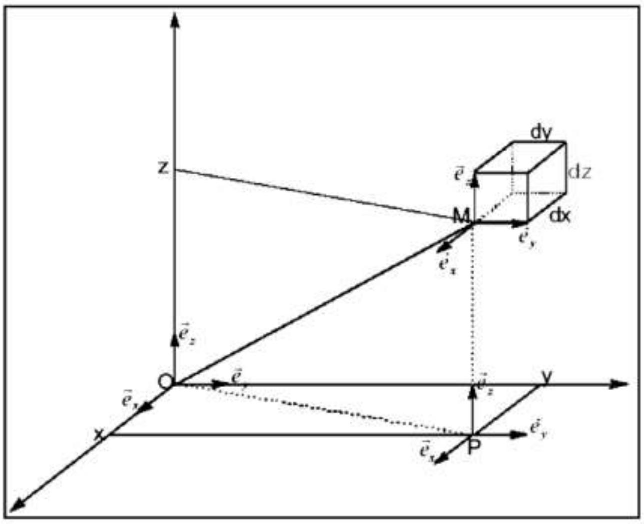
Chapter: Math reminders

In this chapter, we will review the main mathematical concepts we will need to solve the various exercises in this chapter.

1. Elément of length, surface, volume

1.1- Cartesian coordinates

A material point M is assumed to move in a Cartesian coordinate system with origin O. The point M in space is identified by its Cartesian coordinates x, y and z:



'شعاع الموضع' $\overrightarrow{oM} = x\,\vec{i} + y\vec{j} + z\vec{k}$ With $(\vec{\imath}, \vec{\jmath}, k)$ are the unit vectors.

عندما تتغير احداثيات النقطة M بكميات متناهية الصغر dx, dy, dz تنتقل النقطة انتقال عنصري

When the coordinates of point M change infinitesimally: dx, dy, dz the point M undergoes an elementary translation

- **Length element** النتقال العنصري : $d\overrightarrow{0M} = dx\,\vec{i} + dy\vec{j} + dz\vec{k}$
- و منه طول النتقال العنصري dL=$\|\overrightarrow{doM}\| = \sqrt{(dx)^2 + (dy)^2 + (dz)^2}$
- **Surface element** المساحة العنصرية **:**

In the **(xOy)** plane, the surface element generated by the displacement from **M** to **M'** we express the differential surface by calculating the area of a rectangle of length and width dy

$(\perp \boldsymbol{i})$: $\mathbf{dS_x = dy.dz}$

$(\perp \boldsymbol{j})$: $\mathbf{dS_y = dx.dz}$

$(\perp \boldsymbol{k}) : \mathbf{dS_z} = \mathbf{dx.dy}$

1. Infinitesimal volume element

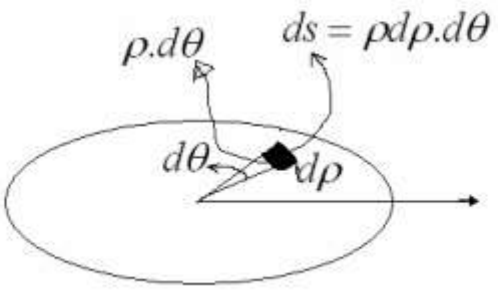
In 3D space, the infinitesimal volume $\boldsymbol{dV}$ generated by the displacement of point **M** is a cube of height dz; $\boldsymbol{dV = dx.dy.dz}$

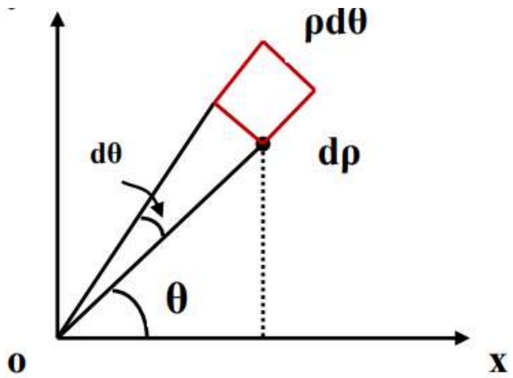
1.2- <u>Polar coordinates$(\boldsymbol{\rho}, \boldsymbol{\theta})$</u>

