Introduction

Fluid dynamics is the science that studies the behavior of Fluids in motion. The fluids studied are considered to be perfect and incompressible (viscosity effects will not be taken into account) $\mu = 0$ et ρ = cte)

V.2 Bernoulli's theorem and its applications

The Bernoulli equation is a fundamental expression in fluid mechanics that expresses the principle of conservation of energy for an incompressible and inviscid fluid flowing in a steady state along a streamline.

V.2.1 Establishment of the Bernoulli equation

In a steady flow of a perfect and incompressible fluid in the gravitational field, we consider an infinitely narrow flow tube so that the quantities of pressure, velocity, density and altitude are constant in the same cross section. Furthermore, we assume that the flow is directed in a single direction.

Hypotheses

- The fluid is perfect (i.e., non-viscous) and incompressible.
- The flow is stationary.
- The volume density of the external forces is derived from a potential: $\vec{F}_V = -\overrightarrow{grad}(U)$

V.2.1.1 Établissement de l'équation de Bernoulli (Sans échange de travail)

The system studied is the fluid between sections SA and SB of the flow tube that moves in the direction from A to B, between which there is no hydraulic machine (pump, turbine, etc.)

During an infinitely small time interval dt, the mass of fluid flowing through section S_A is: : $dm_A = \rho S_A v_A\, dt$. Similarly; the mass of fluid flowing through section S_B is: : $dm_B = \rho S_B v_B\, dt$.

By conservation of mass is $dm_A = dm_B = dm$

From the energy point of view everything happens as if, during the time dt, the mass dm having passed from position A to position B.

By applying the theorem of kinetic energy to the fluid between times t and t+dt (the variation of kinetic energy is equal to the sum of the work of the external forces applied to the system.

$$\Delta E_c = \sum W(\vec{F}_{ext})$$

The change in kinetic energy is as follows:

$$\Delta E_c = \frac{1}{2} dm(v_B^2 - v_A^2)$$

The external forces exerted on the fluid are:

- Surface force: pressure force on the side walls, pressure force on face A and pressure force on face B of the flow tube.
- Volume force: force of the weight
- The work of the weight is:

$$W(P) = -dmg(z_B - z_A)$$

- The work of the pressure force on face A is :

$$W(\vec{F}_A) = P_A S_A v_A dt$$

- The work of the pressure force on face B is:

$$W(\vec{F}_B) = -\,P_B S_B v_B dt$$

The negative sign indicates that the pressure force is exerted in the opposite direction of the displacement.

- The work of the pressure forces on the side walls is:

$$W(\overrightarrow{F_{paroı\ latérales}}) = 0$$

The work is zero because the pressure forces are perpendicular to the displacement.

The kinetic energy theorem is then written:

$$\frac{1}{2} dm(v_B^2 - v_A^2) = -dmg(z_B - z_A) + P_A S_A v_A dt - P_B S_B v_B dt$$

The conservation of mass is written as:

$$dm = \rho\, S_A v_A dt = \rho\, S_B v_B dt$$

Where we get from :

$$\frac{1}{2} dm(v_B^2 - v_A^2) = -dmg(z_B - z_A) + \frac{dm}{\rho}(P_A - P_B)$$

After simply by dm, we arrive at the Bernoulli equation:

$$\frac{P_B}{\rho} + \frac{1}{2} v_B^2 + g z_B = \frac{P_A}{\rho} + \frac{1}{2} v_A^2 + g z_A$$

Or again:

$\frac{P}{\rho}+\frac{1}{2}v^2+gz=cste$ *(*)……General form of the Bernoulli equation*

- Bernoulli's equation therefore reflects the conservation of energy along a streamline.

V.2.1.2 Interpretation of the Bernoulli equation

- Writing in terms of energy

$\frac{P}{\rho}+\frac{1}{2}v^2+gz=cste$ **;** All terms are expressed in (J/Kg) or (m^2/s^2)

$\frac{P}{\rho}$ **:** Pressure potential energy per unit mass.

$\frac{1}{2}v^2$: Kinetic energy per unit mass.

gz: Potential energy per unit mass.

- Bernoulli's theorem shows that the total energy then remains constant along a streamline (there is no loss of energy).

- Writing in terms of pressure

Multiplying the equation * by ρ, we obtain :

$P+\frac{1}{2}\rho v^2+\rho gz=cste$ All terms are expressed in Pascal

P **:** is the static pressure (or local pressure).

