**Exercice 1.** Résoudre avec l’algorithme du simplexe en deux phases le programme linéaire (P).

$$P≡\left\{\begin{array}{c}Max(Z)=2x1+3x2+x3\\x1+x2+x3 \leq 40\\2x1+x2 - x3 \geq 10\\-x2+x3 \geq 10\\x1, x2, x3 \geq 0\end{array}\right.$$

1. forme standard.

$$P≡\left\{\begin{array}{c}Max(Z)=2x1+3x2+x3\\x1+x2+x3 +x4= 40\\2x1+x2 - x3 –x5= 10\\-x2+x3 –x6= 10\\x1, x2, x3 , x4, x5, x6\geq 0\end{array}\right.$$

1. forme artificielle.

$$P≡\left\{\begin{array}{c}Max(Z)=2x1+3x2+x3\\x1+x2+x3 +x4= 40\\2x1+x2 - x3 –x5+x7= 10\\-x2+x3 –x6+x8= 10\\x1, x2, x3 , x4, x5, x6\geq 0\end{array}\right.$$

2. **Phase I :**

**2.1 La table initiale**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | $$x1\downright $$ | **x2** | **x3** | **x4** | **x5** | **x6** | **x7** | **x8** | **bi** |
| **x4** | **1** | **1** | **1** | **1** | **0** | **0** | **0** | **0** | **40** |
| $\leftarrow $**x7** | **2** | **1** | **-1** | **0** | **-1** | **0** | **1** | **0** | **10** |
| **x8** | **0** | **-1** | **1** | **0** | **0** | **-1** | **0** | **1** | **10** |
| **W** | **2** | **0** | **0** | **0** | **-1** | **-1** | **0** | **0** | **20** |
| **Z** | **2** | **3** | **1** | **0** | **0** | **0** | **0** | **0** | **0** |

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | $$x1$$ | **x2** | $$x3\downright $$ | **x4** | **x5** | **x6** | **x7** | **x8** | **bi** |
| **x4** | **0** | **1/2** | **3/2** | **1** | **1/2** | **0** | **-1/2** | **0** | **35** |
| **x1** | **1** | **1/2** | **-1/2** | **0** | **-1/2** | **0** | **1/2** | **0** | **5** |
| $\leftarrow $**x8** | **0** | **-1** | **1** | **0** | **0** | **-1** | **0** | **1** | **10** |
| **W** | **0** | **-1** | **1** | **0** | **0** | **-1** | **-1** | **0** | **10** |
| **Z** | **0** | **2** | **2** | **0** | **1** | **0** | **-1** | **0** | **-10** |

**1.2 Première itération**

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | $$x1$$ | **x2** | $$x3$$ | **x4** | **x5** | **x6** | **x7** | **x8** | **bi** |
| **x4** | **0** | **2** | **0** | **1** | **1/2** | **3/2** | **-1/2** | **-3/2** | **20** |
| **x1** | **1** | **0** | **0** | **0** | **-1/2** | **-1/2** | **1/2** | **1/2** | **10** |
| **x3** | **0** | **-1** | **1** | **0** | **0** | **-1** | **0** | **1** | **10** |
| **W** | **0** | **0** | **0** | **0** | **0** | **0** | **-1** | **-1** | **0** |
| **Z** | **0** | **4** | **0** | **0** | **1** | **2** | **-1** | **-2** | **-30** |

**1.3 Deuxième itération**

La fonction objective atteint la valeur nulle et toutes les variables artificielles sont exclues de la base, donc on passe à la deuxième phase considérant la solution initiale retournée par la première.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | $$x1$$ | $$x2\downright $$ | $$x3$$ | **x4** | **x5** | **x6** | **bi** |
| $\leftarrow $**x4** | **0** | **2** | **0** | **1** | **1/2** | **3/2** | **20** |
| **x1** | **1** | **0** | **0** | **0** | **-1/2** | **-1/2** | **10** |
| **x3** | **0** | **-1** | **1** | **0** | **0** | **-1** | **10** |
| **Z** | **0** | **4** | **0** | **0** | **1** | **2** | **-30** |

**II. Phase II.**

**II.1 Table initiale**

**II.2 Première itération**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | $$x1$$ | $$x2$$ | $$x3$$ | **x4** | **x5** | **x6** | **bi** |
| **x2** | **0** | **1** | **0** | **1/2** | **1/4** | **3/4** | **10** |
| **x1** | **1** | **0** | **0** | **0** | **-1/2** | **-1/2** | **10** |
| **x3** | **0** | **0** | **1** | **1/2** | **1/4** | **-1/4** | **20** |
| **Z** | **0** | **0** | **0** | **-2** | **0** | **-1** | **-70** |

Tous les coefficients dans la ligne de la fonction objective sont négatifs ou nuls donc la solution est optimale avec :

x1\*=10, x2\*=10, x3\*=20, x4\*=x5\*=x6\*=0, Z\*=-Z=70.

