Exam Correction:

Exercise 01:

1) $(p \wedge \varphi) \Rightarrow \varphi$

The truth table:

p	φ	$p \wedge \varphi$	$(p \wedge \varphi) \Rightarrow \varphi$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

1,5

2) $(p \vee \bar{\varphi}) \Rightarrow (\bar{p} \wedge \varphi)$

2

p	φ	$\bar{p}$	$\bar{\varphi}$	$p \vee \bar{\varphi}$	$\bar{p} \vee \varphi$	$(p \vee \bar{\varphi}) \Rightarrow (\bar{p} \wedge \varphi)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	F	T	T
F	F	T	T	T	F	F

Exercise 02 :

1) $2h^2 + 2h + 1 = 0$ length of h ?

$\Delta = 4 - 8 = -4$

$\sqrt{\Delta} = i2$

$$\begin{cases} h_1 = \dfrac{-2+2i}{4} = -\dfrac{1}{2} + \dfrac{i}{2} \\ h_2 = \dfrac{-2-2i}{4} = -\dfrac{1}{2} - \dfrac{i}{2} \end{cases}$$

the length of h

$$|h_1| = |h_2| = \sqrt{h\bar{h}} = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{1}{\sqrt{2}}$$

1,5

2) $h_1 = |h_1| \left(\cos(\theta_1) + i \sin(\theta_2)\right)$ $h_2 = |h_2| \left(\cos(\theta_2) + i \sin(\theta_2)\right)$

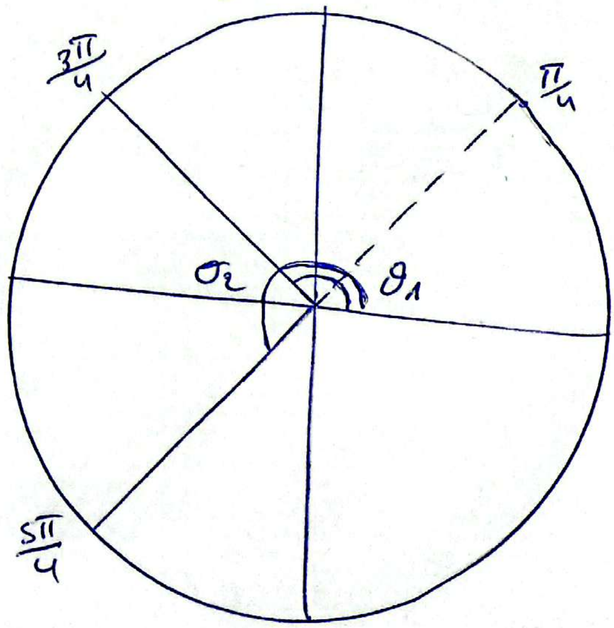
$\cos(\theta_1) = \dfrac{-\frac{1}{2}}{\frac{1}{\sqrt{2}}} = -\dfrac{1}{\sqrt{2}}$ $\cos(\theta_2) = \dfrac{-\frac{1}{2}}{\frac{1}{\sqrt{2}}} = -\dfrac{1}{\sqrt{2}}$

$\sin(\theta_1) = \dfrac{+\frac{1}{2}}{\frac{1}{\sqrt{2}}} = \dfrac{1}{\sqrt{2}}$ $\sin(\theta_2) = \dfrac{-\frac{1}{2}}{\frac{1}{\sqrt{2}}} = -\dfrac{1}{\sqrt{2}}$

$$h_1 = \frac{1}{\sqrt{2}} \left(\cos\left(\frac{3\pi}{4}\right) + i \sin\left(\frac{3\pi}{4}\right) \right)$$

$$h_2 = \frac{1}{\sqrt{2}} \left(\cos\left(\frac{5\pi}{4}\right) + i \sin\left(\frac{5\pi}{4}\right) \right)$$

02



Exercise 03 :

1) is f injective: $f^{-1}(x_1) = f^{-1}(x_2) \overset{?}{\Longrightarrow} x_1 = x_2$

$f^{-1}(x_1) = f^{-1}(x_2) \Rightarrow 2^{x_1} = 2^{x_2} \Rightarrow x_1 = x_2$

$\forall x_1, x_2 \in \mathbb{R}$ f^{-1} is injective. 01

2) we have $y = 2^x \Rightarrow \ln(y) = x \ln(2)$

$$x = \frac{\ln(y)}{\ln(2)}$$

We know that $\ln(y)$ is defined if $y > 0$ and not for $y \in \mathbb{R}$

there are no preimages x as $y \in \mathbb{R}^-$

$\Rightarrow f^{-1}(x)$ is not surjective. 1,5

3) The function $f^{-1}(x)$ is surjective if $y = f^{-1}(x) \in \mathbb{R}^+_*$.