$\frac{1}{2}\rho v^2$: is the dynamic pressure (or kinetic pressure).

ρgz: is the gravity pressure (or gauge pressure).

$P+\frac{1}{2}\rho v^2+\rho gz$ **is the total pressure.**

- This form of the Bernoulli equation expresses the conservation of total pressure along a streamline (there is no pressure loss).

Writing in terms of height

Dividing the equation * by ρ g, we obtain :

$\frac{P}{\rho g}+\frac{1}{2g}v^2+z=cste$; all terms are expressed in (m)

$\frac{P}{\rho g}$: height due to pressure or pressure load

$\frac{1}{2g}v^2$: height due to speed

z : piezometric height.

$\frac{P}{\rho g}+\frac{1}{2g}v^2+z=H$ **: total load or total manometric height**

- Bernoulli's equation shows that the total head remains constant along a streamline (there is no head loss in the flow of a perfect fluid).

Equation de Bernoulli (avec échange de travail)

We take **Figure 5** the fluid passes through a hydraulic machine when it passes (during its movement) from **(A)** to **(B)**. This machine is characterized by an absorbed (or delivered) power $\mathbf{P_{absorbed}}$ **or** $\mathbf{P_{delivered}}$, a power exchanged with the fluid **($\mathbf{P_{machine,\,fluid}}$)** and an efficiency η.

- We assume algebraically $\mathbf{P_{machine,fluid}}$, positive **($\mathbf{P_{(machine\,,fluid)}>0}$)**, if the fluid receives energy from the machine **(pump, fan)**, and the efficiency is given by

$$\boldsymbol{\eta} = \frac{\mathbf{P_{machine\,,fluide}}}{\mathbf{P_{absorbed}}}$$

- We assume algebraically $\mathbf{P_{machine,fluid}}$ negative **($\mathbf{P_{machine,fluid}<0}$)**, if the fluid provides energy to the machine (turbine) and the efficiency $\boldsymbol{\eta}$ is given by:

$$\eta = \frac{\mathbf{P_{délivered}}}{\mathrm{P_{machine\,,fluid}}}$$

The kinetic energy theorem in this case is written:

$$\Delta E_C = W\left(\overrightarrow{F_{weight}}\right) + W\left(\overrightarrow{F_{pressur}}\right) + W\left(\overrightarrow{F_{machine}}\right)$$

$$W\left(\overrightarrow{F_{machin}}\right) = \mathrm{P_{machin\,,fluid}}\ \ dt$$

$$\frac{1}{2}dm(v_B^2 - v_A^2) = -dmg(z_B - z_A) + P_A S_A v_A dt - P_B S_B v_B dt + \mathrm{P_{machine\,,fluid}}\ \ dt$$

Bernoulli's equation therefore becomes:

$$\frac{P_A}{\rho} + \frac{1}{2}v_A^2 + gz_A + \frac{\mathrm{P_{machin\,,fluid}}}{\rho Q_v} = \frac{P_B}{\rho} + \frac{1}{2}v_B^2 + gz_B$$

Where :

$$\frac{P_A}{\rho g} + \frac{1}{2g}v_A^2 + z_A + \frac{\mathrm{P_{machin\,,fluid}}}{\rho g Q_v} = \frac{P_B}{\rho g} + \frac{1}{2g}v_B^2 + z_B$$

with :

$\boldsymbol{Q_v}$: Volume flow in m^3/s

$\mathbf{P_{machin\,,fluid}}$: Power exchanged with the fluid in Watt.

