Rigid body Planar kinetics
Impulse and momentum

The objective of this lecture is revisit the principle of impulse and momentum and conservation of momentum. It can be applied to problems involving both linear and angular motion, and demonstrate how to apply them to solve rigid body planar kinetic problems involving force, moment, velocity and time. It can also be used to analyze the mechanics of impact.

الهدف من هذه المحاضرة هو إعادة النظر في مبدأ الدفع وكمية الحركة وحفظ كمية الحركة. يمكن تطبيقها على المسائل التي تتضمن الحركة الخطية والزاوية على حد سواء، وتوضيح كيفية تطبيقها لحل مسائل الحركية المستوية للجسم الصلب تتضمن القوة والعزم والسرعة والزمن. كما يمكن استخدامها لتحليل ميكانيكا التصادم.

Rigid body Planar kinetics
Impulse and momentum

- As we have seen before, when we move from particle systems to rigid body systems, we will need to not only deal with forces, positions, velocities, and accelerations, but we will also need to deal with **moments**, **orientations**, **angular velocities**, and **angular accelerations**.
- This remains the case with impulse-momentum methods, where we will add an angular counterpart to impulse as well as an angular counterpart to momentum

A dent in an automotive fender can be removed using an impulse tool, which delivers a force over a very short time interval.

How can we determine the magnitude of the linear impulse applied to the fender?

Could you analyze a carpenter's hammer striking a nail in the same fashion?

Sure! When a stake is struck by a sledgehammer, a large impulsive force is delivered to the stake and drives it into the ground.

If we know the initial speed of the sledgehammer and the duration of impact, how can we determine the magnitude of the impulsive force delivered to the stake?

Linear Impulse

- The **linear impulse** of a constant magnitude force will be equal to the magnitude of the force times the duration of the time. $\overrightarrow{L} = (\vec{F})$ (t).
- For non constant magnitudes, the linear impulse of any force is equal to the integral of the force function over magnitude of the force times the duration of the set time period . $\overrightarrow{L} = \int \vec{F}\,(t)\,dt$.
- These are vector quantities, and the direction of the force will be the direction of the impulse.

Angular Impulse

- The **angular impulse** of a constant magnitude moment will be equal to the magnitude of the moment times the duration of the time.

$$\overrightarrow{H} = (\overrightarrow{M})\,(t).$$

- For non constant magnitudes, the angular impulse of any moment is equal to the integral of the moment function over magnitude of the force times the duration of the set time period .

$$\overrightarrow{H} = \int \overrightarrow{M}(t)dt\,.$$

- These are vector quantities, and the direction of the moment will be the direction of the impulse vector.

Linear Momentum

- The linear momentum of a body will be the mass of the body times the velocity of that body.

$$m * \vec{v}$$

- Unlike an impulse which needs to occur over time, the momentum is instantaneous.
- The momentum has a direction as well. The direction of the momentum wil be the instantaneous direction of the velocity at that point.

Angular Momentum

- The **Angular momentum** of a body at any given instant will be the mass moment of inertia of the body times the angular velocity of that body.

$$I * \overrightarrow{\omega}$$

- Just as linear momentum, angular momentum has a direction as well. The direction of the angular momentum vector will be simply be the direction of the angular velocity vector.
- We can take the angular momentum about any point, thought we will usually take the angular momentum about one of four points.
- The center of mass of the body.
- A fixed axis of rotation.
- The instantaneous center of zero velocity.
- The point of impact in a collision.

Angular Momentum about a Fixed Axis or the Instant Center .

To find the angular momentum of a body rotating about a permanently fixed axis, or about instant center .

$$I_O * \overrightarrow{\omega} \text{ or } I_{IC} * \overrightarrow{\omega}$$

Use the parallel axis theorem for this adjustment

$$I_O = I_G + m\,r^2 \text{ or } I_{IC} = I_G + mr^2$$

Angular Momentum about any other point

- If we want to take the angular momentum about any other point. We will include both **rotational** terms and **translational** terms.
- Momentum due to rotation will be.