We denote each point **M** in the plane by its distance from the principle: **ρ** and the angle **θ** between the position ray $\overrightarrow{\boldsymbol{oM}}$ and the axis **(ox)**.

$$\overrightarrow{\boldsymbol{OM}} = \boldsymbol{\rho}\overrightarrow{\boldsymbol{u_\rho}} \quad ; \quad \boldsymbol{\rho} = \|\overrightarrow{\boldsymbol{OM}}\| \quad ; \quad \boldsymbol{\theta} = (\overrightarrow{\boldsymbol{ox}} ; \overrightarrow{\boldsymbol{oM}})$$

Knowing that: $x = \rho cos\theta$; $y = \rho sin\theta$; $\rho = \sqrt{x^2 + y^2}$; $tg\theta = \frac{y}{x} \Rightarrow \theta = arctg\frac{y}{x}$.





Polar coordinate change domain:$(\boldsymbol{0 \leq \rho \leq R; 0 \leq \theta \leq 2\pi})$

When the coordinates of the point M change infinitesimally: **dθ**, **dρ**, the point **M** undergoes an elementary translation:

The length element: $\overrightarrow{\boldsymbol{d0M}}$: $\overrightarrow{doM} = d\rho\overrightarrow{u_\rho} + \rho d\theta\overrightarrow{u_\theta}$

L'élément de surface : $\boldsymbol{dS = \rho.d\rho.d\theta}$

The area of a circle of radius R: $S = \iint dS = \iint \rho d\rho d\theta = \int_0^R \rho d\rho . \int_0^R d\theta = \frac{\rho^2}{2}\Big]_0^{2\pi} \theta]_0^R = \pi R^2$

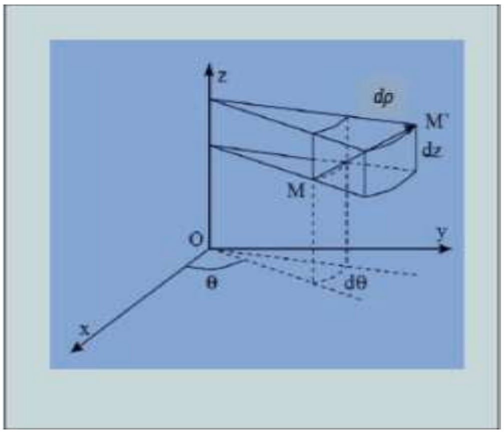
1.3- <u>Cylindrical coordinates (ρ, θ, Z)</u>

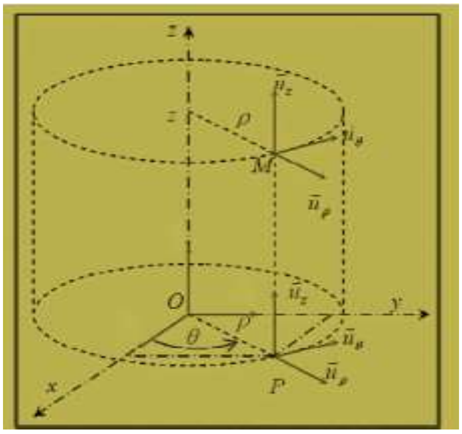
Given a cylindrical coordinate system **(ρ, θ, Z)** with base$(\overrightarrow{\boldsymbol{u_\rho}}, \overrightarrow{\boldsymbol{u_\theta}}, \overrightarrow{\boldsymbol{k}})$, the position of a point "M" is: $\overrightarrow{\boldsymbol{oM}} = \boldsymbol{\rho}\overrightarrow{\boldsymbol{u_\rho}} + \boldsymbol{z}\overrightarrow{k}$

When the coordinates of point M change infinitesimally: $\boldsymbol{d\rho, d\theta, dz}$ the point M undergoes an elementary translation

Length element $\overrightarrow{\boldsymbol{d0M}}$ is: $\overrightarrow{\boldsymbol{d0M}} = \boldsymbol{\rho}\overrightarrow{\boldsymbol{u_\rho}} + \boldsymbol{\rho d\theta}\overrightarrow{\boldsymbol{u_\theta}} + \boldsymbol{dz}\overrightarrow{\boldsymbol{k}}$**.**

dL is arc of length: $dL = \|\overrightarrow{d0M}\| = \sqrt{(d\rho)^2 + (\rho d\theta)^2 + (dz)^2}$





