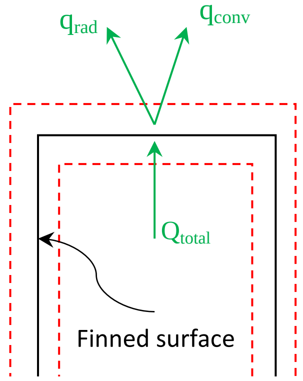
Abd El Hafid Boussouf University Center – Mila
Institute of Science and Technology
Process Engineering

January 2025

Heat Transfer Final Exam

(Model Correction)



Problem n° 01 (08 points):

1) The energy balance on the finned surface in terms of: Q_{total}, q_{conv} and q_{rad} :

Taking a control surface from the finned surface:

$E_{in} + E_g - E_{out} = E_{st}$ (Where: $E_g = 0$, and $E_{st} = 0$)

Give $E_{in} = E_{out}$ then, $\boldsymbol{Q_{total} = q_{conv} + q_{rad}}$ **1**

2) The total surface area ($A_{s,\ total}$) for heat transfer :

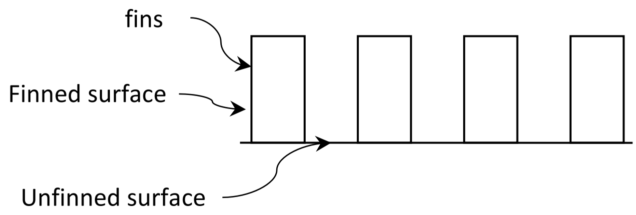
$A_{s;\,total} = A_{s;\,finned} + A_{s;\,unfinned}$ **0.5**

$A_{s;\,finned} = 7 \times \big(2 \times (5 \times 100)\big) + \big(2 \times (2 \times 5)\big) + (2 \times 100)\big)$

$A_{s;\,finned} = 85.4\ cm^2 = 0.0085\ \mathrm{m}^2$ **0.5**

$A_{s;\,unfinned} = (10 \times 6.2) - (7 \times 0.2 \times 10)$

$A_{s;\,unfinned} = 48\ \mathrm{cm}^2 = 0.0048\ \mathrm{m}^2$ **0.5**



Give: $\boldsymbol{A_{s;\,total} = 0.00133\ m^2}$ **1**

3) The radiation heat transfer rate (q_{rad}):

We have: $q_{rad} = \varepsilon . A_{s,total} . \sigma . (T_s^4 - T_{surr}^4)$ **0.5**

Then, $q_{rad} = 0.9 \times 0.0133 \times 5.67 \times 10^{-8}((60+273)^4 - (25+273)^4)$, give : $q_{rad} \approx 3\ W$ **0.5**

4) The heat transfer coefficient (h)

The convection heat transfer coefficient can be determined from *Newton's law of cooling* relation for a finned surface.

$q_{conv} = \eta \times h \times A_s \times (T_\infty - T_s)$ → $h = \frac{q_{conv}}{\eta \times A_s \times (T_\infty - T_s)}$ → $h = \frac{q_{conv}}{0.90 \times 0.0133 \times (60-25)}$ **0.5**

Where: $q_{conv} = Q_{total} - q_{rad} = 12\ \mathrm{W} - 3\ \mathrm{W} = 9\ \mathrm{W}$ **0.5**

Then $h = \frac{9}{0.90 \times 0.0133 \times (60-25)}$ give: $\boldsymbol{h \approx 21.50\ (\frac{W}{m^2}.°C)}$ **0.5**

5) The minimum free-stream velocity (U_{min}) the fan needs to supply to avoid overheating:

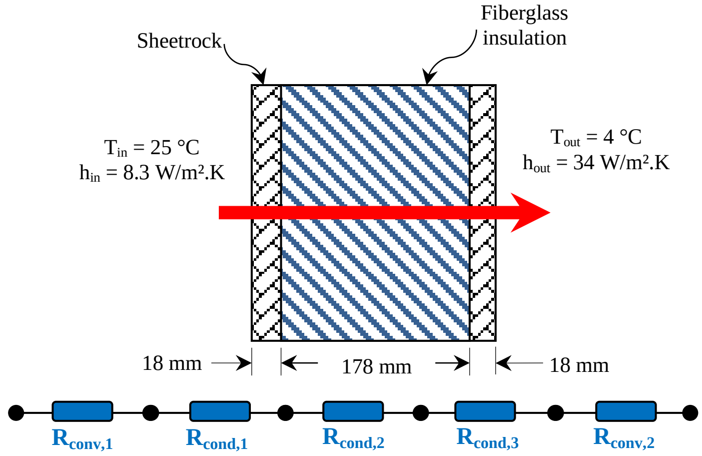
Starting from heat transfer coefficient, *Nusselt* number, *Reynolds* number and finally free-stream velocity will be determined. We assume the flow is laminar over the entire finned surface of the transformer.