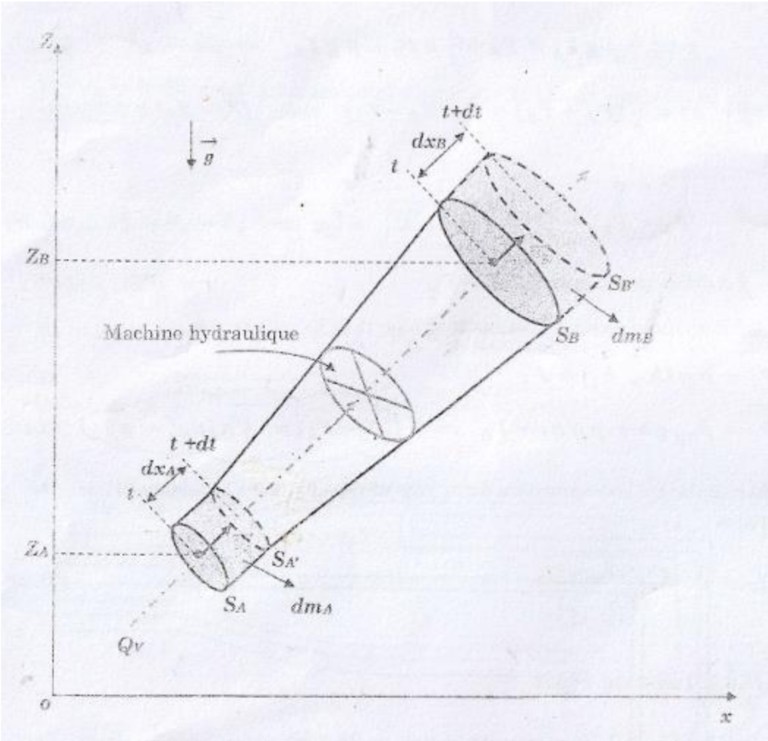

Figure 5 : Energy conservation principle for one-dimensional steady flow with work exchange

Application to flow and speed measurements

Orifice flow –Torricelli formula-

- We consider a large reservoir with a free surface S_A filled with an incompressible and perfect fluid in which a narrow orifice S_B is drilled. The flow is considered to be in steady state.

  Calculer la vitesse découlement V_B à la sortie de l'orifice et le débit de l'écoulement Q_v

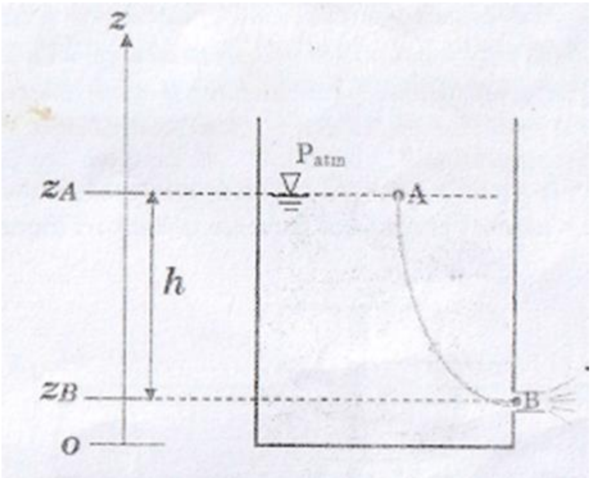

Figure: Flow of a liquid through an orifice

Solve the exercise in the lesson

Venturi tube

It is a behavior consisting of a convergent followed by a divergent. It is equipped with both ports to measure local pressures using a liquid manometer. This pipe is used to measure the flow rate. The Venturi is an ideal device for measuring flow, but it is expensive.

We seek to express the flow rate Qv of a permanent flow of an incompressible fluid (ρ=cste) and perfect which crosses the Venturi.

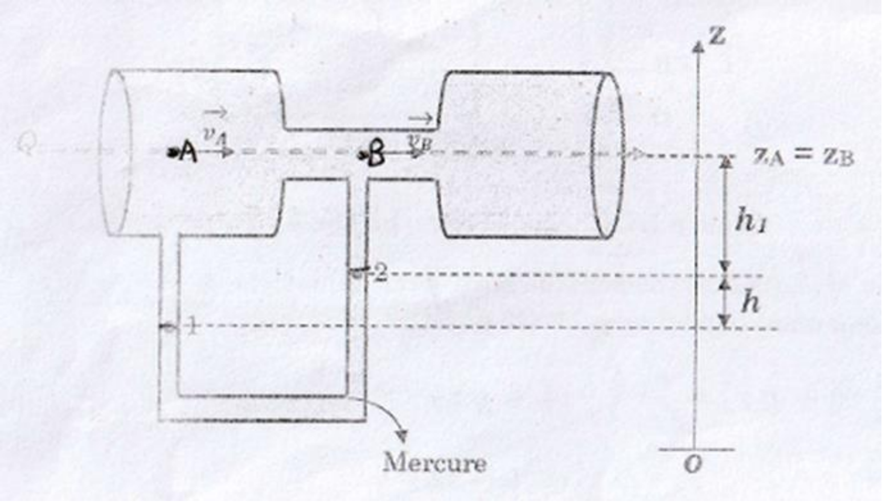

Figure: Venturi tube

Solve the exercise in the lesson