Cylindrical coordinate change domain $:(0 \leq \rho \leq R; 0 \leq \theta \leq 2\pi\, ; 0 \leq z \leq L)$

Surface element (Cylinder lateral surface):

$(\perp\overrightarrow{u\rho})\, ;\, dS = \rho d\theta . dz$ $(\perp\overrightarrow{u_\theta})\ ;\ dS = d\rho . dz$ $(\perp\vec{k})\ ;\ dS = \rho d\rho . d\theta$

- ✓ Lateral surface of a cylinpder with base radius R and height L:

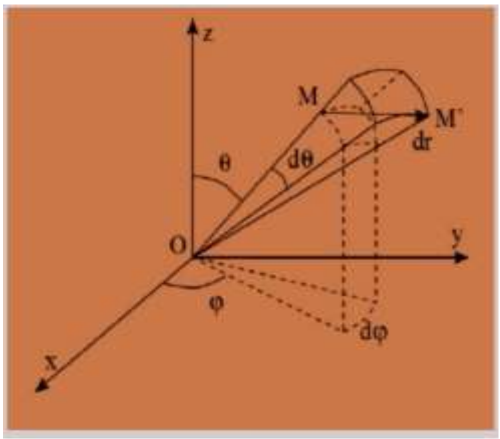
$$S = \iint dS = \iint \rho . d\theta . dz = R \int_0^{2\pi} d\theta \int_0^L dz \qquad (\rho = R)$$

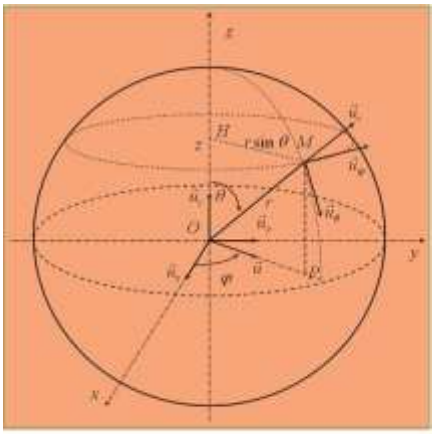
$$S = 2\pi RL$$

Infinitesimal volume element : $dV = \rho d\rho d\theta dz$

1.4- Spherical coordinates (r, θ, φ)

Given a spherical coordinate system(r, θ, φ) of base $(\overrightarrow{u_r}, \overrightarrow{u_\theta}, \overrightarrow{u_\varphi})$, the position of a point "**M**" is : $\overrightarrow{OM} = r\,\overrightarrow{u_r}$.





Spherical coordinate change domain: $(r \geq 0; \quad 0 \leq \theta \leq \pi \ ; \ 0 \leq \varphi \leq 2\pi)$

When the coordinates of point M change infinitesimally: $dr, d\theta, d\varphi$ the point M undergoes an elementary translation

2. **Length element $d\overrightarrow{0M}$: $d\overrightarrow{OM} = dr\,\overrightarrow{u_r} + rd\theta\,\overrightarrow{u_\theta} + rsin\theta d\varphi\,\overrightarrow{u_\varphi}$**

$$dL = \left\|d\overrightarrow{0M}\right\| = \sqrt{(dr)^2 + (rd\theta)^2 + (rsin\theta d\varphi)^2}$$

3. **Surface element dS :**

   $(\perp \overrightarrow{u_r})$; $dS = r^2 d\theta . sin\theta . d\varphi$

   $(\perp \overrightarrow{u_\theta})$; $dS = rdr . sin\theta . d\varphi$

   $(\perp \overrightarrow{u_\varphi})$; $dS = rdr . d\theta$

4. **Infinitesimal volume element : $dV = r^2\, dr\, sin\theta\, d\theta\, d\varphi$**

Exercice :

Calculate the volume of a cylinder V of radius R and height H.