**Exercice 2**

1) forme standard.

Max Y= -3x1-4x2-5x3

$$\left\{\begin{array}{c}x1+2x2+3x3-x\&4 =5\\2x1+2x2+x3-x\&5 =6\\ \\x\&1 , x2,x3 , x4,x5\geq 0\\ \\ \end{array}\right. $$

**1.1 programme artificiel**

Max Y = Max Y= -3x1-4x2-5x3

Max w =-w1-w2

$$\left\{\begin{array}{c}x1+2x2+3x3-x\&4 +w1=5\\2x1+2x2+x3-x\&5 +w2=6\\ \\x\&1 , x2,x3 , x4,x5\geq 0\\w1,w2\geq 0\\ \\ \end{array}\right.$$

**2. Phase I :**

**2.1 La table initiale**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **X1** | **X2** | **X3** | **X4** | **X5** | **W1** | **W2** | **bi** |
| **W1** | **1** | **2** | **3** | **-1** | **0** | **1** | **0** | **5** |
| **W2** | **2** | **2** | **1** | **0** | **-1** | **0** | **1** | **6** |
| **W** | **3** | **4** | **4** | **-1** | **-1** | **0** | **0** | **11** |
| **Z** | **-3** | **-4** | **-5** | **0** | **0** | **0** | **0** | **0** |

La ligne de W est calculée en faisant la somme : LW=LW+Lw1+Lw2 afin d’exprimer W uniquement par les variables de base.

**2.2 Première itération :**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **X1** | **X2** | **X3** | **X4** | **X5** | **W1** | **W2** | **bi** |
| **X2** | **1/2** | **1** | **3/2** | **-1/2** | **0** | **1/2** | **0** | **5/2** |
| **W2** | **1** | **0** | **-2** | **1** | **-1** | **-1** | **1** | **1** |
| **W** | **1** | **0** | **-2** | **1** | **-1** | **-2** | **0** | **1** |
| **Y** | **-1** | **0** | **1** | **-2** | **0** | **2** | **0** | **10** |

**2.3 Deuxième itération**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | **X1** | **X2** | **X3** | **X4** | **X5** | **W1** | **W2** | **bi** |
| **X2** | **0** | **1** | **5/2** | **-1** | **1/2** | **1** | **-1/2** | **2** |
| **X1** | **1** | **0** | **-2** | **1** | **-1** | **-1** | **1** | **1** |
| **W** | **1** | **0** | **0** | **0** | **0** | **-1** | **-1** | **0** |
| **Y** | **0** | **0** | **-1** | **-1** | **-1** | **1** | **1** | **11** |

**3. Phase II**

**3.1 Table initiale**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **X1** | **X2** | **X3** | **X4** | **X5** | **bi** |
| **X2** | **0** | **1** | **5/2** | **-1** | **1/2** | **2** |
| **X1** | **1** | **0** | **-2** | **1** | **-1** | **1** |
| **Y** | **0** | **0** | **-1** | **-1** | **-1** | **11** |

X2\*=2, x1\*=1, x3\*=x4\*=x5\*=0, Y=-11=> Z=11.