$$\overrightarrow{H_G} = I_G * \overrightarrow{\omega}$$

A similar expression to **1** can be derived for the angular momentum if we start from the principle of conservation of angular momentum, $\dot{\boldsymbol{H}}_{\boldsymbol{G}} = \boldsymbol{M}_G$. Here, $\boldsymbol{H}_G$ = $\boldsymbol{I_G}$ $\boldsymbol{\omega}$ is the angular momentum about the center of mass, and $\boldsymbol{M_G}$ is the moment of all externally applied forces about the center of mass. Integrating between times $\boldsymbol{t_1}$ and $\boldsymbol{t_2}$, we have,

$$(\boldsymbol{H_G})_2 - (\boldsymbol{H_G})_1 = \int_{t1}^{t2} \boldsymbol{M}_G \, dt$$

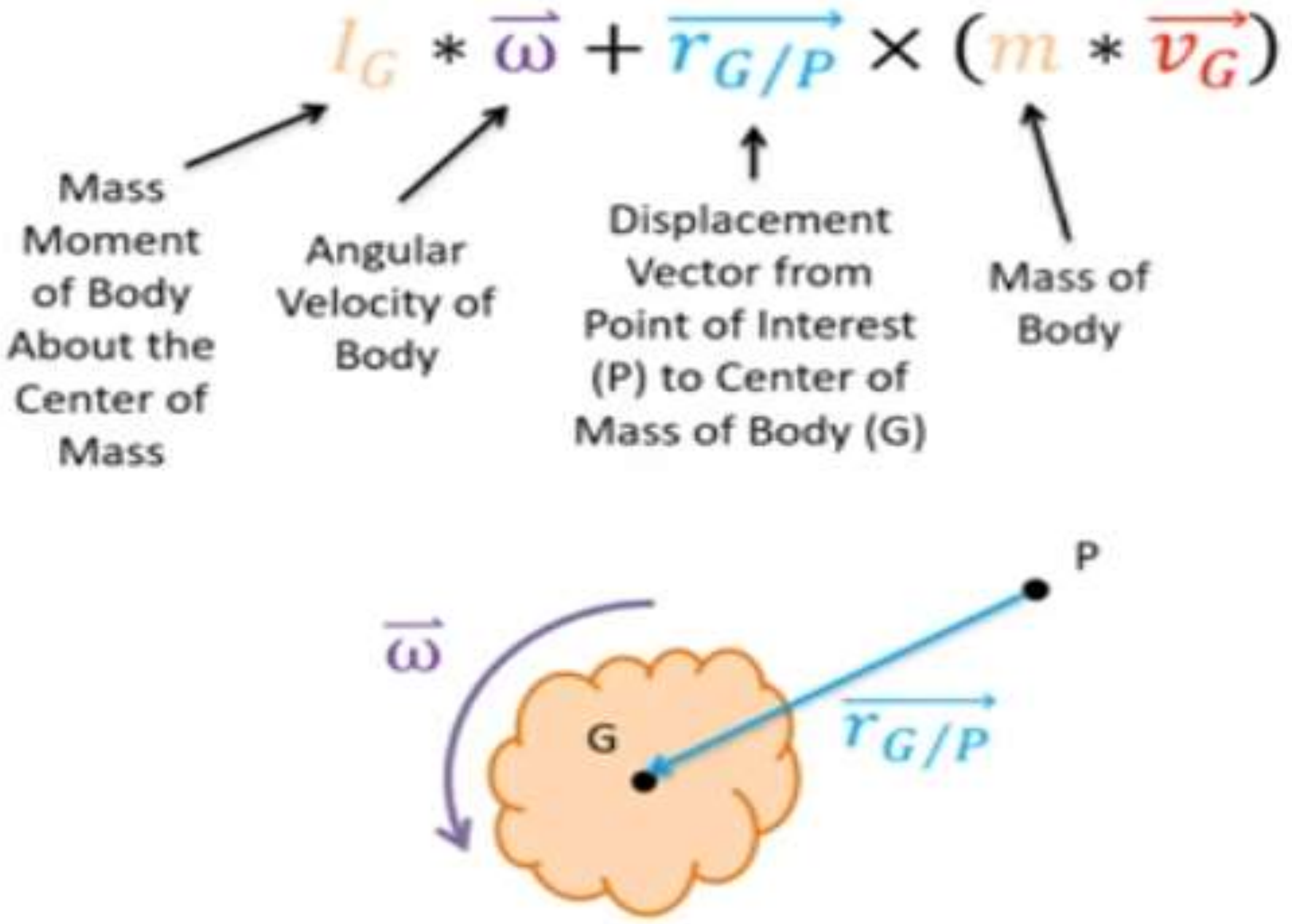
- Angular Momentum due to translation will be.