Solution

We take a volume element a small cube with coordinates $(r, d\theta, dz)$ in the plane the length element is: $dV = dr . rd\theta . dz$

The triple integral of V : $V = \iiint rdr . d\theta . dz$

Separation of variables : $V = \int_0^R rdr \int_0^{2\pi} d\theta \int_0^H dz = \left[\frac{r^2}{2}\right]_0^R [\theta]_0^{2\pi} [z]_0^H$

$$V = \pi R^2 H \text{ (is the volume of a cylinder of height H)}$$

Exercice

Calculate the volume of a sphere **V** with radius **R**. Triple volume integral

$$V = \int_0^R r^2 dr \int_0^H sin\theta d\theta \int_0^{2\pi} d\varphi$$

Separation of variables: $V = \left[\frac{r^3}{3}\right]_0^R [cos\theta]_0^{\pi} [\varphi]_0^{2\pi} = \frac{R^3}{3} 2\ 2\pi$

$V = \frac{4}{3}\pi R^3$ **(V is the volume of a sphere of radius R)**.

2. Operators

2.1 Gradient

- **Cartisian coordinates** $(\vec{i}, \vec{j}, \vec{k})$

The vector quantity ∇ represents the **nabla operator** defined by: $\vec{\nabla} = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$.

Let f(x,y,z) be a scalar field.

The gradient of a scalar field **f(x,y,z)** is the vector:

$$\overrightarrow{grad}f = \vec{\nabla}.f = \frac{\partial f}{\partial x}\vec{i} + \frac{\partial f}{\partial y}\vec{j} + \frac{\partial f}{\partial z}\vec{k}$$

- **Cylindrical coordinates** $(\overrightarrow{U_r}, \overrightarrow{U_\theta}, \vec{k})$

The vector quantity ∇ represents the **nabla operator** defined by: $\vec{\nabla} = \frac{\partial}{\partial r}\overrightarrow{U_r} + \frac{1}{r}\frac{\partial}{\partial \theta}\overrightarrow{U_\theta} + \frac{\partial}{\partial z}\vec{k}$

$$\overrightarrow{grad}f = \vec{\nabla}.f = \frac{\partial f}{\partial r}\overrightarrow{U_r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\overrightarrow{U_\theta} + \frac{\partial f}{\partial z}\vec{k}$$

- **Spherical coordinates** $(\overrightarrow{U_r}, \overrightarrow{U_\theta}, \overrightarrow{U_\varphi})$

The vector quantity ∇ represents the **nabla operator** defined by: $\vec{\nabla} = \frac{\partial}{\partial r}\overrightarrow{U_r} + \frac{1}{r}\frac{\partial}{\partial \theta}\overrightarrow{U_\theta} + \frac{1}{rsin\theta}\frac{\partial}{\partial \varphi}\overrightarrow{U_\varphi}$

$$\overrightarrow{grad}f = \vec{\nabla}.f = \frac{\partial f}{\partial r}\overrightarrow{U_r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\overrightarrow{U_\theta} + \frac{1}{rsin\theta}\frac{\partial f}{\partial \varphi}\overrightarrow{U_\varphi}$$

2.3- Divergence

Consider a vector field $\vec{\boldsymbol{A}}$:

We call divergence of the vector A, the scalar: $\boldsymbol{div}\,\vec{A} = \vec{\boldsymbol{\nabla}}.\vec{A}$

- **In Cartesian coordinates**

$$\vec{A} = A_X(x,y,z)\vec{\imath} + A_Y(x,y,z)\vec{\jmath} + A_z(x,y,z)\vec{k}$$

$$\boldsymbol{div}\,\vec{A} = \frac{\boldsymbol{\partial} A_X}{\boldsymbol{\partial x}} + \frac{\boldsymbol{\partial} A_Y}{\boldsymbol{\partial y}} + \frac{\boldsymbol{\partial} A_z}{\boldsymbol{\partial z}}$$

- **In cylindrical coordinates** $(\boldsymbol{A_r}, \boldsymbol{A_\theta}, \boldsymbol{A_z})$:

$$\vec{A} = A_\rho\overrightarrow{U_\rho} + A_\theta\overrightarrow{U_\theta} + A_z\vec{k}$$

$$\boldsymbol{div}\,\vec{\boldsymbol{A}} = \frac{\boldsymbol{1}}{\boldsymbol{\rho}}\left[\frac{\boldsymbol{\partial}(\boldsymbol{\rho A_\rho})}{\boldsymbol{\partial\rho}} + \frac{\boldsymbol{\partial}(\boldsymbol{A_\theta})}{\boldsymbol{\partial\theta}} + \frac{\boldsymbol{\partial}(\boldsymbol{\rho A_z})}{\boldsymbol{\partial z}}\right]$$

