Chapter 4 : Diffusion phenomena

1-Introduction

Diffusion is known as a means of exchanging solute molecules from one compartment to another across a membrane. It tends to uniform the distribution of particles (ions or non-dissociable molecules), occurring in the direction of decreasing concentrations, meaning the particles move from an area of high concentration to an area of low concentration.

2-Definition

The diffusion phenomenon across a semi-permeable membrane, under the influence of a concentration gradient, involves the movement of solute from the more concentrated medium to the less concentrated medium. Diffusion is the movement of solute particles.

Theoretical Aspect of the Diffusion Phenomenon - Fick's Law

Fick expressed the law that governs the diffusion phenomenon with an equation that aligns well with the one governing the heat propagation phenomenon in a conductor. It is assumed that:

- Diffusion occurs in a single direction in space.
- The concentration of the diffusing solute is constant across the entire x-axis plane.

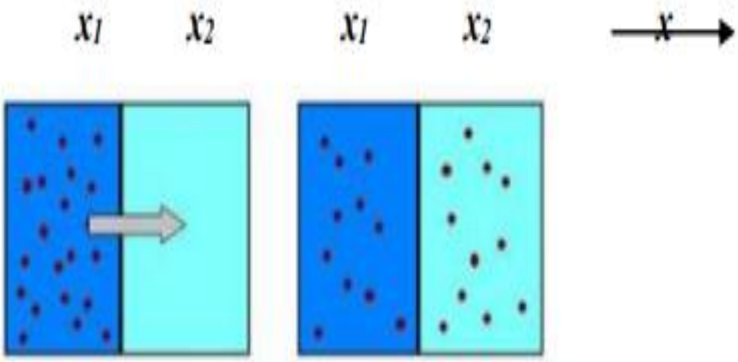

Fig: Diffusion phenomena.

Let Δm be the mass of the solute that diffuses over the time interval Δt from point x_1 to x_2 in the compartment

$$\Delta m = -D \cdot \frac{C_2 - C_1}{x_2 - x_1} \cdot \Delta t \cdot S$$

Thus

$$\Delta m = -D \cdot S \cdot \frac{\Delta C}{\Delta x} \cdot \Delta t \quad \Rightarrow \quad \frac{\Delta m}{\Delta t} = -D \ \frac{\Delta C}{\Delta x} \ S$$

Where:

$\frac{\Delta C}{\Delta x}$: Negative concentration gradient ($\Delta C<0$, $\Delta x>0$)

S: Cross-sectional area of the tube

D: Diffusion coefficient

Units:

Δm: kg; g;

ΔC: kg/m^3; g/cm^3; mole/m^3; mole/cm^3

Δx: m; cm

Δt: s

D: m^2/s; cm^2/s

$\boldsymbol{\frac{\Delta m}{\Delta t}}$:Diffusion flux or the mass of solute that has moved during the time Δt from point

X_A with weight concentration C_A to point X_B with weight concentration C_B

It is also possible to express the number of moles diffu sing (instead of the mass of molecules) per unit of time:

$$\frac{\Delta n}{\Delta t} = -D \ S \frac{\Delta c^M}{\Delta x}$$

$\frac{\Delta n}{\Delta t}$: Molar Flow Rate in moles/s.

The diffusion coefficient D depends on the experimental conditions of diffusion, such as temperature, the nature of the solute, and the nature of the solvent.

$$D = \frac{k_B\, T}{f}$$

Where

k_B is the Boltzmann constant (1.38×10^{-23}J/K)

T is the temperature in Kelvin (K),

f is the friction coefficient, which is defined as follows: if the particle moves with velocity v in a liquid medium, it must overcome a frictional force F that is proportional to v:

F= f v

In the particular case where the particle is spherical with radius r, it can be shown (Stokes' law) that:

$$F = 6\,\pi\,\eta\,r\,v$$

Thus, the friction coefficient is $\boldsymbol{f=6\pi\eta r}$

The diffusion coefficient D is proportional to the temperature T. Therefore, if the temperature and diffusion coefficient increase, the diffusion flux $\frac{\Delta m}{\Delta t}$ also increases.

Consider two solutes with radii r_1 and r_2 diffusing in two media with viscosities η_1 and η_2, and two temperatures T_1 and T_2, respectively.

We have:

$$D_1 = \frac{k\,T_1}{6\,\pi\,\eta_1 r_1}\ , \quad D_2 = \frac{k\,T_2}{6\,\pi\,\eta_2 r_2}$$

$$\frac{D_2}{D_1} = \frac{T_2}{T_1}\,\frac{\eta_2}{\eta_1}\,\frac{r_1}{r_2}$$

$$\Longrightarrow \quad D_2 = \frac{T_2}{T_1}\,\frac{\eta_2}{\eta_1}\,\frac{r_1}{r_2}\,D_1$$

Consider the case of two solutes, 1 and 2, which are different but diffuse under the same experimental conditions (η_1=η_2, T_1=T_2), so.

$$D_2 = \frac{r_1}{r_2} D_1$$

Consider the case where the volumetric density of the solute molecules 1 is equal to that of the 2nd solute ($\rho_1 \approx \rho_2$).

We have $\boldsymbol{\rho} = \frac{m}{v}$ With V= 3/4πr ^{3} (V: volume of a spherical molecule)

$$M = N\,m \quad \Longrightarrow \quad \rho = \frac{m}{V} = \frac{m}{\frac{4}{3}\pi r^3} \quad \Longrightarrow \quad r = \sqrt[3]{\frac{M}{\frac{4}{3}\pi N \rho}}$$

We then obtain for the two solutes:

$$\frac{r_1}{r_2} = \sqrt[3]{\frac{M_1}{M_2}}$$

since $\rho_1 \approx \rho_2$.

Thus, for two solutes with molar masses M_1 and M_2 diffusing under the same experimental conditions, the diffusion coefficient D_2 is given by the following formula:

$$D_2 = \sqrt[3]{\frac{M_1}{M_2}} D_1$$