$Nu = \frac{hL}{k} = \frac{21.5 \times 0.1}{0.02681} = 80.20$ **0.5** With, $Nu = 0.664 Re_L^{0.5} Pr^{\frac{1}{3}}$ → $Re_L = \frac{Nu^2}{0.664^2 \times Pr^{\frac{2}{3}}} = \frac{(80.20)^2}{(0.664)^2 \times (0.7248)^{2/3}} \approx 1.8 \times 10^4$ **0.5**

Then, $Re_L = \frac{U_{min} L}{\nu}$ **0.5** → $U_{min} = \frac{Re_L \times \nu}{L} = \frac{1.8 \times 10^4 \times 1.726 \times 10^{-5}}{0.1}$ → $\boldsymbol{U_{min} = 3.11\ m/s}$ **0.5**

Problem n° 02 (04 points):

1) Draw the thermal resistance circuit for this wall :



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2) Determine the thermal resistance of the wall :

$$R_{wall} = R_{conv,1} + R_{cond,1} + R_{cond,2} + R_{cond,3} + R_{conv,2}$$

$$R_{wall} = \frac{1}{h_{in}.A} + \frac{L_{sheetrock}}{k_{sheetrock}.A} + \frac{L_{fiberglass}}{k_{fiberglass}.A} + \frac{L_{sheetrock}}{k_{sheetrock}.A} + \frac{1}{h_{out}.A}$$ **1**

$$R_{wall} = \frac{1}{8.3\times(5\times3)} + \frac{0.018}{0.15\times(5\times3)} + \frac{0.178}{0.02\times(5\times3)} + \frac{0.018}{0.15\times(5\times3)} + \frac{1}{h_{out}\times(5\times3)}$$

$$R_{wall} = 0.008 + 0.008 + 0.5933 + 0.008 + 0.002 \quad \rightarrow \quad \boldsymbol{R_{wall} \approx 0.6193\ (°C/W)}$$ **1**

3) Calculate the heat transfer rate through the wall :

$$q = \frac{T_{\infty,in} - T_{\infty,out}}{R_{wall}} = \frac{25-4}{0.6193} \quad \rightarrow \quad \boldsymbol{q = 33.91\ (W)}$$ **1**

Problem n° 03 (08 points):

1) The variation of temperature $\frac{T(x) - T(\infty)}{T(b) - T(\infty)}$ along the spoon :

Assuming that the heat transfer from the tip of the spoon is **negligible**, then, the variation of temperature will be given from **Table 1**, by:

$$\frac{T(x) - T(\infty)}{T(b) - T(\infty)} = \frac{\cosh m(L-x)}{\cosh mL}$$ **2** (Equation 01)

2) Determine the temperature difference (**$\Delta T = T_b - T_{tip}$**) across the exposed surface of the spoon :

Calculating T_{tip} means that: **x = L = 7 *in*.**

Substituting in (Equation 01),

$$\frac{T(L) - T(\infty)}{T(b) - T(\infty)} = \frac{\cosh m(L-L)}{\cosh mL} \quad \rightarrow \quad T_{tip} = \left(\frac{\cosh m(L-L)}{\cosh mL}\right) \times \left(T(b) - T(\infty)\right) + T(\infty)$$ **1**

$$m = \sqrt{\frac{h.P}{k.A_c}} = \sqrt{\frac{3\times2\times(\frac{0.5+0.08}{12})}{8.7\times(\frac{0.5\times0.08}{12})}} \rightarrow \mathbf{m = 10\ ft^{-1}}$$ (Inches to feet: 1 ft = 12 in)

$$T_{tip} = \left(\frac{\cosh(0)}{\cosh(10\times\frac{7}{12})}\right)\times(200-75)+75 = \left(\frac{1}{\cosh(5.83)}\right)\times(125)+75 = \left(\frac{1}{170.18}\right)\times(125)+75$$ 1

$\cosh(5.83) = 170.18$ (Using **Table 2)**.

$\mathbf{T_{tip} = 75.73\ °F}$ → $\Delta T = T_b - T_{tip} = 200 - 75.73 = \mathbf{124.27\ °F}$ 1

3) Determine the fin heat transfer rate ($\mathbf{q_f}$) :

From Table 1: $q_f = M.\tanh(mL)$ 1

Where: $M = \sqrt{hPkA_c}\theta_b = \sqrt{3\times2(\frac{0.5+0.08}{12})\times8.7\times(\frac{0.5\times0.08}{12})}\times125 \rightarrow M = 11.4$ 1

$\tanh(mL) = \tanh\left(10\times\frac{7}{12}\right) = \tanh(5.83) = 0.999$

$\mathbf{q_f = 11.38\ Btu/h}$ 1