- **In spherical coordinates** $(\boldsymbol{A_r}, \boldsymbol{A_\theta}, \boldsymbol{A_\varphi})$:

$$\vec{A} = A_r\overrightarrow{U_r} + A_\theta\overrightarrow{U_\theta} + A_\varphi\overrightarrow{U_\varphi}$$

$$\boldsymbol{div}\,\vec{\boldsymbol{A}} = \frac{\boldsymbol{1}}{\boldsymbol{r^2 sin\theta}}\left[\frac{\boldsymbol{\partial}(\boldsymbol{r^2 sin\theta A_r})}{\boldsymbol{\partial r}} + \frac{\boldsymbol{\partial}(\boldsymbol{r sin\theta A_\theta})}{\boldsymbol{\partial\theta}} + \frac{\boldsymbol{\partial}(\boldsymbol{r A_\varphi})}{\boldsymbol{\partial\varphi}}\right]$$

2.2- Rotational

Given a field of vectors$\vec{A}$, we call rotational of the vector the vector product of the operator $\vec{\nabla}$ and the vector $\overrightarrow{rot}\vec{A} = \vec{\nabla}\wedge\vec{A}$

- **Cartisian coordinates**

$$\vec{A} = A_X(x,y,z)\vec{\imath} + A_Y(x,y,z)\,\vec{\jmath} + A_z(x,y,z)\vec{k}$$

$$\overrightarrow{rot}\vec{A} = \vec{\nabla}\wedge\vec{A} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_X & A_Y & A_z \end{vmatrix}$$

$$= \left(\frac{\partial Z}{\partial y} - \frac{\partial Y}{\partial z}\right)\vec{\imath} + \left(\frac{\partial X}{\partial z} - \frac{\partial Z}{\partial x}\right)\vec{\jmath} + \left(\frac{\partial Y}{\partial x} - \frac{\partial X}{\partial y}\right)\vec{k}$$

- **Cylindrical coordinates**

$$\overrightarrow{rot}\vec{A} = \vec{\nabla} \wedge \vec{A} = \begin{vmatrix} \frac{\overrightarrow{u_\rho}}{\rho} & \overrightarrow{u_\theta} & \frac{\vec{k}}{\rho} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\theta & A_z \end{vmatrix}$$

$$= \left[\frac{\partial A_Z}{\partial \theta} - \frac{\partial \rho A_\theta}{\partial Z}\right]\frac{\overrightarrow{u_\rho}}{\rho} - \left[\frac{\partial A_Z}{\partial \rho} - \frac{\partial A_\rho}{\partial Z}\right]\overrightarrow{u_\theta} + \left[\frac{\partial \rho A_\theta}{\partial \rho} - \frac{\partial A_\rho}{\partial \theta}\right]\frac{\vec{k}}{\rho}$$

- **Spherical coordinates**

$$\overrightarrow{rot}\vec{A} = \vec{\nabla} \wedge \vec{A} = \begin{vmatrix} \frac{\overrightarrow{u_r}}{r^2 sin\theta} & \frac{\overrightarrow{u_\theta}}{r\ sin\theta} & \frac{\overrightarrow{u_\varphi}}{r} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \varphi} \\ A_r & rA_\theta & rsin\theta\ A_\varphi \end{vmatrix}$$

$$= \left[\frac{\partial(rsin\theta A_\varphi)}{\partial \theta} - \frac{\partial(rA_\theta)}{\partial \varphi}\right]\frac{\overrightarrow{u_\rho}}{r} - \left[\frac{\partial(rsin\theta A_\varphi)}{\partial r} - \frac{\partial A_r}{\partial \varphi}\right]\frac{\overrightarrow{u_\theta}}{r\ sin\theta} + \left[\frac{\partial r A_\theta}{\partial r} - \frac{\partial A_r}{\partial \theta}\right]\frac{\overrightarrow{u_\varphi}}{r}.$$