$$\overrightarrow{r_{G/P}} \times (m * \overrightarrow{v_G})$$

- Putting in all together we get..

$$I_G * \overrightarrow{\omega} + \overrightarrow{r_{G/P}} \times (m * \overrightarrow{v_G})$$

Angular Momentum about any other point



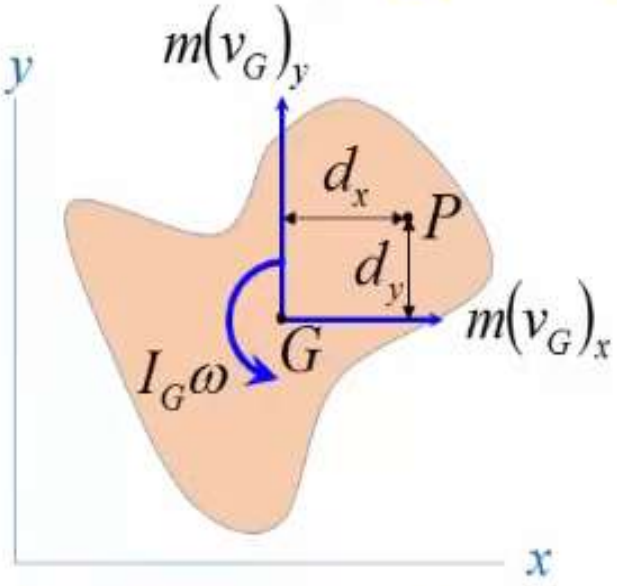
In a similar manner, for rotation about a fixed point O, we can write,

$$(\boldsymbol{H_O})_2 - (\boldsymbol{H_O})_1 = \int_{t1}^{t2} \boldsymbol{M}_O \, dt$$

where $\boldsymbol{M}_O = \boldsymbol{I}_O \, \omega$ the moment of inertia ,$\boldsymbol{I}_{O,}$ refers to the fixed point O, and the external moments are with respect to point O.

Finally, if the external applied moment is zero, then we have conservation of angular momentum, which implies $\omega_2 = \omega_1$

For **rigid body** planar motion:

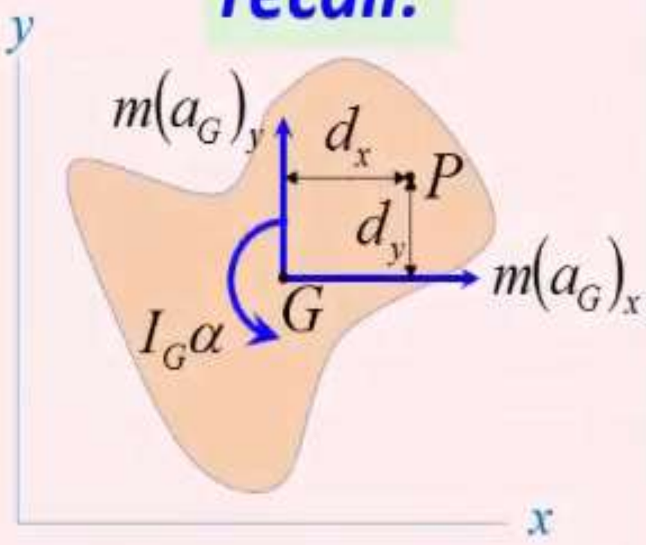


Angular momentum

about point *P*:

$$H_P = m(v_G)_x \cdot d_y - m(v_G)_y \cdot d_x + I_G\omega$$

recall:



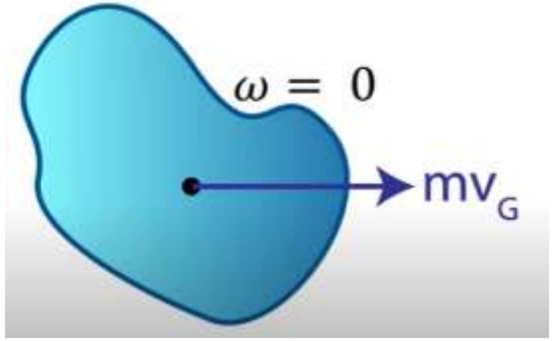
$$\sum (\mathcal{M}_k)_P = m(a_G)_x \cdot d_y - m(a_G)_y \cdot d_x + I_G\alpha$$