Thus the Codomain is restricted as

$$f^{-1} : \mathbb{R} \longrightarrow \mathbb{R}^+_*$$
$$x \longrightarrow f^{-1}(x) = 2^x$$

1,5

4) The inverse function of $f^{-1}(x)$

$$\boxed{f(x) = \frac{\ln(x)}{\ln(2)}}$$

1,5

Exercise 04!

1/ the combination $(\mathbb{C}-\{i\}, *)$ is groupe if the following are satisfied:

1-1 Closure: the operation $*$ is composed of ordinary addition and multiplication and for that

$\forall a, b \in \mathbb{C}-\{i\} : (a*b) \in \mathbb{C}-\{i\}$ 0,75

1-2 Assoaativity! $\forall a, b, c \in \mathbb{C}-\{i\}$

$$\underbrace{(a*b)*c}_{1} \overset{?}{=} \underbrace{a*(b*c)}_{2}$$

$1 = a*b + c + i(a*b)c$

$= \cancel{a} + \cancel{b} + iab + \cancel{c} + i\cancel{ac} + i\cancel{bc} - a\cancel{bc}$

$2 = a*(b*c) = a+(b*c) + ia(b*c)$

$= \cancel{a} + \cancel{b} + \cancel{c} + i\cancel{bc} + i\cancel{ab} + \cancel{iac} - a\cancel{bc}$

$1 = 2 \Rightarrow *$ is Assoaative 1,25

Identity! $\forall a \in \mathbb{C}-\{i\}, \exists e \in \mathbb{C}-\{i\} \,|\, a*e = e*a = a$

$a*e = e*a = \cancel{a} + e + iae = \cancel{a}$

$e(1+ia) = 0$

$e = 0$ or $(1+ia) = 0 \Rightarrow a = -\frac{1}{i} = i$

$a \in \mathbb{C}-\{i\} \Rightarrow a \neq i =$

$\Rightarrow (1+ia) \neq 0$

$\Rightarrow \boxed{e = 0}$ 01

Inverse element! $\forall a \in \mathbb{C}-\{i\}, \exists b \in \mathbb{C}-\{i\} \mid a*b = b*a = e$

$$a*b = b*a = a+b+iab = 0$$

0,75

$$\Rightarrow \boxed{b = a^{-1} = \frac{-a}{(1+ia)}}$$ is defined for all $a \in \mathbb{C}-\{i\}$

the combination $(\mathbb{C}-\{i\}, *)$ satisfies the properties of a group $\Rightarrow$ $(\mathbb{C}-\{i\}, *)$ is a group

2) is $*$ distributive over $\#$

$$\forall a,b,c \in \mathbb{C}-\{i\} \quad \underbrace{a*(b\#c)}_{1} \overset{?}{=} \underbrace{(a*b)\#(b*c)}_{2}$$

$$1 = a*(b\#c) = a+(b\#c)+ia(b\#c)$$
$$= a+b+c-i+ia(b+c-i)$$
$$= a+b+c-i+iab+iac+a$$
$$= 2a+b+c+iab+iac-i$$

02

$$2 = (a*b)\#(b*c) = (a*b)+(b*c)-i$$
$$= a+b+iab+b+c+ibc-i$$
$$= a+b+c+iab+ibc-i$$

$1 \neq 2 \Rightarrow$ $*$ is not distributive over $*$

3/ $a \# e = e \# a = a$

$\Rightarrow \cancel{\alpha} + e - i = \alpha$

$\Rightarrow \boxed{e = i}$ we have $e \notin \mathbb{C} - \{i\}$ 1,75

$\Rightarrow$ # doesnot satisfy identity element.

This imply that the inverse element is not defined (does not exist)

4/ $(\mathbb{C} - \{i\}, \#)$ is not a group becaus # does not satisfy Identity and inverse properties.

1