Some formulas:

- The following relation is always checked : $\overrightarrow{rot}\left(\overrightarrow{grad}f\right) = 0$ is explained as follows ; when $A = \overrightarrow{grad}f \Rightarrow \overrightarrow{rot}A = 0$
- $div(\overrightarrow{rot}A) = 0$
- $div\left(f\vec{A}\right) = f\ div\ \vec{A} + A\overrightarrow{grad}f$
- $\overrightarrow{rot}(f\ \vec{A}) = \overrightarrow{grad}f \wedge \vec{A} + f\overrightarrow{rot}\vec{A}$
- $div\left(\vec{A} \wedge \vec{B}\right) = \vec{B}\ \overrightarrow{rot}\ \vec{A} - \vec{A}\ \overrightarrow{rot}\ \vec{B}$

3- The integrations.

3.1- Définitions

The primitive function of function f (x) is another function (x) such that: $F'(x) = (x)$

Exemples 1.

$F(x) = \frac{x^3}{3} + 3\frac{y^2}{2} + z^2$ Is the primitive function of $f(x) = x^2 + 3y + 2z$.

$(x) = -2\cos x + 2x$ is the primitive function of $(x) = 2sinx + 2$.

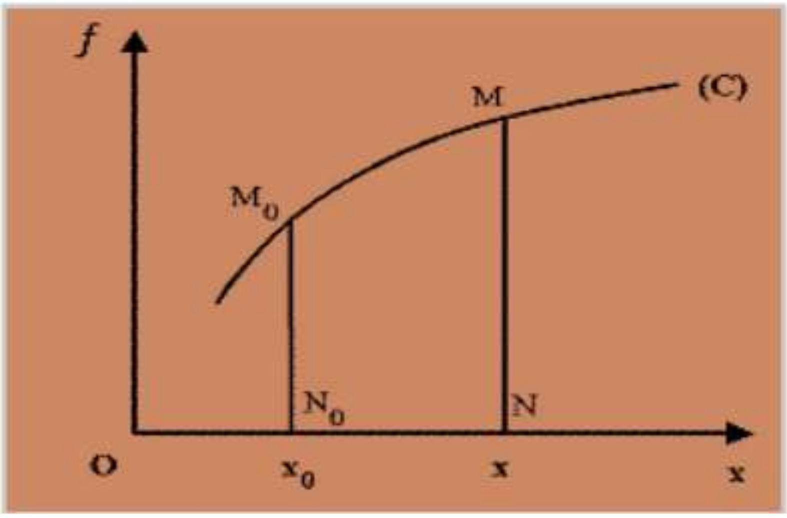
3.2- Simple integral

Let the lines be $\boldsymbol{N_0M_0}$with fixed abscissa $\boldsymbol{X_0}$ and NM with variable abscissa **X**. The area $(\boldsymbol{N_0M_0MN})$ represents the definite integral of the function $\boldsymbol{(x)}$ between points $\boldsymbol{X_0}$ and **X**

$$The\ area(N_0M_0MN) = \int_{X_0}^{X} f(x)dx$$

$$The\ area\ (N_0M_0MN) = F(x) + constant$$

F(x) is the primitive function of (x)



The set of primitives of f(x) is called the indefinite integral where: $\int f(x)dx = F(x) + constant$

3.1- Multiple integral

The multiple integral is a form of integral that applies to functions of several real variables.

Exemples

- For a function with two variables: $\iint (x, y)dxdy$
- For a function with three variables : $\iiint (x, y, z)dxdydz$

3.2- Methods for calculating integrals

3.2.1- Primitives

Let $\boldsymbol{f}$ be a continuous function on an interval $[\boldsymbol{a}, \boldsymbol{b}]$ of $\mathbb{R}$. If its primitive F is easy to determine, then we can use the well-known formula:

$$\int_a^b f(x)dx = F(b) - F(a)$$

The usual primitives are given in the table below :

$f(x)$	$F(x)$
$x^\alpha, \alpha \neq -1$	$\frac{x^{\alpha+1}}{\alpha+1}$
$\frac{1}{x}$	$\ln\|x\|$
e^x	e^x
$\sin x$	$-\cos x$
$\cos x$	$\sin x$
$\tan x$	$-\ln\|\cos x\|$