kinetic moment about point P

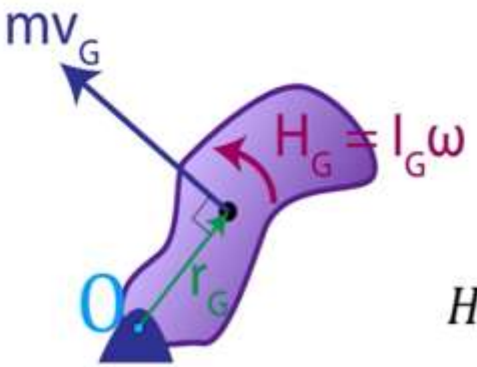
Momentum

A–Translation

$$L = mv_G$$
$$H_G = 0$$

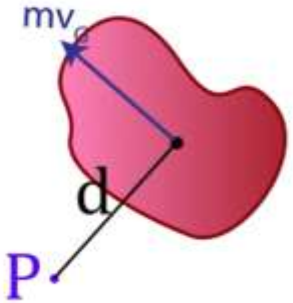


B–Rotation about a fixed axis



$$L = mv_G$$
$$H_G = I_G\omega$$
$$H_O = I_G\omega + r_G(mv_G)$$

C– General Plane Motion



$$L = mv_G$$
$$H_G = I_G\omega$$
$$H_P = I_G\omega + d(mv_G)$$

Translation:

$$H_G = 0$$

Rotation about a fixed axis:

$$H_O = I_O\omega$$

Try to derive this yourself.

center of rotation

General plane motion:

$$H_{IC} = I_{IC}\omega$$

Try to derive this yourself.

instantaneous center of zero velocity

Conservation of momentum

No (or negligible) net **external linear** impulse:

Linear momentum: $$\sum m(\mathbf{v}_G)_1 = \sum m(\mathbf{v}_G)_2$$

No (or negligible) net **external angular** impulse about point *P*:

Angular momentum: $$\sum (H_P)_1 = \sum (H_P)_2$$

Principle of impulse and momentum

$$m(v_{Gx})_1 + \sum \int_{t_1}^{t_2} F_x \, dt = m(v_{Gx})_2$$

$$m(v_{Gy})_1 + \sum \int_{t_1}^{t_2} F_y \, dt = m(v_{Gy})_2$$

$$I_G \omega_1 + \sum \int_{t_1}^{t_2} M_G \, dt = I_G \omega_2$$

Initial Linear Momentum + Linear Impulses = Final Linear Momentum

$$\underbrace{m(v_{Gx})_1}_{\text{Initial Linear Momentum}} + \underbrace{\sum \int_{t_1}^{t_2} F_x \, dt}_{\text{Linear Impulses}} = \underbrace{m(v_{Gx})_2}_{\text{Final Linear Momentum}}$$

$$\underbrace{m(v_{Gy})_1}_{\text{Initial Linear Momentum}} + \underbrace{\sum \int_{t_1}^{t_2} F_y \, dt}_{\text{Linear Impulses}} = \underbrace{m(v_{Gy})_2}_{\text{Final Linear Momentum}}$$

Initial Angular Momentum + Angular Impulses = Final Angular Momentum

$$\underbrace{I_G \omega_1}_{\text{Initial Angular Momentum}} + \underbrace{\sum \int_{t_1}^{t_2} M_G \, dt}_{\text{Angular Impulses}} = \underbrace{I_G \omega_2}_{\text{Final Angular Momentum}}$$

For **rigid body** planar motion:

Principle of linear impulse and momentum

$$\mathbf{L}_1 + \sum \mathbf{I}_{1-2} = \mathbf{L}_2$$

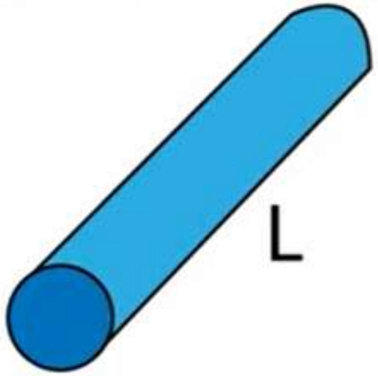
Linear impulse: $\mathbf{I} = \int \mathbf{F} dt$