**Exercice 3.**

Soit le programme linéaire suivant :

Min Z=3$x\_{1}$+$2x\_{2}$+$x\_{3}$
$$\left\{\begin{array}{c}x\_{1}+(4/5)x\_{3}\geq 4\\2x\_{1}+3x\_{2}+4x\_{3}\geq 12\\x\_{1},x\_{2},x\_{3}\geq 0\end{array}\right.$$

1. La résolution du programme par la méthode du deux phases
	1. **la forme standard**

Max Z’=-3$x\_{1}$-$2x\_{2}$-$x\_{3}$ (Z’=-Z)
$$\left\{\begin{array}{c}x\_{1}+(4/5)x\_{3}-x\_{4}=4\\2x\_{1}+3x\_{2}+4x\_{3}-x\_{5}=12\\x\_{1},x\_{2},x\_{3}\geq 0\\x\_{4},x\_{5}\geq 0\end{array}\right.$$

* 1. **la forme artificielle**

Max Z’=-3$x\_{1}$-$2x\_{2}$-$x\_{3}$ (Z’=-Z)

Max W=-$w\_{1}$-$w\_{2}$
$$\left\{\begin{array}{c}x\_{1}+(4/5)x\_{3}-x\_{4}+w\_{1}=4\\2x\_{1}+3x\_{2}+4x\_{3}-x\_{5}+w\_{2}=12\\x\_{1},x\_{2},x\_{3}\geq 0\\x\_{4},x\_{5}\geq 0\end{array}\right.$$

* 1. **la phase 1**

le tableau initial

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | X1 | X2 | X3 | X4 | X5 | W1 | W2 | sm |
| W1 | 1 | 0 | 4/5 | -1 | 0 | 1 | 0 | 4 |
| W2 | 2 | 3 | 4 | 0 | -1 | 0 | 1 | 12 |
| W | 3 | 3 | 24/5 | -1 | -1 | 0 | 0 | 16 |
| Z’ | -3 | -2 | -1 | 0 | 0 | 0 | 0 | 0 |

Itération 1

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | X1 | X2 | X3 | X4 | X5 | W1 | W2 | sm |
| W1 | 3/5 | -3/5 | 0 | -1 | 1/5 | 1 | -1/5 | 8/5 |
| X3 | 1/2 | 3/4 | 1 | 0 | -1/4 | 0 | 1/4 | 3 |
| W | 3/5 | -3/5 | 0 | -1 | 1/5 | 0 | -6/5 | 8/5 |
| Z’ | -5/2 | -5/4 | 0 | 0 | -1/4 | 0 | 1/4 | 3 |

Itération 2

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | X1 | X2 | X3 | X4 | X5 | W1 | W2 | sm |
| x1 | 1 | -1 | 0 | -5/3 | 1/3 | -5/3 | -1/3 | 8/3 |
| X2 | 0 | 5/4 | 1 | 5/6 | -5/12 | -5/6 | 5/12 | 5/3 |
| W | 0 | 0 | 0 | 0 | 0 | -1 | -1 | 0 |
| Z’ | 0 | -15/4 | 0 | -25/6 | 7/12 | 25/6 | -7/12 | 29/3 |

W atteint la valeur nulle et toutes les variables artificielle ont été exclus de la base donc la phase 1 est termine.

**Phase 2.**

Tableau initial

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | X1 | X2 | X3 | X4 | X5 | sm |
| x1 | 1 | -1 | 0 | -5/3 | 1/3 | 8/3 |
| X2 | 0 | 5/4 | 1 | 5/6 | -5/12 | 5/3 |
| Z’ | 0 | -15/4 | 0 | -25/6 | 7/12 | 29/3 |

Itération 1

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | X1 | X2 | X3 | X4 | X5 | sm |
| X5 | 3 | -3 | 0 | -5 | 1 | 8 |
| X3 | 5/4 | 0 | 1 | -5/4 | 0 | 5 |
| Z’ | -7/4 | -2 | 0 | -5/4 | 0 | 5 |

Tous les coefficients sont négatifs ou nuls donc la solution optimale est :

X5\*=8,x3\*=5, x1\*=x2\*=x4\*=0, Z\*=Z’=5.

**Exercice 4.**

Max Z=12$x\_{1}$+15$x\_{2}$+10 $x\_{3}$
$$\left\{\begin{array}{c}x\_{1}+x\_{2}+2x\_{3}\geq 10\\15x\_{1}+6x\_{2}-5x\_{3}\leq 30\\x\_{1}+3x\_{2}+5x\_{3}\leq 18\\x\_{1},x\_{2},x\_{3}\geq 0\end{array}\right.$$

La méthode big-M :

La M-forme :