$f(x)$	$F(x)$
$\operatorname{sh} x$	$\operatorname{ch} x$
$\operatorname{ch} x$	$\operatorname{sh} x$
$\operatorname{th} x$	$\ln(\operatorname{ch} x)$
$\frac{1}{\sqrt{1-x^2}}$	$\arcsin x$
$\frac{1}{\sqrt{x^2+b}}, b \neq 0$	$\ln\left\|x + \sqrt{x^2+b}\right\|$
$\frac{1}{x^2+1}$	$\arctan x$

Exemples

1- $\int \frac{dx}{x} = ln|x| + c$

2- $\int_0^{2\pi} sin\theta d\theta = [-cos\theta]_0^{2\pi} = 1$

3.2.2- Intégration par parties

Soit f et g des fonctions définies sur un intervalle I, nous avons :

$$\int f'(x)g(x)dx = f(x)g(x) - \int f(x)\, g'(x)dx$$

Exemples 1.

Calculate the following integral: $\int_0^t t^2 e^t dt$

In this integral, we pose: $f'(x) = e^t \quad et \quad g(x) = t^2$

Let's calculate f(x) et $g'(x)$: $f(x) = e^t \quad et \quad g(x) = 2t$

$$\int_0^1 \boldsymbol{t^2} e^{t} dt = [\boldsymbol{t^2} e^{t}]_0^1 - \mathbf{2} \underbrace{\int_0^1 \boldsymbol{t} e^{t} dt}_{I_1}$$

Let's apply integration by parts a second time to calculate the integral I_1: Let's say:

$u'(x) = e^t \quad et \quad u(x) = e^t \quad , v(x) = t \quad \Rightarrow \quad v'(x) = 1$

$$\int_0^1 \boldsymbol{te^t dt} = [\boldsymbol{te^t}]_0^1 - \int_0^1 \boldsymbol{e^t dt} = \boldsymbol{e - e + 1 = 1}$$

So :

$$\int_0^1 \boldsymbol{t^2} e^{t} dt = [\boldsymbol{t^2} e^{t}]_0^1 - \mathbf{2} = \boldsymbol{e} - \mathbf{2}$$

3- <u>Calculate $\int x \sin x\, dx$</u>

We pose: $(x) = x \Rightarrow g'(x) = 1$ et $f'(x) = sinx \Rightarrow (x) = -cosx$

$$\int x \sin x\, dx = -xcosx + \int \cos x\, dx$$

$$\boldsymbol{\int x \sin x\, dx = -xcosx + sinx + C}$$

Exercice

Apply integration by change of variables to calculate:

$$\int_0^{\frac{\pi}{3}} x\, cosx\, dx \quad ; \quad \int x^2 . \ln x\, dx$$

Solution

1) <u>For the calculation of $\int_0^{\frac{\pi}{3}} x\, cosx\, dx$</u> :

We ask $'(x) = \cos(x)$ and $g(x) = x$

$(x) = \int \cos(x)\, dx \quad \Rightarrow \quad f(x) = \sin x$ and $g'(x) = 1$

Applying integration by parts: $\int f'(x)g(x)dx = f(x)g(x) - \int f(x)\, g'(x)dx$

$$\int_0^{\frac{\pi}{3}} x\, cosx\, dx = [x \sin(x)]_0^{\frac{\pi}{3}} - \int_0^{\frac{\pi}{3}} sinx\, dx = \frac{\pi\sqrt{3}}{6} + [cos(x)]_0^{\frac{\pi}{3}} = \frac{\pi\sqrt{3}}{6} - \frac{1}{2}$$

2) <u>For the calculation of $\int x^2 . \ln x\, dx$</u> :

We ask $f'(x) = x^2$ and $(x) = \ln(x)$

$(x) = \int x^2 dx = \frac{x^3}{3}$ end $g'(x) = \frac{1}{x}$

Applying integration by parts: $\int f'(x)g(x)dx = f(x)g(x) - \int f(x)\, g'(x)dx$

$$\int x^2 . \ln x\, dx = \frac{x^3 \ln(x)}{3} - \int \frac{x^2}{3} dx = \frac{x^3 \ln(x)}{3} - \frac{x^3}{9} = \frac{x^3}{9}\left(\ln(x) - \frac{1}{3}\right) + c$$

3.2.3- Integration by change of variables

Integration by change of variables is a method of integration, which consists of considering a new variable of integration in order to replace a function of the initial variable of integration.