Principle of angular impulse and momentum

$$(H_P)_1 + \sum \int_{t_1}^{t_2} M_P dt = (H_P)_2$$

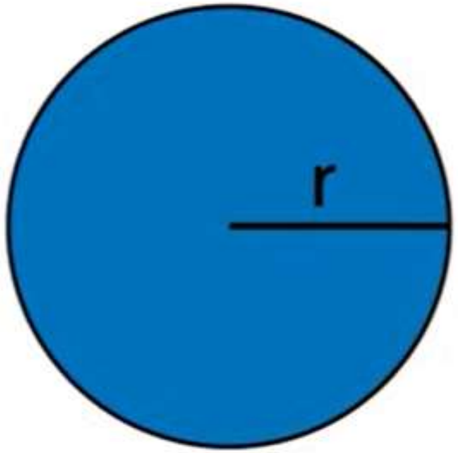
Mass Moment of Inertia

About Center of Uniform Rod



$$I = \frac{1}{12} m(L^2)$$

Uniform Disk



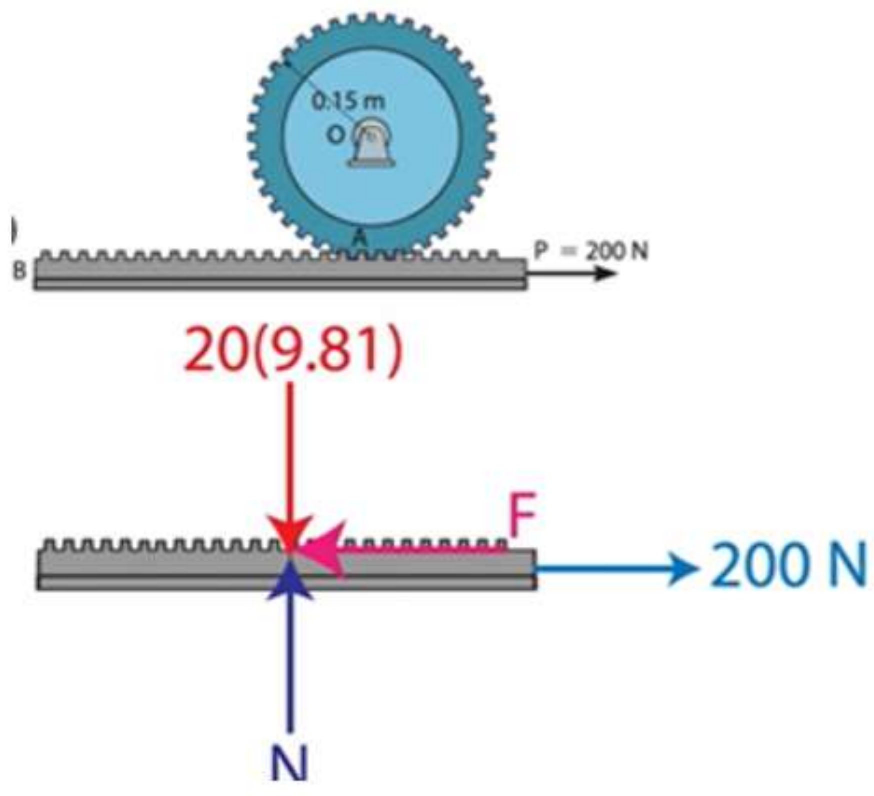
$$I = \frac{1}{2} m(r^2)$$

Radius of Gyration

$$I = mk^2$$

Example 8

The ***30 kg*** gear ***A*** has a radius of gyration about its center of mass ***O*** of ***k = 125 mm***. If the ***20 kg*** gear rack ***B*** is subjected to a force of ***P = 200 N***. determine the time required for the gear to obtain an angular velocity of ***20*** rad/s, starting from rest. The contact surface between the gear rack and the horizontal plane is smooth.



$$\xrightarrow{+} \quad m(v_B)_1 + \sum \int_{t_1}^{t_2} F_x\, dt = m(v_B)_2$$

$$0 + 200(t) - F(t) = 20(3)$$

$$F(t) = 200t - 60$$

$$I_O\omega_1 + \sum \int_{t_1}^{t_2} M_O\, dt = I_O\omega_2$$

$$0 + F(0.15)(t) = \underline{(0.46875)(20)}$$

$$I_O = mk^2$$

$$I_O = (30)(0.125^2)$$

$$I_O = 0.46875\ \mathrm{kg \cdot m^2}$$

$$F(t) = 200t - 60$$

$$0 + F(0.15)(t) = (0.46875)(20)$$

$$F = 102.04\ \mathrm{N}$$

$$\boxed{t = 0.6125\ \mathrm{s}}$$