Max Z=12$x\_{1}$+15$x\_{2}$+10 $x\_{3}$-m$x\_{7}$
$\left\{\begin{array}{c}x\_{1}+x\_{2}+2x\_{3}-x\_{4}+x\_{7}=10\\15x\_{1}+6x\_{2}-5x\_{3}+x\_{5}=30\\x\_{1}+3x\_{2}+5x\_{3}+x\_{6}=18\\x\_{1},x\_{2},x\_{3},x\_{4},x\_{5},x\_{6},x\_{7}\geq 0\end{array}\right.$

**Table initiale**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| vb | x1 | x2 | X3$\downright $ | x4 | X5 | X6 | X7 | b |
| X7 | 1 | 1 | 2 | -1 | 0 | 0 | 1 | 10 |
| X5 | 15 | 6 | -5 | 0 | 1 | 0 | 0 | 30 |
| <-X6 | 1 | 3 | 5 | 0 | 0 | 1 | 0 | 18 |
| Z | 12+m | 15+m | 10+2m | -m | 0 | 0 | 0 | 10m |

**Première itération**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| vb | x1$\downright $ | x2 | x3 | x4 | X5 | X6 | X7 | b |
| X7 | 3/5 | -1/5 | 0 | -1 | 0 | -2/5 | 1 | 14/5 |
| <-X5 | 16 | 9 | 0 | 0 | 1 | 1 | 0 | 48 |
| X3 | 1/5 | 3/5 | 1 | 0 | 0 | 1/5 | 0 | 18/5 |
| Z | 10+3m/5 | 9-m/5 | 0 | -m | 0 | -2-2m/5 | 0 | -36+14m/5 |

**Deuxième itération**

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| vb | x1$\downright $ | x2 | x3 | x4 | X5 | X6 | X7 | b |
| X7 | 0 | -19/15 | 0 | -1 | -3/80 | -7/16 | 1 | 1 |
| X1 | 1 | 9/16 | 0 | 0 | 1/16 | 1/16 | 0 | 3 |
| X3 | 0 | 39/80 | 1 | 0 | -1/80 | 3/16 | 0 | 3 |
| Z | 0 | 54/16-43m/80 | 0 | -m | -10/16-3m/80 | -21/8-7m/6 | 0 | -66+m |

Tous les coefficients dans la ligne de la fonction objective sont négatifs ou nuls alors qu’on n’arrive pas à exclure la variable artificielle x7 de la base donc, le problème n’admet pas de solution réalisable.

**Exercice 5.**

La m-forme

Max Z=-2$x\_{1}$-$x\_{2}$-4 $x\_{3}-mx\_{4}-mx\_{5}$
$$\left\{\begin{array}{c}x\_{1}+x\_{2}+2x\_{3}+x\_{4}=3\\2x\_{1}+x\_{2}+3x\_{3}+x\_{5}=5\\x\_{1},x\_{2},x\_{3}, x\_{4},x\_{5}\geq 0\end{array}\right.$$

Tableau initial

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| vb | x1 | x2 | x3↓ | x4 | X5 | b |
| ←X4 | 1 | 1 | 2 | 1 | 0 | 3 |
| X5 | 2 | 1 | 3 | 0 | 1 | 5 |
| Z | 3m-2 | 2m-1 | 5m-4 | 0 | 0 | 8m |

1ere itération

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| vb | x1↓ | x2 | x3↓ | x4 | X5 | b |
| X3 | ½ | ½ | 1 | 1/2 | 0 | 3/2 |
| ←X5 | ½ | -1/2 | 0 | -3/2 | 1 | 1/2 |
| Z | m/2 | -m/2+1 | 0 | -5m/2+2 | 0 | m/2+6 |

2eme itération

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| vb | x1 | x2↓ | x3↓ | x4 | X5 | b |
| ←X3 | 0 | 1 | 1 | 2 | -1 | 1 |
| X1 | 1 | -1 | 0 | -3 | 2 | 1 |
| Z | 0 | 1 | 0 | -m+2 | -m | 6 |

3eme itération

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| vb | x1 | x2 | x3↓ | x4 | X5 | b |
| X2 | 0 | 1 | 1 | 2 | -1 | 1 |
| X1 | 1 | 0 | 1 | -1 | 1 | 2 |
| Z | 0 | 0 | -1 | -m | -m+1 | 5 |

Tous les coefficients dans la ligne de la fonction objective sont négatifs ou nuls donc la solution optimale est :

X1\*=2, x2\*=1, x3\*=0, Z\*=5.