Exemples 1.

- **Calculate the following integral:**

$$\int_0^1 \frac{x^2}{x+1}\, dx$$

Let's ask : $y = x + 1 \Leftrightarrow x = y - 1 \Rightarrow dx = dy,\ 0 \leftarrow x \rightarrow 1 \Rightarrow 1 \leftarrow y \rightarrow 2$

We will have :

$$\int_0^1 \frac{x^2}{x+1}\, dx = \int_1^2 \frac{(y-1)^2}{y}\, dy = \int_1^2 y dy - \int_1^2 2dy + \int_1^2 \frac{1}{y} dy$$

$$= \left[\frac{y^2}{2}\right]_1^2 - 2[y]_1^2 + [lny]_1^2 = ln\, 2 - \frac{1}{2}.$$

- **Calculate the following integral: $\int_0^1 \frac{1}{1+\sqrt{x}}\,dx$**

Let's ask $y = \sqrt{x} \Leftrightarrow x = y^2 0 \leftarrow x \rightarrow 1 \Rightarrow 0 \leftarrow y \rightarrow 1$

$$\int_0^1 \frac{1}{1+\sqrt{x}}\,dx = \int_0^1 \frac{2y}{1+y}\,dy = \int_0^1 \left(\frac{2(1+y)}{1+y} - \frac{2}{1+y}\right) dy$$

$$= \int_0^1 2\,dy - \int_0^1 \frac{2}{1+y}\,dy = 2[y]_0^1 - 2[ln(1+y)]_0^1$$

So: $\int_0^1 \frac{1}{1+\sqrt{x}}\,dx = 2(1 - ln(2))$

Exercice

Appliquer l'intégration par changement de variables pour calculer :

$$\int_1^4 \frac{1-\sqrt{x}}{x}dx \quad ; \quad \int_1^e \frac{(lnx)^n}{x}dx \quad ; \quad \int \frac{x}{(x^2-4)^2}dx$$

Solution

1. Calcul de :

$$\int_1^4 \frac{1-\sqrt{x}}{\sqrt{x}}dx$$

Posons : $u = \sqrt{x} \Leftrightarrow x = u^2 \Rightarrow dx = 2udu,$

$$1 \leftarrow x \rightarrow 4 \Rightarrow 1 \leftarrow u \rightarrow 2$$

$$\int_1^4 \frac{1-\sqrt{x}}{\sqrt{x}}dx = \int_1^2 2u\left(\frac{1-u}{u}\right)du = \int_1^2 2du - \int_1^2 2udu$$

$$= [2u - u^2]_1^2 = -1$$

$$\int_1^4 \frac{1-\sqrt{x}}{\sqrt{x}}dx = -1$$

2. Calcul de :

$$\int_1^e \frac{(\ln x)^n}{x}dx$$

Posons : $u = lnx \Leftrightarrow du = \frac{dx}{x} \Rightarrow dx = xdu, \quad 1 \leftarrow x \rightarrow e \Rightarrow 0 \leftarrow u \rightarrow 1$

$$\int_1^e \frac{(\ln x)^n}{x}dx = \int_0^1 u^n du = \left[\frac{u^{n+1}}{n+1}\right]_0^1 = \frac{1}{n+1}$$

$$\int_1^e \frac{(\ln x)^n}{x}dx = \frac{1}{n+1}$$

3. Calcul de $\int \frac{x}{(x^2-4)^2}dx$:

Posons : $u = x^2 \Leftrightarrow 2xdx = du \Rightarrow xdx = \frac{du}{2}$

$$\int \frac{x}{(x^2-4)^2}dx = \frac{1}{2}\int \frac{1}{(u-4)^2}du = -\frac{1}{2(u-4)}$$

$$\int \frac{x}{(x^2-4)^2}dx = -\frac{1}{2(x^2-4)} + C$$